

Computer algebra independent integration tests

4-Trig-functions/4.3-Tangent/4.3.1.3-d-sin^m-a+b-tanⁿ

Nasser M. Abbasi

May 22, 2020

Compiled on May 22, 2020 at 9:19pm

Contents

1	Introduction	3
1.1	Listing of CAS systems tested	3
1.2	Results	3
1.3	Performance	5
1.4	list of integrals that has no closed form antiderivative	6
1.5	list of integrals solved by CAS but has no known antiderivative	6
1.6	list of integrals solved by CAS but failed verification	6
1.7	Timing	7
1.8	Verification	7
1.9	Important notes about some of the results	7
1.10	Design of the test system	8
2	detailed summary tables of results	11
2.1	List of integrals sorted by grade for each CAS	11
2.2	Detailed conclusion table per each integral for all CAS systems	12
2.3	Detailed conclusion table specific for Rubi results	25
3	Listing of integrals	29
3.1	$\int \frac{\sin^4(x)}{i+\tan(x)} dx$	29
3.2	$\int \frac{\sin^3(x)}{i+\tan(x)} dx$	32
3.3	$\int \frac{\sin^2(x)}{i+\tan(x)} dx$	35
3.4	$\int \frac{\sin(x)}{i+\tan(x)} dx$	38
3.5	$\int \frac{\csc(x)}{i+\tan(x)} dx$	41
3.6	$\int \frac{\csc^2(x)}{i+\tan(x)} dx$	44
3.7	$\int \frac{\csc^3(x)}{i+\tan(x)} dx$	47
3.8	$\int \frac{\csc^4(x)}{i+\tan(x)} dx$	50
3.9	$\int \frac{\csc^5(x)}{i+\tan(x)} dx$	53
3.10	$\int \frac{\csc^6(x)}{i+\tan(x)} dx$	57
3.11	$\int \sin^5(c+dx)(a+b\tan(c+dx)) dx$	60
3.12	$\int \sin^4(c+dx)(a+b\tan(c+dx)) dx$	63
3.13	$\int \sin^3(c+dx)(a+b\tan(c+dx)) dx$	67
3.14	$\int \sin^2(c+dx)(a+b\tan(c+dx)) dx$	73

3.15	$\int \sin(c + dx)(a + b \tan(c + dx)) dx$	76
3.16	$\int \csc(c + dx)(a + b \tan(c + dx)) dx$	80
3.17	$\int \csc^2(c + dx)(a + b \tan(c + dx)) dx$	83
3.18	$\int \csc^3(c + dx)(a + b \tan(c + dx)) dx$	85
3.19	$\int \csc^4(c + dx)(a + b \tan(c + dx)) dx$	88
3.20	$\int \csc^5(c + dx)(a + b \tan(c + dx)) dx$	91
3.21	$\int \csc^6(c + dx)(a + b \tan(c + dx)) dx$	95
3.22	$\int \sin^4(c + dx)(a + b \tan(c + dx))^2 dx$	98
3.23	$\int \sin^3(c + dx)(a + b \tan(c + dx))^2 dx$	104
3.24	$\int \sin^2(c + dx)(a + b \tan(c + dx))^2 dx$	107
3.25	$\int \sin(c + dx)(a + b \tan(c + dx))^2 dx$	111
3.26	$\int \csc(c + dx)(a + b \tan(c + dx))^2 dx$	116
3.27	$\int \csc^2(c + dx)(a + b \tan(c + dx))^2 dx$	119
3.28	$\int \csc^3(c + dx)(a + b \tan(c + dx))^2 dx$	122
3.29	$\int \csc^4(c + dx)(a + b \tan(c + dx))^2 dx$	126
3.30	$\int \csc^5(c + dx)(a + b \tan(c + dx))^2 dx$	129
3.31	$\int \csc^6(c + dx)(a + b \tan(c + dx))^2 dx$	133
3.32	$\int \sin^3(c + dx)(a + b \tan(c + dx))^3 dx$	136
3.33	$\int \sin^2(c + dx)(a + b \tan(c + dx))^3 dx$	140
3.34	$\int \sin(c + dx)(a + b \tan(c + dx))^3 dx$	145
3.35	$\int \csc(c + dx)(a + b \tan(c + dx))^3 dx$	149
3.36	$\int \csc^2(c + dx)(a + b \tan(c + dx))^3 dx$	152
3.37	$\int \csc^3(c + dx)(a + b \tan(c + dx))^3 dx$	155
3.38	$\int \csc^4(c + dx)(a + b \tan(c + dx))^3 dx$	159
3.39	$\int \csc^5(c + dx)(a + b \tan(c + dx))^3 dx$	162
3.40	$\int \csc^6(c + dx)(a + b \tan(c + dx))^3 dx$	167
3.41	$\int \sin^3(c + dx)(a + b \tan(c + dx))^4 dx$	170
3.42	$\int \sin^2(c + dx)(a + b \tan(c + dx))^4 dx$	174
3.43	$\int \sin(c + dx)(a + b \tan(c + dx))^4 dx$	180
3.44	$\int \csc(c + dx)(a + b \tan(c + dx))^4 dx$	184
3.45	$\int \csc^2(c + dx)(a + b \tan(c + dx))^4 dx$	187
3.46	$\int \csc^3(c + dx)(a + b \tan(c + dx))^4 dx$	190
3.47	$\int \csc^4(c + dx)(a + b \tan(c + dx))^4 dx$	194
3.48	$\int \csc^5(c + dx)(a + b \tan(c + dx))^4 dx$	197
3.49	$\int \csc^6(c + dx)(a + b \tan(c + dx))^4 dx$	202
3.50	$\int \csc^7(c + dx)(a + b \tan(c + dx))^4 dx$	205
3.51	$\int \frac{\sin^5(c+dx)}{a+b \tan(c+dx)} dx$	210
3.52	$\int \frac{\sin^4(c+dx)}{a+b \tan(c+dx)} dx$	215
3.53	$\int \frac{\sin^3(c+dx)}{a+b \tan(c+dx)} dx$	219
3.54	$\int \frac{\sin^2(c+dx)}{a+b \tan(c+dx)} dx$	223
3.55	$\int \frac{\sin(c+dx)}{a+b \tan(c+dx)} dx$	227
3.56	$\int \frac{\csc(c+dx)}{a+b \tan(c+dx)} dx$	230
3.57	$\int \frac{\csc^2(c+dx)}{a+b \tan(c+dx)} dx$	233
3.58	$\int \frac{\csc^3(c+dx)}{a+b \tan(c+dx)} dx$	236
3.59	$\int \frac{\csc^4(c+dx)}{a+b \tan(c+dx)} dx$	240
3.60	$\int \frac{\csc^6(c+dx)}{a+b \tan(c+dx)} dx$	243
3.61	$\int \frac{\sin^6(c+dx)}{(a+b \tan(c+dx))^2} dx$	246
3.62	$\int \frac{\sin^4(c+dx)}{(a+b \tan(c+dx))^2} dx$	251
3.63	$\int \frac{\sin^2(c+dx)}{(a+b \tan(c+dx))^2} dx$	256

3.64	$\int \frac{\csc^2(c+dx)}{(a+b \tan(c+dx))^2} dx$	260
3.65	$\int \frac{\csc^4(c+dx)}{(a+b \tan(c+dx))^2} dx$	263
3.66	$\int \frac{\csc^6(c+dx)}{(a+b \tan(c+dx))^2} dx$	266
3.67	$\int \frac{\sin^6(c+dx)}{(a+b \tan(c+dx))^3} dx$	270
3.68	$\int \frac{\sin^4(c+dx)}{(a+b \tan(c+dx))^3} dx$	276
3.69	$\int \frac{\sin^2(c+dx)}{(a+b \tan(c+dx))^3} dx$	281
3.70	$\int \frac{\csc^2(c+dx)}{(a+b \tan(c+dx))^3} dx$	285
3.71	$\int \frac{\csc^4(c+dx)}{(a+b \tan(c+dx))^3} dx$	288
3.72	$\int \frac{\csc^6(c+dx)}{(a+b \tan(c+dx))^3} dx$	291
3.73	$\int \frac{\sin^4(c+dx)}{(a+b \tan(c+dx))^4} dx$	295
3.74	$\int \frac{\sin^2(c+dx)}{(a+b \tan(c+dx))^4} dx$	300
3.75	$\int \frac{\csc^2(c+dx)}{(a+b \tan(c+dx))^4} dx$	305
3.76	$\int \frac{\csc^4(c+dx)}{(a+b \tan(c+dx))^4} dx$	308
3.77	$\int \frac{\csc^6(c+dx)}{(a+b \tan(c+dx))^4} dx$	312
3.78	$\int \frac{\csc(x)}{1+\tan(x)} dx$	316
3.79	$\int \sin^m(c+dx)(a+b \tan(c+dx))^3 dx$	319
3.80	$\int \sin^m(c+dx)(a+b \tan(c+dx))^2 dx$	322
3.81	$\int \sin^m(c+dx)(a+b \tan(c+dx)) dx$	325
3.82	$\int \frac{\sin^m(c+dx)}{a+b \tan(c+dx)} dx$	328
3.83	$\int \sin^m(c+dx)(a+b \tan(c+dx))^n dx$	332
3.84	$\int \sin^4(c+dx)(a+b \tan(c+dx))^n dx$	334
3.85	$\int \sin^2(c+dx)(a+b \tan(c+dx))^n dx$	338
3.86	$\int \csc^2(c+dx)(a+b \tan(c+dx))^n dx$	341
3.87	$\int \csc^4(c+dx)(a+b \tan(c+dx))^n dx$	344
3.88	$\int \sin^3(c+dx)(a+b \tan(c+dx))^n dx$	347
3.89	$\int \sin(c+dx)(a+b \tan(c+dx))^n dx$	349
3.90	$\int \csc(c+dx)(a+b \tan(c+dx))^n dx$	351
3.91	$\int \csc^3(c+dx)(a+b \tan(c+dx))^n dx$	353

4 Listing of Grading functions

355

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [91]. This is test number [102].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric₂F₁ functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (91)	% 0. (0)
Mathematica	% 98.9 (90)	% 1.1 (1)
Maple	% 91.21 (83)	% 8.79 (8)
Maxima	% 81.32 (74)	% 18.68 (17)
Fricas	% 91.21 (83)	% 8.79 (8)
Sympy	% 5.49 (5)	% 94.51 (86)
Giac	% 83.52 (76)	% 16.48 (15)

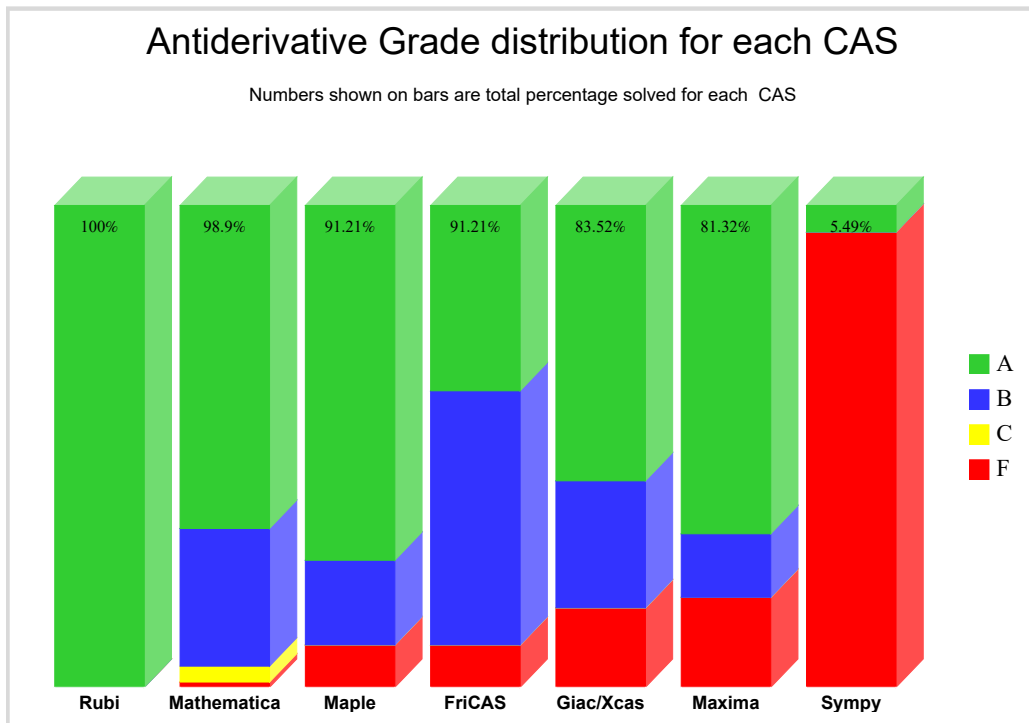
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

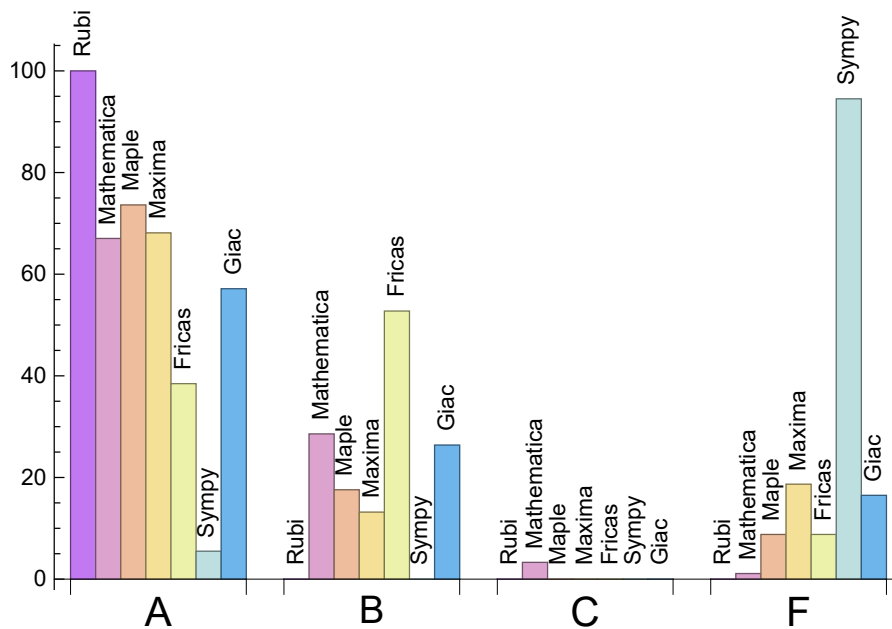
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	67.03	28.57	3.3	1.1
Maple	73.63	17.58	0.	8.79
Maxima	68.13	13.19	0.	18.68
Fricas	38.46	52.75	0.	8.79
Sympy	5.49	0.	0.	94.51
Giac	57.14	26.37	0.	16.48

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.32	136.4	0.95	109.	1.
Mathematica	2.34	277.16	1.88	162.	1.63
Maple	0.09	222.58	1.48	141.	1.32
Maxima	1.23	239.86	1.61	151.5	1.43
Fricas	2.35	672.3	5.02	444.	4.75
Sympy	0.26	27.2	0.69	31.	0.65
Giac	1.7	685.49	6.92	243.5	2.15

1.4 list of integrals that has no closed form antiderivative

{83, 88, 89, 90, 91}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {30, 39, 48}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

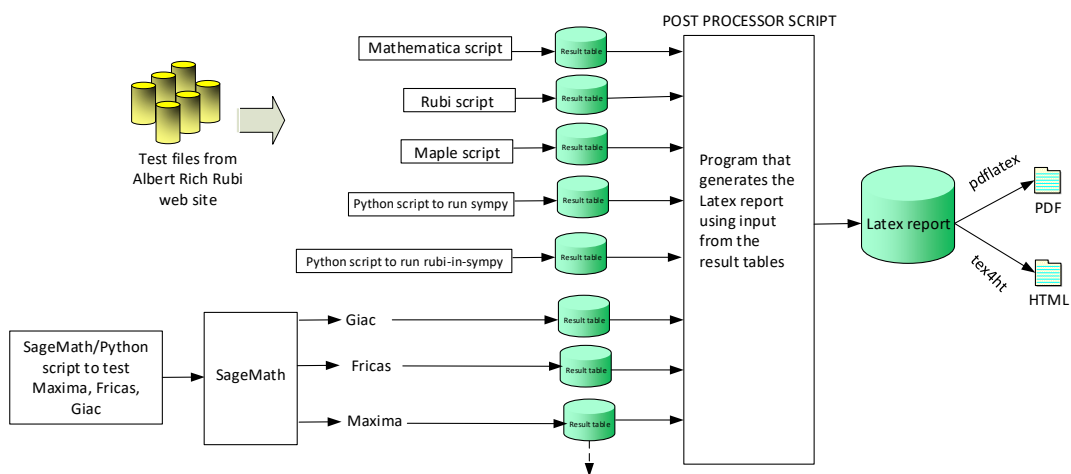
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 19, 21, 23, 25, 29, 31, 33, 36, 38, 42, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 72, 73, 74, 79, 80, 81, 83, 85, 86, 87, 88, 89, 90, 91 }

B grade: { 7, 9, 22, 24, 26, 27, 28, 30, 32, 34, 35, 37, 39, 40, 41, 43, 44, 46, 48, 66, 70, 71, 75, 76, 77, 84 }

C grade: { 18, 20, 78 }

F grade: { 82 }

2.1.3 Maple

A grade: { 1, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 64, 65, 66, 70, 71, 72, 75, 76, 77, 78, 83, 88, 89, 90, 91 }

B grade: { 2, 4, 7, 24, 33, 42, 52, 54, 61, 62, 63, 67, 68, 69, 73, 74 }

C grade: { }

F grade: { 79, 80, 81, 82, 84, 85, 86, 87 }

2.1.4 Maxima

A grade: { 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 54, 57, 59, 60, 64, 65, 66, 70, 71, 72, 75, 76, 77, 83, 88, 89, 90, 91 }

B grade: { 5, 7, 9, 61, 62, 63, 67, 68, 69, 73, 74, 78 }

C grade: { }

F grade: { 1, 2, 3, 4, 51, 53, 55, 56, 58, 79, 80, 81, 82, 84, 85, 86, 87 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 11, 12, 13, 14, 15, 22, 23, 24, 25, 32, 33, 34, 35, 41, 42, 43, 44, 45, 47, 50, 51, 52, 53, 54, 57, 83, 88, 89, 90, 91 }

B grade: { 5, 7, 8, 9, 10, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 46, 48, 49, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

C grade: { }

F grade: { 79, 80, 81, 82, 84, 85, 86, 87 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 89 }

B grade: { }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91 }

2.1.7 Giac

A grade: { 1, 3, 5, 6, 8, 10, 16, 17, 19, 20, 21, 26, 27, 28, 29, 30, 31, 35, 36, 38, 39, 40, 44, 45, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 70, 71, 72, 75, 76, 77, 78, 88, 89, 90, 91 }

B grade: { 2, 4, 7, 9, 12, 13, 14, 15, 18, 22, 24, 25, 33, 37, 42, 52, 54, 61, 62, 67, 68, 69, 73, 74 }

C grade: { }

F grade: { 11, 23, 32, 34, 41, 43, 79, 80, 81, 82, 83, 84, 85, 86, 87 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	67	66	0	136	51	72
normalized size	1	1.	0.86	0.85	0.	1.74	0.65	0.92
time (sec)	N/A	0.066	0.08	0.075	0.	2.065	0.395	1.306

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	51	81	0	101	37	96
normalized size	1	1.	1.76	2.79	0.	3.48	1.28	3.31
time (sec)	N/A	0.137	0.019	0.051	0.	2.061	0.289	1.298

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	39	47	0	89	31	55
normalized size	1	1.	0.78	0.94	0.	1.78	0.62	1.1
time (sec)	N/A	0.054	0.091	0.041	0.	2.125	0.425	1.341

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	33	47	0	47	17	45
normalized size	1	1.	1.74	2.47	0.	2.47	0.89	2.37
time (sec)	N/A	0.089	0.012	0.035	0.	1.994	0.174	1.34

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	31	21	38	73	0	30
normalized size	1	1.	1.94	1.31	2.38	4.56	0.	1.88
time (sec)	N/A	0.086	0.016	0.033	1.478	2.205	0.	1.333

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	15	20	23	78	0	24
normalized size	1	1.	0.83	1.11	1.28	4.33	0.	1.33
time (sec)	N/A	0.033	0.019	0.033	1.404	2.02	0.	1.32

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	75	42	80	232	0	63
normalized size	1	1.	3.12	1.75	3.33	9.67	0.	2.62
time (sec)	N/A	0.132	0.022	0.039	1.246	2.133	0.	1.385

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	29	15	16	109	0	16
normalized size	1	1.	1.53	0.79	0.84	5.74	0.	0.84
time (sec)	N/A	0.039	0.018	0.038	1.366	1.996	0.	1.417

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	139	58	112	424	0	85
normalized size	1	1.	3.48	1.45	2.8	10.6	0.	2.12
time (sec)	N/A	0.151	0.023	0.043	1.301	2.139	0.	1.402

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	41	28	32	176	0	32
normalized size	1	1.	1.11	0.76	0.86	4.76	0.	0.86
time (sec)	N/A	0.047	0.019	0.041	1.363	1.987	0.	1.388

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	103	113	123	267	0	0
normalized size	1	1.	1.02	1.12	1.22	2.64	0.	0.
time (sec)	N/A	0.076	0.032	0.041	1.389	2.277	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	82	92	117	189	0	1439
normalized size	1	1.	0.99	1.11	1.41	2.28	0.	17.34
time (sec)	N/A	0.168	0.079	0.033	2.159	2.359	0.	1.7

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	71	79	95	197	0	7223
normalized size	1	1.	1.03	1.14	1.38	2.86	0.	104.68
time (sec)	N/A	0.068	0.027	0.035	1.361	2.303	0.	2.784

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	56	58	70	120	0	558
normalized size	1	1.	1.14	1.18	1.43	2.45	0.	11.39
time (sec)	N/A	0.084	0.048	0.035	2.511	2.255	0.	1.437

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	48	45	62	134	0	1669
normalized size	1	1.	1.3	1.22	1.68	3.62	0.	45.11
time (sec)	N/A	0.036	0.02	0.027	1.521	2.3	0.	1.658

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	52	42	62	170	0	66
normalized size	1	1.	2.	1.62	2.38	6.54	0.	2.54
time (sec)	N/A	0.029	0.024	0.036	1.535	2.351	0.	1.413

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	36	26	34	171	0	47
normalized size	1	1.	1.44	1.04	1.36	6.84	0.	1.88
time (sec)	N/A	0.078	0.059	0.077	1.388	2.495	0.	1.364

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	107	75	112	367	0	159
normalized size	1	1.	1.78	1.25	1.87	6.12	0.	2.65
time (sec)	N/A	0.068	0.027	0.082	1.142	2.89	0.	1.512

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	72	60	68	316	0	84
normalized size	1	1.	1.26	1.05	1.19	5.54	0.	1.47
time (sec)	N/A	0.088	0.245	0.081	1.128	2.795	0.	1.362

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	151	109	166	585	0	239
normalized size	1	1.	1.54	1.11	1.69	5.97	0.	2.44
time (sec)	N/A	0.092	0.029	0.084	1.26	2.723	0.	1.427

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	104	94	97	473	0	113
normalized size	1	1.	1.2	1.08	1.11	5.44	0.	1.3
time (sec)	N/A	0.108	0.545	0.085	1.132	2.1	0.	1.384

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	240	204	173	333	0	7808
normalized size	1	1.	2.12	1.81	1.53	2.95	0.	69.1
time (sec)	N/A	0.186	3.197	0.04	1.477	2.063	0.	6.669

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	152	167	140	321	0	0
normalized size	1	1.	1.25	1.37	1.15	2.63	0.	0.
time (sec)	N/A	0.131	1.023	0.041	0.974	1.993	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	162	145	111	240	0	1432
normalized size	1	1.	2.13	1.91	1.46	3.16	0.	18.84
time (sec)	N/A	0.113	2.362	0.039	1.476	2.016	0.	1.879

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	111	108	90	230	0	3830
normalized size	1	1.	1.63	1.59	1.32	3.38	0.	56.32
time (sec)	N/A	0.074	0.439	0.036	1.119	2.322	0.	4.538

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	97	61	81	288	0	100
normalized size	1	1.	2.26	1.42	1.88	6.7	0.	2.33
time (sec)	N/A	0.056	0.233	0.044	1.069	2.434	0.	1.572

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	91	43	53	246	0	69
normalized size	1	1.	2.17	1.02	1.26	5.86	0.	1.64
time (sec)	N/A	0.047	0.544	0.042	1.107	2.282	0.	1.539

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	250	120	165	581	0	232
normalized size	1	1.	2.63	1.26	1.74	6.12	0.	2.44
time (sec)	N/A	0.118	1.951	0.051	1.101	2.778	0.	1.636

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	127	104	93	446	0	123
normalized size	1	1.	1.61	1.32	1.18	5.65	0.	1.56
time (sec)	N/A	0.07	1.409	0.053	1.086	2.394	0.	1.619

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	994	183	252	856	0	363
normalized size	1	1.	6.02	1.11	1.53	5.19	0.	2.2
time (sec)	N/A	0.16	6.2	0.054	1.075	2.974	0.	1.652

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	114	166	140	628	0	177
normalized size	1	1.	0.93	1.36	1.15	5.15	0.	1.45
time (sec)	N/A	0.101	1.51	0.055	1.04	2.428	0.	1.408

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	771	271	234	455	0	0
normalized size	1	1.	3.76	1.32	1.14	2.22	0.	0.
time (sec)	N/A	0.189	6.266	0.053	1.037	2.396	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	203	226	153	342	0	3497
normalized size	1	1.	1.97	2.19	1.49	3.32	0.	33.95
time (sec)	N/A	0.145	4.227	0.052	1.515	2.168	0.	3.227

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	637	193	173	346	0	0
normalized size	1	1.	4.79	1.45	1.3	2.6	0.	0.
time (sec)	N/A	0.116	6.156	0.043	1.118	2.124	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	241	125	150	383	0	194
normalized size	1	1.	2.8	1.45	1.74	4.45	0.	2.26
time (sec)	N/A	0.086	2.26	0.061	1.085	2.497	0.	2.06

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	126	63	76	327	0	95
normalized size	1	1.	1.97	0.98	1.19	5.11	0.	1.48
time (sec)	N/A	0.053	0.976	0.056	1.101	2.047	0.	2.067

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	897	170	231	720	0	410
normalized size	1	1.	6.36	1.21	1.64	5.11	0.	2.91
time (sec)	N/A	0.135	6.183	0.065	1.152	2.74	0.	1.985

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	212	141	132	575	0	180
normalized size	1	1.	1.88	1.25	1.17	5.09	0.	1.59
time (sec)	N/A	0.086	2.207	0.068	1.064	2.018	0.	2.048

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	1229	254	338	1060	0	504
normalized size	1	1.	5.37	1.11	1.48	4.63	0.	2.2
time (sec)	N/A	0.208	6.2	0.068	1.106	2.955	0.	2.15

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	515	230	192	834	0	255
normalized size	1	1.	3.08	1.38	1.15	4.99	0.	1.53
time (sec)	N/A	0.134	1.758	0.069	1.118	2.284	0.	2.032

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	1017	412	294	528	0	0
normalized size	1	1.	3.7	1.5	1.07	1.92	0.	0.
time (sec)	N/A	0.248	6.275	0.059	1.148	2.133	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	263	368	208	431	0	5307
normalized size	1	1.	1.89	2.65	1.5	3.1	0.	38.18
time (sec)	N/A	0.175	6.266	0.057	1.709	2.248	0.	7.192

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	383	309	224	414	0	0
normalized size	1	1.	2.13	1.72	1.24	2.3	0.	0.
time (sec)	N/A	0.157	5.459	0.049	1.093	2.066	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	352	214	188	444	0	261
normalized size	1	1.	2.98	1.81	1.59	3.76	0.	2.21
time (sec)	N/A	0.114	5.132	0.069	1.108	2.565	0.	2.71

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	162	90	97	389	0	116
normalized size	1	1.	1.95	1.08	1.17	4.69	0.	1.4
time (sec)	N/A	0.058	1.16	0.063	1.12	2.049	0.	2.648

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	1128	192	254	818	0	405
normalized size	1	1.	7.01	1.19	1.58	5.08	0.	2.52
time (sec)	N/A	0.157	6.193	0.075	1.174	3.035	0.	2.77

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	188	184	162	633	0	217
normalized size	1	1.	1.37	1.34	1.18	4.62	0.	1.58
time (sec)	N/A	0.104	3.677	0.077	1.07	2.199	0.	2.645

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	1491	317	410	1289	0	647
normalized size	1	1.	5.44	1.16	1.5	4.7	0.	2.36
time (sec)	N/A	0.241	6.287	0.082	1.173	3.983	0.	2.684

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	233	301	231	922	0	317
normalized size	1	1.	1.2	1.55	1.19	4.75	0.	1.63
time (sec)	N/A	0.157	3.919	0.082	1.069	2.634	0.	2.653

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	660	442	522	1661	0	873
normalized size	1	1.	1.64	1.1	1.3	4.13	0.	2.17
time (sec)	N/A	0.306	6.259	0.127	1.154	4.568	0.	2.8

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	289	361	0	840	0	626
normalized size	1	1.	1.05	1.32	0.	3.07	0.	2.28
time (sec)	N/A	0.354	3.124	0.079	0.	2.327	0.	1.342

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	249	565	378	481	0	451
normalized size	1	1.	1.58	3.58	2.39	3.04	0.	2.85
time (sec)	N/A	0.339	2.592	0.068	1.655	2.289	0.	1.196

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	139	205	0	597	0	325
normalized size	1	1.	0.83	1.22	0.	3.55	0.	1.93
time (sec)	N/A	0.223	1.375	0.065	0.	2.278	0.	1.386

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	170	238	194	278	0	248
normalized size	1	1.	1.81	2.53	2.06	2.96	0.	2.64
time (sec)	N/A	0.163	0.723	0.059	1.572	2.123	0.	1.148

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	79	100	0	435	0	159
normalized size	1	1.	0.88	1.11	0.	4.83	0.	1.77
time (sec)	N/A	0.106	0.336	0.05	0.	2.046	0.	1.334

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	75	65	0	450	0	127
normalized size	1	1.	1.14	0.98	0.	6.82	0.	1.92
time (sec)	N/A	0.129	0.105	0.056	0.	2.377	0.	1.436

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	47	53	63	246	0	81
normalized size	1	1.	0.94	1.06	1.26	4.92	0.	1.62
time (sec)	N/A	0.06	0.131	0.062	1.068	2.055	0.	1.208

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	179	162	0	644	0	282
normalized size	1	1.	1.47	1.33	0.	5.28	0.	2.31
time (sec)	N/A	0.307	0.827	0.069	0.	2.683	0.	1.351

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	95	144	131	500	0	194
normalized size	1	1.	0.88	1.33	1.21	4.63	0.	1.8
time (sec)	N/A	0.104	0.456	0.076	1.064	2.154	0.	1.181

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	273	227	902	0	339
normalized size	1	1.	0.89	1.62	1.34	5.34	0.	2.01
time (sec)	N/A	0.151	2.164	0.083	1.095	2.412	0.	1.244

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	526	1211	1079	1388	0	992
normalized size	1	1.	1.77	4.08	3.63	4.67	0.	3.34
time (sec)	N/A	0.914	6.48	0.103	1.796	2.896	0.	1.233

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	373	724	684	986	0	693
normalized size	1	1.	1.72	3.34	3.15	4.54	0.	3.19
time (sec)	N/A	0.562	3.807	0.099	1.53	2.632	0.	1.216

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	246	352	396	640	0	355
normalized size	1	1.	1.66	2.38	2.68	4.32	0.	2.4
time (sec)	N/A	0.295	3.26	0.089	1.556	2.249	0.	1.177

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	109	75	100	663	0	100
normalized size	1	1.	1.51	1.04	1.39	9.21	0.	1.39
time (sec)	N/A	0.065	0.373	0.092	0.987	2.199	0.	1.219

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	244	189	194	1004	0	274
normalized size	1	1.	1.74	1.35	1.39	7.17	0.	1.96
time (sec)	N/A	0.122	2.506	0.153	1.092	2.374	0.	1.298

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	589	343	304	1770	0	448
normalized size	1	1.	2.69	1.57	1.39	8.08	0.	2.05
time (sec)	N/A	0.198	6.231	0.122	1.225	2.59	0.	1.239

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	683	1449	1469	2137	0	1246
normalized size	1	1.	1.79	3.79	3.85	5.59	0.	3.26
time (sec)	N/A	1.432	6.604	0.118	1.933	3.59	0.	1.286

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	501	882	1004	1569	0	794
normalized size	1	1.	1.76	3.09	3.52	5.51	0.	2.79
time (sec)	N/A	0.85	6.438	0.109	1.811	3.095	0.	1.27

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	316	542	625	1152	0	651
normalized size	1	1.	1.53	2.63	3.03	5.59	0.	3.16
time (sec)	N/A	0.396	3.959	0.115	1.849	2.483	0.	1.237

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	241	96	146	1219	0	153
normalized size	1	1.	2.54	1.01	1.54	12.83	0.	1.61
time (sec)	N/A	0.077	2.589	0.119	1.146	2.46	0.	1.27

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	456	234	259	1818	0	320
normalized size	1	1.	2.56	1.31	1.46	10.21	0.	1.8
time (sec)	N/A	0.153	3.227	0.13	1.202	2.599	0.	1.346

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	494	410	379	2344	0	516
normalized size	1	1.	1.86	1.55	1.43	8.85	0.	1.95
time (sec)	N/A	0.239	4.675	0.137	1.219	3.015	0.	1.238

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	564	1215	1346	2412	0	1218
normalized size	1	1.	1.54	3.32	3.68	6.59	0.	3.33
time (sec)	N/A	1.37	5.419	0.123	1.896	4.005	0.	1.35

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	395	668	894	1789	0	867
normalized size	1	1.	1.5	2.53	3.39	6.78	0.	3.28
time (sec)	N/A	0.571	3.594	0.12	1.712	3.002	0.	1.324

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	259	117	189	1901	0	174
normalized size	1	1.	2.23	1.01	1.63	16.39	0.	1.5
time (sec)	N/A	0.088	2.064	0.131	1.14	2.846	0.	1.225

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	528	278	308	2732	0	300
normalized size	1	1.	2.58	1.36	1.5	13.33	0.	1.46
time (sec)	N/A	0.173	2.034	0.198	1.123	3.255	0.	1.334

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	673	476	439	3507	0	578
normalized size	1	1.	2.24	1.59	1.46	11.69	0.	1.93
time (sec)	N/A	0.271	1.634	0.165	1.24	4.431	0.	1.361

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	41	26	68	203	0	59
normalized size	1	1.	1.58	1.	2.62	7.81	0.	2.27
time (sec)	N/A	0.074	0.041	0.025	1.85	2.354	0.	1.296

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	205	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.45	2.52	0.392	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	166	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.268	1.194	0.248	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	0.257	0.488	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	765	765	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	4.173	12.792	0.164	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.463	3.184	0.644	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	910	0	0	0	0	0
normalized size	1	1.	2.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.803	6.579	0.556	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	270	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.367	1.088	0.473	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.875	0.239	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	78	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	1.244	0.229	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.979	3.069	0.542	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.855	2.185	0.041	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.487	1.52	0.23	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.7	2.852	0.189	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of

the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [5] had the largest ratio of [0.6364]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.	13	0.308
2	A	9	7	1.	13	0.538
3	A	5	4	1.	13	0.308
4	A	8	6	1.	11	0.546
5	A	8	7	1.	11	0.636
6	A	3	2	1.	13	0.154
7	A	8	7	1.	13	0.538
8	A	4	3	1.	13	0.231
9	A	9	8	1.	13	0.615
10	A	4	3	1.	13	0.231
11	A	8	5	1.	19	0.263
12	A	6	4	1.	19	0.21
13	A	8	5	1.	19	0.263
14	A	5	4	1.	19	0.21
15	A	6	5	1.	17	0.294
16	A	4	2	1.	17	0.118
17	A	3	1	1.	19	0.053
18	A	7	6	1.	19	0.316
19	A	3	1	1.	19	0.053
20	A	9	6	1.	19	0.316
21	A	3	1	1.	19	0.053
22	A	8	6	1.	21	0.286
23	A	11	7	1.	21	0.333
24	A	6	6	1.	21	0.286
25	A	9	7	1.	19	0.368
26	A	6	4	1.	19	0.21
27	A	3	2	1.	21	0.095
28	A	10	7	1.	21	0.333
29	A	3	2	1.	21	0.095
30	A	13	9	1.	21	0.429
31	A	3	2	1.	21	0.095
32	A	16	8	1.	21	0.381
33	A	7	6	1.	21	0.286
34	A	13	8	1.	19	0.421
35	A	8	5	1.	19	0.263
36	A	3	2	1.	21	0.095
37	A	12	7	1.	21	0.333
38	A	3	2	1.	21	0.095
39	A	17	9	1.	21	0.429
40	A	3	2	1.	21	0.095
41	A	19	8	1.	21	0.381

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	7	6	1.	21	0.286
43	A	16	9	1.	19	0.474
44	A	10	5	1.	19	0.263
45	A	3	2	1.	21	0.095
46	A	14	9	1.	21	0.429
47	A	3	2	1.	21	0.095
48	A	21	9	1.	21	0.429
49	A	3	2	1.	21	0.095
50	A	25	9	1.	21	0.429
51	A	13	9	1.	21	0.429
52	A	8	6	1.	21	0.286
53	A	10	9	1.	21	0.429
54	A	7	6	1.	21	0.286
55	A	6	6	1.	19	0.316
56	A	6	5	1.	19	0.263
57	A	3	2	1.	21	0.095
58	A	15	11	1.	21	0.524
59	A	3	2	1.	21	0.095
60	A	3	2	1.	21	0.095
61	A	9	6	1.	21	0.286
62	A	8	6	1.	21	0.286
63	A	7	6	1.	21	0.286
64	A	3	2	1.	21	0.095
65	A	3	2	1.	21	0.095
66	A	3	2	1.	21	0.095
67	A	9	6	1.	21	0.286
68	A	8	6	1.	21	0.286
69	A	7	6	1.	21	0.286
70	A	3	2	1.	21	0.095
71	A	3	2	1.	21	0.095
72	A	3	2	1.	21	0.095
73	A	8	6	1.	21	0.286
74	A	7	6	1.	21	0.286
75	A	3	2	1.	21	0.095
76	A	3	2	1.	21	0.095
77	A	3	2	1.	21	0.095
78	A	6	5	1.	9	0.556
79	A	8	5	1.	21	0.238
80	A	6	5	1.	21	0.238
81	A	5	4	1.	19	0.21
82	A	14	6	1.	21	0.286
83	A	0	0	0.	0	0.
84	A	7	4	1.	21	0.19
85	A	6	4	1.	21	0.19

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.	21	0.095
87	A	4	4	1.	21	0.19
88	A	0	0	0.	0	0.
89	A	0	0	0.	0	0.
90	A	0	0	0.	0	0.
91	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1

$$\int \frac{\sin^4(x)}{i+\tan(x)} dx$$

Optimal. Leaf size=78

$$-\frac{ix}{16} - \frac{i}{8(-\tan(x)+i)} - \frac{3i}{16(\tan(x)+i)} - \frac{1}{32(-\tan(x)+i)^2} - \frac{5}{32(\tan(x)+i)^2} + \frac{i}{24(\tan(x)+i)^3}$$

[Out] $(-I/16)*x - 1/(32*(I - \text{Tan}[x])^2) - (I/8)/(I - \text{Tan}[x]) + (I/24)/(I + \text{Tan}[x])^3 - 5/(32*(I + \text{Tan}[x])^2) - ((3*I)/16)/(I + \text{Tan}[x])$

Rubi [A] time = 0.0659966, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3516, 848, 88, 203}

$$-\frac{ix}{16} - \frac{i}{8(-\tan(x)+i)} - \frac{3i}{16(\tan(x)+i)} - \frac{1}{32(-\tan(x)+i)^2} - \frac{5}{32(\tan(x)+i)^2} + \frac{i}{24(\tan(x)+i)^3}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^4/(I + Tan[x]),x]

[Out] $(-I/16)*x - 1/(32*(I - \text{Tan}[x])^2) - (I/8)/(I - \text{Tan}[x]) + (I/24)/(I + \text{Tan}[x])^3 - 5/(32*(I + \text{Tan}[x])^2) - ((3*I)/16)/(I + \text{Tan}[x])$

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 848

Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p], x]

$x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x] \&\& \text{IntegersQ}[m, n] \&\& (\text{IntegerQ}[p] \parallel (\text{GtQ}[m, 0] \&\& \text{GeQ}[n, -1]))$

Rule 203

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_ \text{Symbol}] :> \text{Simp}[(1 \cdot \text{ArcTan}[\text{Rt}[b, 2] \cdot x] / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{\sin^4(x)}{i + \tan(x)} dx &= \text{Subst} \left(\int \frac{x^4}{(i+x)(1+x^2)^3} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \frac{x^4}{(-i+x)^3(i+x)^4} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(\frac{1}{16(-i+x)^3} - \frac{i}{8(-i+x)^2} - \frac{i}{8(i+x)^4} + \frac{5}{16(i+x)^3} + \frac{3i}{16(i+x)^2} - \frac{i}{16(1+x^2)} \right) dx, x, \tan(x) \right) \\ &= -\frac{1}{32(i - \tan(x))^2} - \frac{i}{8(i - \tan(x))} + \frac{i}{24(i + \tan(x))^3} - \frac{5}{32(i + \tan(x))^2} - \frac{3i}{16(i + \tan(x))} - \frac{1}{16} i \text{Subst} \\ &= -\frac{ix}{16} - \frac{1}{32(i - \tan(x))^2} - \frac{i}{8(i - \tan(x))} + \frac{i}{24(i + \tan(x))^3} - \frac{5}{32(i + \tan(x))^2} - \frac{3i}{16(i + \tan(x))} \end{aligned}$$

Mathematica [A] time = 0.0800587, size = 67, normalized size = 0.86

$$\frac{\sec(x) (-32 \sin(x) - 27 \sin(3x) + 5 \sin(5x) - 56i \cos(x) - 9i \cos(3x) + i \cos(5x) + 24 \tan^{-1}(\tan(x))(\cos(x) - i \sin(x)))}{384(\tan(x) + i)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^4/(I + Tan[x]), x]

[Out] (Sec[x]*((-56*I)*Cos[x] - (9*I)*Cos[3*x] + I*Cos[5*x] + 24*ArcTan[Tan[x]])*(Cos[x] - I*Sin[x]) - 32*Sin[x] - 27*Sin[3*x] + 5*Sin[5*x])/((384*(I + Tan[x])))

Maple [A] time = 0.075, size = 66, normalized size = 0.9

$$\frac{\frac{i}{8}}{\tan(x) - i} - \frac{1}{32(\tan(x) - i)^2} - \frac{\ln(\tan(x) - i)}{32} + \frac{\frac{i}{24}}{(i + \tan(x))^3} - \frac{\frac{3i}{16}}{i + \tan(x)} - \frac{5}{32(i + \tan(x))^2} + \frac{\ln(i + \tan(x))}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^4/(I+tan(x)), x)

[Out] 1/8*I/(tan(x)-I)-1/32/(tan(x)-I)^2-1/32*ln(tan(x)-I)+1/24*I/(I+tan(x))^3-3/16*I/(I+tan(x))-5/32/(I+tan(x))^2+1/32*ln(I+tan(x))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [A] time = 2.06533, size = 136, normalized size = 1.74

$$\frac{1}{384} \left(-24i x e^{4ix} - 2e^{10ix} + 9e^{8ix} - 12e^{6ix} - 18e^{2ix} + 3 \right) e^{-4ix}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="fricas")

[Out] 1/384*(-24*I*x*e^(4*I*x) - 2*e^(10*I*x) + 9*e^(8*I*x) - 12*e^(6*I*x) - 18*e^(2*I*x) + 3)*e^(-4*I*x)

Sympy [A] time = 0.395444, size = 51, normalized size = 0.65

$$-\frac{ix}{16} - \frac{e^{6ix}}{192} + \frac{3e^{4ix}}{128} - \frac{e^{2ix}}{32} - \frac{3e^{-2ix}}{64} + \frac{e^{-4ix}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**4/(I+tan(x)),x)

[Out] -I*x/16 - exp(6*I*x)/192 + 3*exp(4*I*x)/128 - exp(2*I*x)/32 - 3*exp(-2*I*x)/64 + exp(-4*I*x)/128

Giac [A] time = 1.3062, size = 72, normalized size = 0.92

$$\frac{3i \tan(x)^4 + 21 \tan(x)^3 + 13i \tan(x)^2 + 11 \tan(x) + 8i}{48 (\tan(x) + i)^3 (\tan(x) - i)^2} + \frac{1}{32} \log(\tan(x) + i) - \frac{1}{32} \log(\tan(x) - i)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="giac")

[Out] -1/48*(3*I*tan(x)^4 + 21*tan(x)^3 + 13*I*tan(x)^2 + 11*tan(x) + 8*I)/((tan(x) + I)^3*(tan(x) - I)^2) + 1/32*log(tan(x) + I) - 1/32*log(tan(x) - I)

3.2 $\int \frac{\sin^3(x)}{i+\tan(x)} dx$

Optimal. Leaf size=29

$$\frac{\sin^5(x)}{5} - \frac{1}{5}i\cos^5(x) + \frac{1}{3}i\cos^3(x)$$

[Out] (I/3)*Cos[x]^3 - (I/5)*Cos[x]^5 + Sin[x]^5/5

Rubi [A] time = 0.136883, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3518, 3108, 3107, 2565, 14, 2564, 30}

$$\frac{\sin^5(x)}{5} - \frac{1}{5}i\cos^5(x) + \frac{1}{3}i\cos^3(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^3/(I + Tan[x]),x]

[Out] (I/3)*Cos[x]^3 - (I/5)*Cos[x]^5 + Sin[x]^5/5

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*cos[e + f*x] + b*sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> Dist[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*cos[c + d*x] + a*sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILtQ[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2564

Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{\sin^3(x)}{i + \tan(x)} dx &= \int \frac{\cos(x) \sin^3(x)}{i \cos(x) + \sin(x)} dx \\
 &= - \left(i \int \cos(x) (\cos(x) + i \sin(x)) \sin^3(x) dx \right) \\
 &= - \left(i \int (\cos^2(x) \sin^3(x) + i \cos(x) \sin^4(x)) dx \right) \\
 &= - \left(i \int \cos^2(x) \sin^3(x) dx \right) + \int \cos(x) \sin^4(x) dx \\
 &= i \operatorname{Subst} \left(\int x^2 (1 - x^2) dx, x, \cos(x) \right) + \operatorname{Subst} \left(\int x^4 dx, x, \sin(x) \right) \\
 &= \frac{\sin^5(x)}{5} + i \operatorname{Subst} \left(\int (x^2 - x^4) dx, x, \cos(x) \right) \\
 &= \frac{1}{3} i \cos^3(x) - \frac{1}{5} i \cos^5(x) + \frac{\sin^5(x)}{5}
 \end{aligned}$$

Mathematica [A] time = 0.019013, size = 51, normalized size = 1.76

$$\frac{\sin(x)}{8} - \frac{1}{16} \sin(3x) + \frac{1}{80} \sin(5x) + \frac{1}{8} i \cos(x) + \frac{1}{48} i \cos(3x) - \frac{1}{80} i \cos(5x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^3/(I + Tan[x]), x]

[Out] (I/8)*Cos[x] + (I/48)*Cos[3*x] - (I/80)*Cos[5*x] + Sin[x]/8 - Sin[3*x]/16 + Sin[5*x]/80

Maple [B] time = 0.051, size = 81, normalized size = 2.8

$$-\frac{i}{4} \left(\tan\left(\frac{x}{2}\right) - i \right)^{-2} + \frac{1}{6} \left(\tan\left(\frac{x}{2}\right) - i \right)^{-3} + \frac{1}{8} \left(\tan\left(\frac{x}{2}\right) - i \right)^{-1} + i \left(\tan\left(\frac{x}{2}\right) + i \right)^{-4} + \frac{2}{5} \left(\tan\left(\frac{x}{2}\right) + i \right)^{-5} - \frac{2}{3} \left(\tan\left(\frac{x}{2}\right) + i \right)^{-6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^3/(I+tan(x)), x)

[Out] -1/4*I/(tan(1/2*x)-I)^2+1/6/(tan(1/2*x)-I)^3+1/8/(tan(1/2*x)-I)+I/(tan(1/2*x)+I)^4+2/5/(tan(1/2*x)+I)^5-2/3/(tan(1/2*x)+I)^3-1/8/(tan(1/2*x)+I)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [A] time = 2.06113, size = 101, normalized size = 3.48

$$\frac{1}{240} (-3i e^{8ix} + 10i e^{6ix} + 30i e^{2ix} - 5i) e^{-3ix}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="fricas")

[Out] 1/240*(-3*I*e^(8*I*x) + 10*I*e^(6*I*x) + 30*I*e^(2*I*x) - 5*I)*e^(-3*I*x)

Sympy [A] time = 0.289148, size = 37, normalized size = 1.28

$$-\frac{ie^{5ix}}{80} + \frac{ie^{3ix}}{24} + \frac{ie^{-ix}}{8} - \frac{ie^{-3ix}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**3/(I+tan(x)),x)

[Out] -I*exp(5*I*x)/80 + I*exp(3*I*x)/24 + I*exp(-I*x)/8 - I*exp(-3*I*x)/48

Giac [B] time = 1.29783, size = 96, normalized size = 3.31

$$\frac{-3i \tan\left(\frac{1}{2}x\right)^2 - 12 \tan\left(\frac{1}{2}x\right) + 5i}{24\left(-i \tan\left(\frac{1}{2}x\right) - 1\right)^3} - \frac{15 \tan\left(\frac{1}{2}x\right)^4 + 60i \tan\left(\frac{1}{2}x\right)^3 - 10 \tan\left(\frac{1}{2}x\right)^2 - 20i \tan\left(\frac{1}{2}x\right) + 7}{120\left(\tan\left(\frac{1}{2}x\right) + i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="giac")

[Out] -1/24*(-3*I*tan(1/2*x)^2 - 12*tan(1/2*x) + 5*I)/(-I*tan(1/2*x) - 1)^3 - 1/120*(15*tan(1/2*x)^4 + 60*I*tan(1/2*x)^3 - 10*tan(1/2*x)^2 - 20*I*tan(1/2*x) + 7)/(tan(1/2*x) + I)^5

3.3 $\int \frac{\sin^2(x)}{i+\tan(x)} dx$

Optimal. Leaf size=50

$$-\frac{ix}{8} - \frac{i}{8(-\tan(x)+i)} - \frac{i}{4(\tan(x)+i)} - \frac{1}{8(\tan(x)+i)^2}$$

[Out] $(-I/8)*x - (I/8)/(I - \text{Tan}[x]) - 1/(8*(I + \text{Tan}[x])^2) - (I/4)/(I + \text{Tan}[x])$

Rubi [A] time = 0.0539709, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3516, 848, 88, 203}

$$-\frac{ix}{8} - \frac{i}{8(-\tan(x)+i)} - \frac{i}{4(\tan(x)+i)} - \frac{1}{8(\tan(x)+i)^2}$$

Antiderivative was successfully verified.

[In] `Int[Sin[x]^2/(I + Tan[x]),x]`

[Out] $(-I/8)*x - (I/8)/(I - \text{Tan}[x]) - 1/(8*(I + \text{Tan}[x])^2) - (I/4)/(I + \text{Tan}[x])$

Rule 3516

`Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]`

Rule 848

`Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))`

Rule 88

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))`

Rule 203

`Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x)}{i + \tan(x)} dx &= \text{Subst} \left(\int \frac{x^2}{(i+x)(1+x^2)^2} dx, x, \tan(x) \right) \\
&= \text{Subst} \left(\int \frac{x^2}{(-i+x)^2(i+x)^3} dx, x, \tan(x) \right) \\
&= \text{Subst} \left(\int \left(-\frac{i}{8(-i+x)^2} + \frac{1}{4(i+x)^3} + \frac{i}{4(i+x)^2} - \frac{i}{8(1+x^2)} \right) dx, x, \tan(x) \right) \\
&= -\frac{i}{8(i - \tan(x))} - \frac{1}{8(i + \tan(x))^2} - \frac{i}{4(i + \tan(x))} - \frac{1}{8} i \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) \\
&= -\frac{ix}{8} - \frac{i}{8(i - \tan(x))} - \frac{1}{8(i + \tan(x))^2} - \frac{i}{4(i + \tan(x))}
\end{aligned}$$

Mathematica [A] time = 0.091249, size = 39, normalized size = 0.78

$$-\frac{i(-3i \sin(2x) + \cos(2x) + 2 \tan^{-1}(\tan(x))(\tan(x) + i) + 3)}{16(\tan(x) + i)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^2/(I + Tan[x]),x]

[Out] ((-I/16)*(3 + Cos[2*x] - (3*I)*Sin[2*x] + 2*ArcTan[Tan[x]]*(I + Tan[x])))/(I + Tan[x])

Maple [A] time = 0.041, size = 47, normalized size = 0.9

$$\frac{\frac{i}{8}}{\tan(x) - i} - \frac{\ln(\tan(x) - i)}{16} - \frac{\frac{i}{4}}{i + \tan(x)} - \frac{1}{8(i + \tan(x))^2} + \frac{\ln(i + \tan(x))}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2/(I+tan(x)),x)

[Out] 1/8*I/(tan(x)-I)-1/16*ln(tan(x)-I)-1/4*I/(I+tan(x))-1/8/(I+tan(x))^2+1/16*ln(I+tan(x))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [A] time = 2.12543, size = 89, normalized size = 1.78

$$\frac{1}{32} \left(-4i x e^{(2ix)} + e^{(6ix)} - 2 e^{(4ix)} - 2 \right) e^{(-2ix)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="fricas")

[Out] 1/32*(-4*I*x*e^(2*I*x) + e^(6*I*x) - 2*e^(4*I*x) - 2)*e^(-2*I*x)

Sympy [A] time = 0.425476, size = 31, normalized size = 0.62

$$-\frac{ix}{8} + \frac{e^{4ix}}{32} - \frac{e^{2ix}}{16} - \frac{e^{-2ix}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**2/(I+tan(x)),x)

[Out] -I*x/8 + exp(4*I*x)/32 - exp(2*I*x)/16 - exp(-2*I*x)/16

Giac [A] time = 1.3412, size = 55, normalized size = 1.1

$$-\frac{i \tan(x)^2 + 3 \tan(x) + 2i}{8(\tan(x) + i)^2(\tan(x) - i)} + \frac{1}{16} \log(\tan(x) + i) - \frac{1}{16} \log(\tan(x) - i)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="giac")

[Out] -1/8*(I*tan(x)^2 + 3*tan(x) + 2*I)/((tan(x) + I)^2*(tan(x) - I)) + 1/16*log(tan(x) + I) - 1/16*log(tan(x) - I)

3.4 $\int \frac{\sin(x)}{i+\tan(x)} dx$

Optimal. Leaf size=19

$$\frac{\sin^3(x)}{3} + \frac{1}{3}i\cos^3(x)$$

[Out] (I/3)*Cos[x]^3 + Sin[x]^3/3

Rubi [A] time = 0.0893613, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {3518, 3108, 3107, 2565, 30, 2564}

$$\frac{\sin^3(x)}{3} + \frac{1}{3}i\cos^3(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(I + Tan[x]),x]

[Out] (I/3)*Cos[x]^3 + Sin[x]^3/3

Rule 3518

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*cos[e + f*x] + b*sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3108

Int[cos[(c_.) + (d_.)*(x_.)]^(m_.)*sin[(c_.) + (d_.)*(x_.)]^(n_.)*(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)]^(p_.), x_Symbol] :> Dist[a^p*b^p, Int[(Cos[c + d*x]^m*sin[c + d*x]^n)/(b*cos[c + d*x] + a*sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILtQ[p, 0]

Rule 3107

Int[cos[(c_.) + (d_.)*(x_.)]^(m_.)*sin[(c_.) + (d_.)*(x_.)]^(n_.)*(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)]^(p_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]

Rule 2565

Int[(cos[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{i + \tan(x)} dx &= \int \frac{\cos(x) \sin(x)}{i \cos(x) + \sin(x)} dx \\
&= -i \int \cos(x)(\cos(x) + i \sin(x)) \sin(x) dx \\
&= -\left(i \int (\cos^2(x) \sin(x) + i \cos(x) \sin^2(x)) dx\right) \\
&= -\left(i \int \cos^2(x) \sin(x) dx\right) + \int \cos(x) \sin^2(x) dx \\
&= i \operatorname{Subst}\left(\int x^2 dx, x, \cos(x)\right) + \operatorname{Subst}\left(\int x^2 dx, x, \sin(x)\right) \\
&= \frac{1}{3}i \cos^3(x) + \frac{\sin^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.01213, size = 33, normalized size = 1.74

$$\frac{\sin(x)}{4} - \frac{1}{12} \sin(3x) + \frac{1}{4}i \cos(x) + \frac{1}{12}i \cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(I + Tan[x]), x]
```

```
[Out] (I/4)*Cos[x] + (I/12)*Cos[3*x] + Sin[x]/4 - Sin[3*x]/12
```

Maple [B] time = 0.035, size = 47, normalized size = 2.5

$$\frac{1}{2} \left(\tan\left(\frac{x}{2}\right) - i \right)^{-1} + i \left(\tan\left(\frac{x}{2}\right) + i \right)^{-2} + \frac{2}{3} \left(\tan\left(\frac{x}{2}\right) + i \right)^{-3} - \frac{1}{2} \left(\tan\left(\frac{x}{2}\right) + i \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(I+tan(x)), x)
```

```
[Out] 1/2/(tan(1/2*x)-I)+I/(tan(1/2*x)+I)^2+2/3/(tan(1/2*x)+I)^3-1/2/(tan(1/2*x)+
I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(I+tan(x)), x, algorithm="maxima")
```

[Out] Exception raised: RuntimeError

Fricas [A] time = 1.99361, size = 47, normalized size = 2.47

$$\frac{1}{12} (i e^{4ix} + 3i) e^{-ix}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(I+tan(x)),x, algorithm="fricas")

[Out] 1/12*(I*e^(4*I*x) + 3*I)*e^(-I*x)

Sympy [A] time = 0.17362, size = 17, normalized size = 0.89

$$\frac{i e^{3ix}}{12} + \frac{i e^{-ix}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(I+tan(x)),x)

[Out] I*exp(3*I*x)/12 + I*exp(-I*x)/4

Giac [B] time = 1.3401, size = 45, normalized size = 2.37

$$-\frac{i}{2 \left(-i \tan\left(\frac{1}{2}x\right) - 1 \right)} - \frac{3 \tan\left(\frac{1}{2}x\right)^2 - 1}{6 \left(\tan\left(\frac{1}{2}x\right) + i \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(I+tan(x)),x, algorithm="giac")

[Out] -1/2*I/(-I*tan(1/2*x) - 1) - 1/6*(3*tan(1/2*x)^2 - 1)/(tan(1/2*x) + I)^3

3.5 $\int \frac{\csc(x)}{i+\tan(x)} dx$

Optimal. Leaf size=16

$$\sin(x) - i \cos(x) + i \tanh^{-1}(\cos(x))$$

[Out] I*ArcTanh[Cos[x]] - I*Cos[x] + Sin[x]

Rubi [A] time = 0.0864813, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3518, 3108, 3107, 2637, 2592, 321, 206}

$$\sin(x) - i \cos(x) + i \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Csc[x]/(I + Tan[x]),x]

[Out] I*ArcTanh[Cos[x]] - I*Cos[x] + Sin[x]

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] := Dist[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILtQ[p, 0]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] := Int[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{i + \tan(x)} dx &= \int \frac{\cot(x)}{i \cos(x) + \sin(x)} dx \\
&= -i \int \cot(x)(\cos(x) + i \sin(x)) dx \\
&= -i \int (i \cos(x) + \cos(x) \cot(x)) dx \\
&= -i \int \cos(x) \cot(x) dx + \int \cos(x) dx \\
&= \sin(x) + i \operatorname{Subst} \left(\int \frac{x^2}{1-x^2} dx, x, \cos(x) \right) \\
&= -i \cos(x) + \sin(x) + i \operatorname{Subst} \left(\int \frac{1}{1-x^2} dx, x, \cos(x) \right) \\
&= i \tanh^{-1}(\cos(x)) - i \cos(x) + \sin(x)
\end{aligned}$$

Mathematica [A] time = 0.015746, size = 31, normalized size = 1.94

$$\sin(x) - i \cos(x) - i \log \left(\sin \left(\frac{x}{2} \right) \right) + i \log \left(\cos \left(\frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(I + Tan[x]), x]
```

```
[Out] (-I)*Cos[x] + I*Log[Cos[x/2]] - I*Log[Sin[x/2]] + Sin[x]
```

Maple [A] time = 0.033, size = 21, normalized size = 1.3

$$2 (\tan(x/2) + i)^{-1} - i \ln \left(\tan \left(\frac{x}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(I+tan(x)), x)
```

```
[Out] 2/(tan(1/2*x)+I)-I*ln(tan(1/2*x))
```

Maxima [B] time = 1.47837, size = 38, normalized size = 2.38

$$\frac{2}{\frac{\sin(x)}{\cos(x)+1} + i} - i \log \left(\frac{\sin(x)}{\cos(x)+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(I+tan(x)),x, algorithm="maxima")

[Out] 2/(sin(x)/(cos(x) + 1) + I) - I*log(sin(x)/(cos(x) + 1))

Fricas [B] time = 2.20494, size = 73, normalized size = 4.56

$$-i e^{ix} + i \log(e^{ix} + 1) - i \log(e^{ix} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(I+tan(x)),x, algorithm="fricas")

[Out] -I*e^(I*x) + I*log(e^(I*x) + 1) - I*log(e^(I*x) - 1)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(x)}{\tan(x) + i} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(I+tan(x)),x)

[Out] Integral(csc(x)/(tan(x) + I), x)

Giac [A] time = 1.33305, size = 30, normalized size = 1.88

$$-\frac{2i}{-i \tan\left(\frac{1}{2}x\right) + 1} - i \log\left(-i \tan\left(\frac{1}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(I+tan(x)),x, algorithm="giac")

[Out] -2*I/(-I*tan(1/2*x) + 1) - I*log(-I*tan(1/2*x))

3.6 $\int \frac{\csc^2(x)}{i+\tan(x)} dx$

Optimal. Leaf size=18

$$ix + i \cot(x) + \log(\tan(x)) + \log(\cos(x))$$

[Out] I*x + I*Cot[x] + Log[Cos[x]] + Log[Tan[x]]

Rubi [A] time = 0.0333998, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3516, 44}

$$ix + i \cot(x) + \log(\tan(x)) + \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^2/(I + Tan[x]), x]

[Out] I*x + I*Cot[x] + Log[Cos[x]] + Log[Tan[x]]

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(x)}{i+\tan(x)} dx &= \text{Subst} \left(\int \frac{1}{x^2(i+x)} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(\frac{1}{-i-x} - \frac{i}{x^2} + \frac{1}{x} \right) dx, x, \tan(x) \right) \\ &= ix + i \cot(x) + \log(\cos(x)) + \log(\tan(x)) \end{aligned}$$

Mathematica [A] time = 0.0191565, size = 15, normalized size = 0.83

$$ix + i \cot(x) + \log(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^2/(I + Tan[x]), x]

[Out] I*x + I*Cot[x] + Log[Sin[x]]

Maple [A] time = 0.033, size = 20, normalized size = 1.1

$$-\ln(i + \tan(x)) + \ln(\tan(x)) + \frac{i}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^2/(I+tan(x)),x)

[Out] -ln(I+tan(x))+ln(tan(x))+I/tan(x)

Maxima [A] time = 1.40432, size = 23, normalized size = 1.28

$$\frac{i}{\tan(x)} - \log(\tan(x) + i) + \log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="maxima")

[Out] I/tan(x) - log(tan(x) + I) + log(tan(x))

Fricas [A] time = 2.02021, size = 78, normalized size = 4.33

$$\frac{(e^{(2i x)} - 1) \log(e^{(2i x)} - 1) - 2}{e^{(2i x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="fricas")

[Out] ((e^(2*I*x) - 1)*log(e^(2*I*x) - 1) - 2)/(e^(2*I*x) - 1)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(x)}{\tan(x) + i} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)**2/(I+tan(x)),x)

[Out] Integral(csc(x)**2/(tan(x) + I), x)

Giac [A] time = 1.32035, size = 24, normalized size = 1.33

$$\frac{i}{\tan(x)} - \log(\tan(x) + i) + \log(|\tan(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(1+tan(x)),x, algorithm="giac")
```

```
[Out] 1/tan(x) - log(tan(x) + 1) + log(abs(tan(x)))
```


3.7 $\int \frac{\csc^3(x)}{i+\tan(x)} dx$

Optimal. Leaf size=24

$$-\csc(x) - \frac{1}{2}i \tanh^{-1}(\cos(x)) + \frac{1}{2}i \cot(x) \csc(x)$$

[Out] $(-I/2)*\text{ArcTanh}[\text{Cos}[x]] - \text{Csc}[x] + (I/2)*\text{Cot}[x]*\text{Csc}[x]$

Rubi [A] time = 0.13226, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3518, 3108, 3107, 2606, 8, 2611, 3770}

$$-\csc(x) - \frac{1}{2}i \tanh^{-1}(\cos(x)) + \frac{1}{2}i \cot(x) \csc(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[x]^3/(I + \text{Tan}[x]), x]$

[Out] $(-I/2)*\text{ArcTanh}[\text{Cos}[x]] - \text{Csc}[x] + (I/2)*\text{Cot}[x]*\text{Csc}[x]$

Rule 3518

$\text{Int}[\sin[(e_.) + (f_.)(x_.)]^{(m_.)} * ((a_.) + (b_.)\tan[(e_.) + (f_.)(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Int}[(\text{Sin}[e + f*x]^{m*(a*\text{Cos}[e + f*x] + b*\text{Sin}[e + f*x])^n} / \text{Cos}[e + f*x]^n, x] /; \text{FreeQ}\{a, b, e, f\}, x] \&\& \text{IntegerQ}[(m - 1)/2] \&\& \text{ILtQ}[n, 0] \&\& ((\text{LtQ}[m, 5] \&\& \text{GtQ}[n, -4]) \|\ (\text{EqQ}[m, 5] \&\& \text{EqQ}[n, -1]))$

Rule 3108

$\text{Int}[\cos[(c_.) + (d_.)(x_.)]^{(m_.)} * \sin[(c_.) + (d_.)(x_.)]^{(n_.)} * (\cos[(c_.) + (d_.)(x_.)] * (a_.) + (b_.)\sin[(c_.) + (d_.)(x_.)])^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[a^p * b^p, \text{Int}[(\text{Cos}[c + d*x]^{m*\text{Sin}[c + d*x]^n} / (b*\text{Cos}[c + d*x] + a*\text{Sin}[c + d*x])^p, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{EqQ}[a^2 + b^2, 0] \&\& \text{ILtQ}[p, 0]$

Rule 3107

$\text{Int}[\cos[(c_.) + (d_.)(x_.)]^{(m_.)} * \sin[(c_.) + (d_.)(x_.)]^{(n_.)} * (\cos[(c_.) + (d_.)(x_.)] * (a_.) + (b_.)\sin[(c_.) + (d_.)(x_.)])^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\cos[c + d*x]^{m*\text{Sin}[c + d*x]^n * (a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^p, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{IGtQ}[p, 0]$

Rule 2606

$\text{Int}[(a_.)\sec[(e_.) + (f_.)(x_.)]^{(m_.)} * ((b_.)\tan[(e_.) + (f_.)(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[a/f, \text{Subst}[\text{Int}[(a*x)^{(m-1)} * (-1 + x^2)^{(n-1)/2}, x], x, \text{Sec}[e + f*x]], x] /; \text{FreeQ}\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n-1)/2] \&\& !(\text{IntegerQ}[m/2] \&\& \text{LtQ}[0, m, n + 1])$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\csc^3(x)}{i + \tan(x)} dx &= \int \frac{\cot(x) \csc^2(x)}{i \cos(x) + \sin(x)} dx \\
 &= -\left(i \int \cot(x) \csc^2(x) (\cos(x) + i \sin(x)) dx \right) \\
 &= -\left(i \int (i \cot(x) \csc(x) + \cot^2(x) \csc(x)) dx \right) \\
 &= -\left(i \int \cot^2(x) \csc(x) dx \right) + \int \cot(x) \csc(x) dx \\
 &= \frac{1}{2} i \cot(x) \csc(x) + \frac{1}{2} i \int \csc(x) dx - \text{Subst}\left(\int 1 dx, x, \csc(x)\right) \\
 &= -\frac{1}{2} i \tanh^{-1}(\cos(x)) - \csc(x) + \frac{1}{2} i \cot(x) \csc(x)
 \end{aligned}$$

Mathematica [B] time = 0.021679, size = 75, normalized size = 3.12

$$-\frac{1}{2} \tan\left(\frac{x}{2}\right) - \frac{1}{2} \cot\left(\frac{x}{2}\right) + \frac{1}{8} i \csc^2\left(\frac{x}{2}\right) - \frac{1}{8} i \sec^2\left(\frac{x}{2}\right) + \frac{1}{2} i \log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{2} i \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(I + Tan[x]), x]
```

```
[Out] -Cot[x/2]/2 + (I/8)*Csc[x/2]^2 - (I/2)*Log[Cos[x/2]] + (I/2)*Log[Sin[x/2]] - (I/8)*Sec[x/2]^2 - Tan[x/2]/2
```

Maple [B] time = 0.039, size = 42, normalized size = 1.8

$$-\frac{1}{2} \tan\left(\frac{x}{2}\right) - \frac{i}{8} \left(\tan\left(\frac{x}{2}\right)\right)^2 + \frac{i}{8} \left(\tan\left(\frac{x}{2}\right)\right)^{-2} - \frac{1}{2} \left(\tan\left(\frac{x}{2}\right)\right)^{-1} + \frac{i}{2} \ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(I+tan(x)), x)
```

```
[Out] -1/2*tan(1/2*x)-1/8*I*tan(1/2*x)^2+1/8*I/tan(1/2*x)^2-1/2/tan(1/2*x)+1/2*I*ln(tan(1/2*x))
```

Maxima [B] time = 1.24577, size = 80, normalized size = 3.33

$$-\frac{\left(\frac{4 \sin(x)}{\cos(x)+1} - i\right)(\cos(x) + 1)^2}{8 \sin(x)^2} - \frac{\sin(x)}{2(\cos(x) + 1)} - \frac{i \sin(x)^2}{8(\cos(x) + 1)^2} + \frac{1}{2} i \log\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="maxima")

[Out] $-1/8*(4*\sin(x)/(\cos(x) + 1) - I)*(\cos(x) + 1)^2/\sin(x)^2 - 1/2*\sin(x)/(\cos(x) + 1) - 1/8*I*\sin(x)^2/(\cos(x) + 1)^2 + 1/2*I*\log(\sin(x)/(\cos(x) + 1))$

Fricas [B] time = 2.13258, size = 232, normalized size = 9.67

$$\frac{(-i e^{4ix} + 2i e^{2ix} - i) \log(e^{ix} + 1) + (i e^{4ix} - 2i e^{2ix} + i) \log(e^{ix} - 1) - 6i e^{3ix} + 2i e^{ix}}{2(e^{4ix} - 2e^{2ix} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="fricas")

[Out] $1/2*((-I*e^{4*I*x} + 2*I*e^{2*I*x} - I)*\log(e^{I*x} + 1) + (I*e^{4*I*x} - 2*I*e^{2*I*x} + I)*\log(e^{I*x} - 1) - 6*I*e^{3*I*x} + 2*I*e^{I*x})/(e^{4*I*x} - 2*e^{2*I*x} + 1)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(x)}{\tan(x) + i} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)**3/(I+tan(x)),x)

[Out] Integral(csc(x)**3/(tan(x) + I), x)

Giac [B] time = 1.3854, size = 63, normalized size = 2.62

$$-\frac{1}{8}i \tan\left(\frac{1}{2}x\right)^2 - \frac{6i \tan\left(\frac{1}{2}x\right)^2 + 4 \tan\left(\frac{1}{2}x\right) - i}{8 \tan\left(\frac{1}{2}x\right)^2} + \frac{1}{2}i \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right) - \frac{1}{2} \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="giac")

[Out] $-1/8*I*\tan(1/2*x)^2 - 1/8*(6*I*\tan(1/2*x)^2 + 4*\tan(1/2*x) - I)/\tan(1/2*x)^2 + 1/2*I*\log(\text{abs}(\tan(1/2*x))) - 1/2*\tan(1/2*x)$

3.8 $\int \frac{\csc^4(x)}{i+\tan(x)} dx$

Optimal. Leaf size=19

$$-\frac{\cot^2(x)}{2} + \frac{1}{3}i \cot^3(x)$$

[Out] $-\text{Cot}[x]^2/2 + (I/3)*\text{Cot}[x]^3$

Rubi [A] time = 0.038832, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3516, 848, 43}

$$-\frac{\cot^2(x)}{2} + \frac{1}{3}i \cot^3(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[x]^4/(I + \text{Tan}[x]), x]$

[Out] $-\text{Cot}[x]^2/2 + (I/3)*\text{Cot}[x]^3$

Rule 3516

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[b/f, \text{Subst}[\text{Int}[(x^m*(a + x)^n)/(b^2 + x^2)^{(m/2 + 1)}, x], x, b*\text{Tan}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 848

$\text{Int}[((d_.) + (e_.)*(x_.))^{(m_.)}*((f_.) + (g_.)*(x_.))^{(n_.)}*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[(d + e*x)^{(m + p)}*(f + g*x)^n*(a/d + (c*x)/e)^p, x] /;$ FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

Rule 43

$\text{Int}[((a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{\csc^4(x)}{i + \tan(x)} dx &= \text{Subst} \left(\int \frac{1 + x^2}{x^4(i + x)} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \frac{-i + x}{x^4} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(-\frac{i}{x^4} + \frac{1}{x^3} \right) dx, x, \tan(x) \right) \\ &= -\frac{1}{2} \cot^2(x) + \frac{1}{3}i \cot^3(x) \end{aligned}$$

Mathematica [A] time = 0.0175136, size = 29, normalized size = 1.53

$$-\frac{1}{3}i \cot(x) - \frac{\csc^2(x)}{2} + \frac{1}{3}i \cot(x) \csc^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^4/(I + Tan[x]), x]

[Out] (-I/3)*Cot[x] - Csc[x]^2/2 + (I/3)*Cot[x]*Csc[x]^2

Maple [A] time = 0.038, size = 15, normalized size = 0.8

$$-\frac{1}{2(\tan(x))^2} + \frac{i}{3(\tan(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^4/(I+tan(x)), x)

[Out] -1/2/tan(x)^2+1/3*I/tan(x)^3

Maxima [A] time = 1.36622, size = 16, normalized size = 0.84

$$-\frac{i(-3i \tan(x) - 2)}{6 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^4/(I+tan(x)), x, algorithm="maxima")

[Out] -1/6*I*(-3*I*tan(x) - 2)/tan(x)^3

Fricas [B] time = 1.99596, size = 109, normalized size = 5.74

$$\frac{2(6e^{4ix} - 3e^{2ix} + 1)}{3(e^{6ix} - 3e^{4ix} + 3e^{2ix} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^4/(I+tan(x)), x, algorithm="fricas")

[Out] 2/3*(6*e^(4*I*x) - 3*e^(2*I*x) + 1)/(e^(6*I*x) - 3*e^(4*I*x) + 3*e^(2*I*x) - 1)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(I+tan(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41731, size = 16, normalized size = 0.84

$$-\frac{3 \tan(x) - 2i}{6 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/6*(3*tan(x) - 2*I)/tan(x)^3
```

3.9 $\int \frac{\csc^5(x)}{i+\tan(x)} dx$

Optimal. Leaf size=40

$$-\frac{\csc^3(x)}{3} - \frac{1}{8}i \tanh^{-1}(\cos(x)) + \frac{1}{4}i \cot(x) \csc^3(x) - \frac{1}{8}i \cot(x) \csc(x)$$

[Out] $(-I/8)*\text{ArcTanh}[\text{Cos}[x]] - (I/8)*\text{Cot}[x]*\text{Csc}[x] - \text{Csc}[x]^3/3 + (I/4)*\text{Cot}[x]*\text{Csc}[x]^3$

Rubi [A] time = 0.150726, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3518, 3108, 3107, 2606, 30, 2611, 3768, 3770}

$$-\frac{\csc^3(x)}{3} - \frac{1}{8}i \tanh^{-1}(\cos(x)) + \frac{1}{4}i \cot(x) \csc^3(x) - \frac{1}{8}i \cot(x) \csc(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[x]^5/(I + \text{Tan}[x]), x]$

[Out] $(-I/8)*\text{ArcTanh}[\text{Cos}[x]] - (I/8)*\text{Cot}[x]*\text{Csc}[x] - \text{Csc}[x]^3/3 + (I/4)*\text{Cot}[x]*\text{Csc}[x]^3$

Rule 3518

$\text{Int}[\sin[(e_.) + (f_.)(x_.)]^{(m_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)(x_.)])^{(n_.)}, x_Symbol] := \text{Int}[(\text{Sin}[e + f*x]^{m*(a*\text{Cos}[e + f*x] + b*\text{Sin}[e + f*x])^n})/\text{Cos}[e + f*x]^n, x] /;$ FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3108

$\text{Int}[\cos[(c_.) + (d_.)(x_.)]^{(m_.)}*\sin[(c_.) + (d_.)(x_.)]^{(n_.)}*(\cos[(c_.) + (d_.)(x_.)]*(a_.) + (b_.)*\sin[(c_.) + (d_.)(x_.)])^{(p_.)}, x_Symbol] := \text{Dist}[a^p*b^p, \text{Int}[(\text{Cos}[c + d*x]^m*\text{Sin}[c + d*x]^n)/(\text{b*Cos}[c + d*x] + \text{a*Sin}[c + d*x])^p, x], x] /;$ FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILtQ[p, 0]

Rule 3107

$\text{Int}[\cos[(c_.) + (d_.)(x_.)]^{(m_.)}*\sin[(c_.) + (d_.)(x_.)]^{(n_.)}*(\cos[(c_.) + (d_.)(x_.)]*(a_.) + (b_.)*\sin[(c_.) + (d_.)(x_.)])^{(p_.)}, x_Symbol] := \text{Int}[\text{ExpandTrig}[\cos[c + d*x]^m*\sin[c + d*x]^n*(\text{a*cos}[c + d*x] + \text{b*sin}[c + d*x])^p, x], x] /;$ FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]

Rule 2606

$\text{Int}[(a_.)*\sec[(e_.) + (f_.)(x_.)]^{(m_.)}*((b_.)*\tan[(e_.) + (f_.)(x_.)])^{(n_.)}, x_Symbol] := \text{Dist}[a/f, \text{Subst}[\text{Int}[(a*x)^{(m-1)}*(-1 + x^2)^{((n-1)/2)}, x], x, \text{Sec}[e + f*x]], x] /;$ FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2611

Int[((a_)*sec[(e_) + (f_)*(x_)]^(m_))*((b_)*tan[(e_) + (f_)*(x_)]^(n_)), x_Symbol] := Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]

Rule 3768

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\csc^5(x)}{i + \tan(x)} dx &= \int \frac{\cot(x) \csc^4(x)}{i \cos(x) + \sin(x)} dx \\
 &= - \left(i \int \cot(x) \csc^4(x) (\cos(x) + i \sin(x)) dx \right) \\
 &= - \left(i \int (i \cot(x) \csc^3(x) + \cot^2(x) \csc^3(x)) dx \right) \\
 &= - \left(i \int \cot^2(x) \csc^3(x) dx \right) + \int \cot(x) \csc^3(x) dx \\
 &= \frac{1}{4} i \cot(x) \csc^3(x) + \frac{1}{4} i \int \csc^3(x) dx - \text{Subst} \left(\int x^2 dx, x, \csc(x) \right) \\
 &= -\frac{1}{8} i \cot(x) \csc(x) - \frac{\csc^3(x)}{3} + \frac{1}{4} i \cot(x) \csc^3(x) + \frac{1}{8} i \int \csc(x) dx \\
 &= -\frac{1}{8} i \tanh^{-1}(\cos(x)) - \frac{1}{8} i \cot(x) \csc(x) - \frac{\csc^3(x)}{3} + \frac{1}{4} i \cot(x) \csc^3(x)
 \end{aligned}$$

Mathematica [B] time = 0.0233137, size = 139, normalized size = 3.48

$$-\frac{1}{12} \tan\left(\frac{x}{2}\right) - \frac{1}{12} \cot\left(\frac{x}{2}\right) + \frac{1}{64} i \csc^4\left(\frac{x}{2}\right) - \frac{1}{32} i \csc^2\left(\frac{x}{2}\right) - \frac{1}{64} i \sec^4\left(\frac{x}{2}\right) + \frac{1}{32} i \sec^2\left(\frac{x}{2}\right) + \frac{1}{8} i \log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{8} i \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^5/(I + Tan[x]), x]

[Out] -Cot[x/2]/12 - (I/32)*Csc[x/2]^2 - (Cot[x/2]*Csc[x/2]^2)/24 + (I/64)*Csc[x/2]^4 - (I/8)*Log[Cos[x/2]] + (I/8)*Log[Sin[x/2]] + (I/32)*Sec[x/2]^2 - (I/64)*Sec[x/2]^4 - Tan[x/2]/12 - (Sec[x/2]^2*Tan[x/2])/24

Maple [A] time = 0.043, size = 58, normalized size = 1.5

$$-\frac{1}{8} \tan\left(\frac{x}{2}\right) - \frac{i}{64} \left(\tan\left(\frac{x}{2}\right)\right)^4 - \frac{1}{24} \left(\tan\left(\frac{x}{2}\right)\right)^3 + \frac{i}{64} \left(\tan\left(\frac{x}{2}\right)\right)^{-4} - \frac{1}{8} \left(\tan\left(\frac{x}{2}\right)\right)^{-1} - \frac{1}{24} \left(\tan\left(\frac{x}{2}\right)\right)^{-3} + \frac{i}{8} \ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(x)^5/(I+tan(x)),x)`

[Out] $-1/8*\tan(1/2*x)-1/64*I*\tan(1/2*x)^4-1/24*\tan(1/2*x)^3+1/64*I/\tan(1/2*x)^4-1/8/\tan(1/2*x)-1/24/\tan(1/2*x)^3+1/8*I*\ln(\tan(1/2*x))$

Maxima [B] time = 1.30085, size = 112, normalized size = 2.8

$$\frac{\left(\frac{8 \sin(x)}{\cos(x)+1} + \frac{24 \sin(x)^3}{(\cos(x)+1)^3} - 3i\right)(\cos(x) + 1)^4}{192 \sin(x)^4} - \frac{\sin(x)}{8(\cos(x) + 1)} - \frac{\sin(x)^3}{24(\cos(x) + 1)^3} - \frac{i \sin(x)^4}{64(\cos(x) + 1)^4} + \frac{1}{8}i \log\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)^5/(I+tan(x)),x, algorithm="maxima")`

[Out] $-1/192*(8*\sin(x)/(\cos(x) + 1) + 24*\sin(x)^3/(\cos(x) + 1)^3 - 3*I)*(\cos(x) + 1)^4/\sin(x)^4 - 1/8*\sin(x)/(\cos(x) + 1) - 1/24*\sin(x)^3/(\cos(x) + 1)^3 - 1/64*I*\sin(x)^4/(\cos(x) + 1)^4 + 1/8*I*\log(\sin(x)/(\cos(x) + 1))$

Fricas [B] time = 2.13878, size = 424, normalized size = 10.6

$$\frac{(-3ie^{8ix} + 12ie^{6ix} - 18ie^{4ix} + 12ie^{2ix} - 3i) \log(e^{ix} + 1) + (3ie^{8ix} - 12ie^{6ix} + 18ie^{4ix} - 12ie^{2ix} + 3i) \log(e^{ix} - 1)}{24(e^{8ix} - 4e^{6ix} + 6e^{4ix} - 4e^{2ix} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)^5/(I+tan(x)),x, algorithm="fricas")`

[Out] $1/24*((-3*I*e^{(8*I*x)} + 12*I*e^{(6*I*x)} - 18*I*e^{(4*I*x)} + 12*I*e^{(2*I*x)} - 3*I)*\log(e^{(I*x)} + 1) + (3*I*e^{(8*I*x)} - 12*I*e^{(6*I*x)} + 18*I*e^{(4*I*x)} - 12*I*e^{(2*I*x)} + 3*I)*\log(e^{(I*x)} - 1) + 6*I*e^{(7*I*x)} + 106*I*e^{(5*I*x)} - 22*I*e^{(3*I*x)} + 6*I*e^{(I*x)})/(e^{(8*I*x)} - 4*e^{(6*I*x)} + 6*e^{(4*I*x)} - 4*e^{(2*I*x)} + 1)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)**5/(I+tan(x)),x)`

[Out] Timed out

Giac [B] time = 1.40169, size = 85, normalized size = 2.12

$$-\frac{1}{64}i \tan\left(\frac{1}{2}x\right)^4 - \frac{1}{24} \tan\left(\frac{1}{2}x\right)^3 - \frac{50i \tan\left(\frac{1}{2}x\right)^4 + 24 \tan\left(\frac{1}{2}x\right)^3 + 8 \tan\left(\frac{1}{2}x\right) - 3i}{192 \tan\left(\frac{1}{2}x\right)^4} + \frac{1}{8}i \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right) - \frac{1}{8} \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/64*I*tan(1/2*x)^4 - 1/24*tan(1/2*x)^3 - 1/192*(50*I*tan(1/2*x)^4 + 24*tan(1/2*x)^3 + 8*tan(1/2*x) - 3*I)/tan(1/2*x)^4 + 1/8*I*log(abs(tan(1/2*x))) - 1/8*tan(1/2*x)
```

3.10 $\int \frac{\csc^6(x)}{i+\tan(x)} dx$

Optimal. Leaf size=37

$$\frac{1}{5}i \cot^5(x) - \frac{\cot^4(x)}{4} + \frac{1}{3}i \cot^3(x) - \frac{\cot^2(x)}{2}$$

[Out] $-\text{Cot}[x]^2/2 + (I/3)*\text{Cot}[x]^3 - \text{Cot}[x]^4/4 + (I/5)*\text{Cot}[x]^5$

Rubi [A] time = 0.0469125, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3516, 848, 75}

$$\frac{1}{5}i \cot^5(x) - \frac{\cot^4(x)}{4} + \frac{1}{3}i \cot^3(x) - \frac{\cot^2(x)}{2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[x]^6/(I + \text{Tan}[x]), x]$

[Out] $-\text{Cot}[x]^2/2 + (I/3)*\text{Cot}[x]^3 - \text{Cot}[x]^4/4 + (I/5)*\text{Cot}[x]^5$

Rule 3516

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] :> \text{Dist}[b/f, \text{Subst}[\text{Int}[(x^m*(a + x)^n)/(b^2 + x^2)^{(m/2 + 1)}], x], x, b*\text{Tan}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 848

$\text{Int}[(d_. + (e_.)*(x_.))^{(m_.)*((f_.) + (g_.)*(x_.))^{(n_.)*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] :> \text{Int}[(d + e*x)^{(m + p)}*(f + g*x)^n*(a/d + (c*x)/e)^p, x] /;$ FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

Rule 75

$\text{Int}[(d_.)*(x_.)^{(n_.)*((a_.) + (b_.)*(x_.))*((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /;$ FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p + 2, 0] && GtQ[n + 2*p, 0])

Rubi steps

$$\begin{aligned} \int \frac{\csc^6(x)}{i+\tan(x)} dx &= \text{Subst} \left(\int \frac{(1+x^2)^2}{x^6(i+x)} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \frac{(-i+x)^2(i+x)}{x^6} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(-\frac{i}{x^6} + \frac{1}{x^5} - \frac{i}{x^4} + \frac{1}{x^3} \right) dx, x, \tan(x) \right) \\ &= -\frac{1}{2} \cot^2(x) + \frac{1}{3}i \cot^3(x) - \frac{\cot^4(x)}{4} + \frac{1}{5}i \cot^5(x) \end{aligned}$$

Mathematica [A] time = 0.018521, size = 41, normalized size = 1.11

$$-\frac{2}{15}i \cot(x) - \frac{\csc^4(x)}{4} + \frac{1}{5}i \cot(x) \csc^4(x) - \frac{1}{15}i \cot(x) \csc^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^6/(I + Tan[x]), x]

[Out] ((-2*I)/15)*Cot[x] - (I/15)*Cot[x]*Csc[x]^2 - Csc[x]^4/4 + (I/5)*Cot[x]*Csc[x]^4

Maple [A] time = 0.041, size = 28, normalized size = 0.8

$$-\frac{1}{4(\tan(x))^4} - \frac{1}{2(\tan(x))^2} + \frac{\frac{i}{3}}{(\tan(x))^3} + \frac{\frac{i}{5}}{(\tan(x))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^6/(I+tan(x)), x)

[Out] -1/4/tan(x)^4-1/2/tan(x)^2+1/3*I/tan(x)^3+1/5*I/tan(x)^5

Maxima [A] time = 1.36274, size = 32, normalized size = 0.86

$$\frac{i(30i \tan(x)^3 + 20 \tan(x)^2 + 15i \tan(x) + 12)}{60 \tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^6/(I+tan(x)), x, algorithm="maxima")

[Out] 1/60*I*(30*I*tan(x)^3 + 20*tan(x)^2 + 15*I*tan(x) + 12)/tan(x)^5

Fricas [B] time = 1.98741, size = 176, normalized size = 4.76

$$-\frac{4(30e^{6ix} - 10e^{4ix} + 5e^{2ix} - 1)}{15(e^{10ix} - 5e^{8ix} + 10e^{6ix} - 10e^{4ix} + 5e^{2ix} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^6/(I+tan(x)), x, algorithm="fricas")

[Out] -4/15*(30*e^(6*I*x) - 10*e^(4*I*x) + 5*e^(2*I*x) - 1)/(e^(10*I*x) - 5*e^(8*I*x) + 10*e^(6*I*x) - 10*e^(4*I*x) + 5*e^(2*I*x) - 1)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)**6/(I+tan(x)),x)

[Out] Timed out

Giac [A] time = 1.38777, size = 32, normalized size = 0.86

$$-\frac{30 \tan(x)^3 - 20i \tan(x)^2 + 15 \tan(x) - 12i}{60 \tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^6/(I+tan(x)),x, algorithm="giac")

[Out] -1/60*(30*tan(x)^3 - 20*I*tan(x)^2 + 15*tan(x) - 12*I)/tan(x)^5

3.11 $\int \sin^5(c + dx)(a + b \tan(c + dx)) dx$

Optimal. Leaf size=101

$$-\frac{a \cos^5(c + dx)}{5d} + \frac{2a \cos^3(c + dx)}{3d} - \frac{a \cos(c + dx)}{d} - \frac{b \sin^5(c + dx)}{5d} - \frac{b \sin^3(c + dx)}{3d} - \frac{b \sin(c + dx)}{d} + \frac{b \tanh^{-1}(\sin(c + dx))}{d}$$

[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (2*a*Cos[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]^5)/(5*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) - (b*Sin[c + d*x]^5)/(5*d)

Rubi [A] time = 0.0764194, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3517, 2633, 2592, 302, 206}

$$-\frac{a \cos^5(c + dx)}{5d} + \frac{2a \cos^3(c + dx)}{3d} - \frac{a \cos(c + dx)}{d} - \frac{b \sin^5(c + dx)}{5d} - \frac{b \sin^3(c + dx)}{3d} - \frac{b \sin(c + dx)}{d} + \frac{b \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^5*(a + b*Tan[c + d*x]),x]

[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (2*a*Cos[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]^5)/(5*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) - (b*Sin[c + d*x]^5)/(5*d)

Rule 3517

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \sin^5(c+dx)(a+b\tan(c+dx))dx &= \int (a\sin^5(c+dx)+b\sin^5(c+dx)\tan(c+dx))dx \\
&= a\int \sin^5(c+dx)dx+b\int \sin^5(c+dx)\tan(c+dx)dx \\
&= -\frac{a\operatorname{Subst}\left(\int(1-2x^2+x^4)dx,x,\cos(c+dx)\right)}{d}+\frac{b\operatorname{Subst}\left(\int\frac{x^6}{1-x^2}dx,x,\sin(c+dx)\right)}{d} \\
&= -\frac{a\cos(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos^5(c+dx)}{5d}+\frac{b\operatorname{Subst}\left(\int(-1-x^2-x^4)dx,x,\sin(c+dx)\right)}{d} \\
&= -\frac{a\cos(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d} \\
&= \frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cos(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0317254, size = 103, normalized size = 1.02

$$-\frac{5a\cos(c+dx)}{8d}+\frac{5a\cos(3(c+dx))}{48d}-\frac{a\cos(5(c+dx))}{80d}-\frac{b\sin^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^5*(a + b*Tan[c + d*x]),x]

[Out] (b*ArcTanh[Sin[c + d*x]])/d - (5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*x)])/ (48*d) - (a*Cos[5*(c + d*x)])/(80*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) - (b*Sin[c + d*x]^5)/(5*d)

Maple [A] time = 0.041, size = 113, normalized size = 1.1

$$-\frac{b(\sin(dx+c))^5}{5d}-\frac{b(\sin(dx+c))^3}{3d}-\frac{b\sin(dx+c)}{d}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{8a\cos(dx+c)}{15d}-\frac{a\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^5*(a+b*tan(d*x+c)),x)

[Out] -1/5*b*sin(d*x+c)^5/d-1/3*b*sin(d*x+c)^3/d-b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))-8/15*a*cos(d*x+c)/d-1/5/d*a*cos(d*x+c)*sin(d*x+c)^4-4/15/d*a*cos(d*x+c)*sin(d*x+c)^2

Maxima [A] time = 1.38921, size = 123, normalized size = 1.22

$$\frac{2(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c))a+(6\sin(dx+c)^5+10\sin(dx+c)^3-15\log(\sin(dx+c)))b}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] $-1/30*(2*(3*\cos(dx + c)^5 - 10*\cos(dx + c)^3 + 15*\cos(dx + c))*a + (6*\sin(dx + c)^5 + 10*\sin(dx + c)^3 - 15*\log(\sin(dx + c) + 1) + 15*\log(\sin(dx + c) - 1) + 30*\sin(dx + c))*b)/d$

Fricas [A] time = 2.27656, size = 267, normalized size = 2.64

$$\frac{6 a \cos(dx + c)^5 - 20 a \cos(dx + c)^3 + 30 a \cos(dx + c) - 15 b \log(\sin(dx + c) + 1) + 15 b \log(-\sin(dx + c) + 1) + 2(3 b \cos(dx + c)^5 - 10 b \cos(dx + c)^3 + 15 b \cos(dx + c))}{30 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out] $-1/30*(6*a*\cos(dx + c)^5 - 20*a*\cos(dx + c)^3 + 30*a*\cos(dx + c) - 15*b*\log(\sin(dx + c) + 1) + 15*b*\log(-\sin(dx + c) + 1) + 2*(3*b*\cos(dx + c)^4 - 11*b*\cos(dx + c)^2 + 23*b)*\sin(dx + c))/d$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)**5*(a+b*tan(d*x+c)),x)`

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")`

[Out] Timed out

3.12 $\int \sin^4(c + dx)(a + b \tan(c + dx)) dx$

Optimal. Leaf size=83

$$\frac{\sin^3(c + dx) \cos(c + dx)(a + b \tan(c + dx))}{4d} - \frac{\sin(c + dx) \cos(c + dx)(3a + 4b \tan(c + dx))}{8d} + \frac{3ax}{8} - \frac{b \log(\cos(c + dx))}{d}$$

[Out] (3*a*x)/8 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Tan[c + d*x]))/(4*d) - (Cos[c + d*x]*Sin[c + d*x]*(3*a + 4*b*Tan[c + d*x]))/(8*d)

Rubi [A] time = 0.168075, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {819, 635, 203, 260}

$$\frac{\sin^3(c + dx) \cos(c + dx)(a + b \tan(c + dx))}{4d} - \frac{\sin(c + dx) \cos(c + dx)(3a + 4b \tan(c + dx))}{8d} + \frac{3ax}{8} - \frac{b \log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x]),x]

[Out] (3*a*x)/8 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Tan[c + d*x]))/(4*d) - (Cos[c + d*x]*Sin[c + d*x]*(3*a + 4*b*Tan[c + d*x]))/(8*d)

Rule 819

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) || !ILtQ[m + 2*p + 3, 0])

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
\int \sin^4(c+dx)(a+b \tan(c+dx)) dx &= \frac{\text{Subst}\left(\int \frac{x^{4(a+bx)}}{(1+x^2)^3} dx, x, \tan(c+dx)\right)}{d} \\
&= -\frac{\cos(c+dx) \sin^3(c+dx)(a+b \tan(c+dx))}{4d} + \frac{\text{Subst}\left(\int \frac{x^{2(3a+4bx)}}{(1+x^2)^2} dx, x, \tan(c+dx)\right)}{4d} \\
&= -\frac{\cos(c+dx) \sin^3(c+dx)(a+b \tan(c+dx))}{4d} - \frac{\cos(c+dx) \sin(c+dx)(3a+4b \tan(c+dx))}{8d} \\
&= -\frac{\cos(c+dx) \sin^3(c+dx)(a+b \tan(c+dx))}{4d} - \frac{\cos(c+dx) \sin(c+dx)(3a+4b \tan(c+dx))}{8d} \\
&= \frac{3ax}{8} - \frac{b \log(\cos(c+dx))}{d} - \frac{\cos(c+dx) \sin^3(c+dx)(a+b \tan(c+dx))}{4d} - \frac{\cos(c+dx) \sin(c+dx)(3a+4b \tan(c+dx))}{8d}
\end{aligned}$$

Mathematica [A] time = 0.078587, size = 82, normalized size = 0.99

$$\frac{3a(c+dx)}{8d} - \frac{a \sin(2(c+dx))}{4d} + \frac{a \sin(4(c+dx))}{32d} - \frac{b\left(\frac{1}{4} \cos^4(c+dx) - \cos^2(c+dx) + \log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x]), x]

[Out] (3*a*(c + d*x))/(8*d) - (b*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c + d*x]]))/d - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)

Maple [A] time = 0.033, size = 92, normalized size = 1.1

$$-\frac{b(\sin(dx+c))^4}{4d} - \frac{b(\sin(dx+c))^2}{2d} - \frac{b \ln(\cos(dx+c))}{d} - \frac{a \cos(dx+c)(\sin(dx+c))^3}{4d} - \frac{3a \sin(dx+c) \cos(dx+c)}{8d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c)), x)

[Out] -1/4/d*b*sin(d*x+c)^4-1/2/d*b*sin(d*x+c)^2-b*ln(cos(d*x+c))/d-1/4/d*a*cos(d*x+c)*sin(d*x+c)^3-3/8/d*a*sin(d*x+c)*cos(d*x+c)+3/8*a*x+3/8/d*a*c

Maxima [A] time = 2.15915, size = 117, normalized size = 1.41

$$\frac{3(dx+c)a + 4b \log(\tan(dx+c)^2 + 1) - \frac{5a \tan(dx+c)^3 - 8b \tan(dx+c)^2 + 3a \tan(dx+c) - 6b}{\tan(dx+c)^4 + 2 \tan(dx+c)^2 + 1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)), x, algorithm="maxima")

[Out] 1/8*(3*(d*x + c)*a + 4*b*log(tan(d*x + c)^2 + 1) - (5*a*tan(d*x + c)^3 - 8*b*tan(d*x + c)^2 + 3*a*tan(d*x + c) - 6*b)/(tan(d*x + c)^4 + 2*tan(d*x + c)

$\wedge 2 + 1)) / d$

Fricas [A] time = 2.35933, size = 189, normalized size = 2.28

$$\frac{2b \cos(dx + c)^4 - 3adx - 8b \cos(dx + c)^2 + 8b \log(-\cos(dx + c)) - (2a \cos(dx + c)^3 - 5a \cos(dx + c)) \sin(dx + c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] $-1/8*(2*b*\cos(d*x + c)^4 - 3*a*d*x - 8*b*\cos(d*x + c)^2 + 8*b*\log(-\cos(d*x + c)) - (2*a*\cos(d*x + c)^3 - 5*a*\cos(d*x + c))*\sin(d*x + c))/d$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx)) \sin^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c)),x)

[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**4, x)

Giac [B] time = 1.69996, size = 1439, normalized size = 17.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] $1/32*(12*a*d*x*\tan(d*x)^4*\tan(c)^4 - 16*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^4*\tan(c)^4 + 24*a*d*x*\tan(d*x)^4*\tan(c)^2 + 24*a*d*x*\tan(d*x)^2*\tan(c)^4 + 11*b*\tan(d*x)^4*\tan(c)^4 - 32*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^4*\tan(c)^2 + 12*a*\tan(d*x)^4*\tan(c)^3 - 32*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(c)^4 + 12*a*\tan(d*x)^3*\tan(c)^4 + 12*a*d*x*\tan(d*x)^4 + 48*a*d*x*\tan(d*x)^2*\tan(c)^2 + 6*b*\tan(d*x)^4*\tan(c)^2 - 32*b*\tan(d*x)^3*\tan(c)^3 + 12*a*d*x*\tan(c)^4 + 6*b*\tan(d*x)^2*\tan(c)^4 - 16*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^4 + 20*a*\tan(d*x)^4*\tan(c) - 64*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(c)^2 + 24*a*\tan(d*x)^3*\tan(c)^2 + 24*a*\tan(d*x)^2*\tan(c)^3 - 16*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(c)^4 + 20*a*\tan(d*x)*\tan(c)^4 + 24*a*d*x*\tan(d*x)^2 - 13*b*\tan(d*x)^4 - 64*b*\tan(d*x)^3*\tan(c) + 24*a*d*x*\tan(c)^2 - 36*b*\tan(d*x$

$$\begin{aligned}
&)^2 \tan(c)^2 - 64b \tan(dx) \tan(c)^3 - 13b \tan(c)^4 - 32b \log(4(\tan(c)^2 + 1) / (\tan(dx)^4 \tan(c)^2 - 2 \tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1)) \tan(dx)^2 - 20a \tan(dx)^3 - 24a \tan(dx)^2 \tan(c) - 32b \log(4(\tan(c)^2 + 1) / (\tan(dx)^4 \tan(c)^2 - 2 \tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1)) \tan(c)^2 - 24a \tan(dx) \tan(c)^2 - 20a \tan(c)^3 + 12a dx + 6b \tan(dx)^2 - 32b \tan(dx) \tan(c) + 6b \tan(c)^2 - 16b \log(4(\tan(c)^2 + 1) / (\tan(dx)^4 \tan(c)^2 - 2 \tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1)) - 12a \tan(dx) - 12a \tan(c) + 11b) / (d \tan(dx)^4 \tan(c)^4 + 2d \tan(dx)^4 \tan(c)^2 + 2d \tan(dx)^2 \tan(c)^4 + d \tan(dx)^4 + 4d \tan(dx)^2 \tan(c)^2 + d \tan(c)^4 + 2d \tan(dx)^2 + 2d \tan(c)^2 + d)
\end{aligned}$$

3.13 $\int \sin^3(c + dx)(a + b \tan(c + dx)) dx$

Optimal. Leaf size=69

$$\frac{a \cos^3(c + dx)}{3d} - \frac{a \cos(c + dx)}{d} - \frac{b \sin^3(c + dx)}{3d} - \frac{b \sin(c + dx)}{d} + \frac{b \tanh^{-1}(\sin(c + dx))}{d}$$

[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)

Rubi [A] time = 0.0679413, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3517, 2633, 2592, 302, 206}

$$\frac{a \cos^3(c + dx)}{3d} - \frac{a \cos(c + dx)}{d} - \frac{b \sin^3(c + dx)}{3d} - \frac{b \sin(c + dx)}{d} + \frac{b \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x]),x]

[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)

Rule 3517

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^(n + 1)/2], x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \sin^3(c+dx)(a+b \tan(c+dx)) dx &= \int (a \sin^3(c+dx) + b \sin^3(c+dx) \tan(c+dx)) dx \\
&= a \int \sin^3(c+dx) dx + b \int \sin^3(c+dx) \tan(c+dx) dx \\
&= -\frac{a \operatorname{Subst}\left(\int (1-x^2) dx, x, \cos(c+dx)\right)}{d} + \frac{b \operatorname{Subst}\left(\int \frac{x^4}{1-x^2} dx, x, \sin(c+dx)\right)}{d} \\
&= -\frac{a \cos(c+dx)}{d} + \frac{a \cos^3(c+dx)}{3d} + \frac{b \operatorname{Subst}\left(\int \left(-1-x^2 + \frac{1}{1-x^2}\right) dx, x, \sin(c+dx)\right)}{d} \\
&= -\frac{a \cos(c+dx)}{d} + \frac{a \cos^3(c+dx)}{3d} - \frac{b \sin(c+dx)}{d} - \frac{b \sin^3(c+dx)}{3d} + \frac{b \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sin(c+dx)\right)}{d} \\
&= \frac{b \tanh^{-1}(\sin(c+dx))}{d} - \frac{a \cos(c+dx)}{d} + \frac{a \cos^3(c+dx)}{3d} - \frac{b \sin(c+dx)}{d} - \frac{b \sin^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0267371, size = 71, normalized size = 1.03

$$-\frac{3a \cos(c+dx)}{4d} + \frac{a \cos(3(c+dx))}{12d} - \frac{b \sin^3(c+dx)}{3d} - \frac{b \sin(c+dx)}{d} + \frac{b \tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x]), x]

[Out] (b*ArcTanh[Sin[c + d*x]])/d - (3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/ (12*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)

Maple [A] time = 0.035, size = 79, normalized size = 1.1

$$-\frac{b(\sin(dx+c))^3}{3d} - \frac{b \sin(dx+c)}{d} + \frac{b \ln(\sec(dx+c) + \tan(dx+c))}{d} - \frac{a \cos(dx+c)(\sin(dx+c))^2}{3d} - \frac{2a \cos(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c)), x)

[Out] -1/3*b*sin(d*x+c)^3/d - b*sin(d*x+c)/d + 1/d*b*ln(sec(d*x+c)+tan(d*x+c)) - 1/3/d*a*cos(d*x+c)*sin(d*x+c)^2 - 2/3*a*cos(d*x+c)/d

Maxima [A] time = 1.36148, size = 95, normalized size = 1.38

$$\frac{2(\cos(dx+c)^3 - 3 \cos(dx+c))a - (2 \sin(dx+c)^3 - 3 \log(\sin(dx+c) + 1) + 3 \log(\sin(dx+c) - 1) + 6 \sin(dx+c))b}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)), x, algorithm="maxima")

[Out] 1/6*(2*(cos(d*x + c)^3 - 3*cos(d*x + c))*a - (2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*b)/d

Fricas [A] time = 2.30315, size = 197, normalized size = 2.86

$$\frac{2 a \cos (d x+c)^3-6 a \cos (d x+c)+3 b \log (\sin (d x+c)+1)-3 b \log (-\sin (d x+c)+1)+2\left(b \cos (d x+c)^2-4 b\right) s}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] 1/6*(2*a*cos(d*x + c)^3 - 6*a*cos(d*x + c) + 3*b*log(sin(d*x + c) + 1) - 3*b*log(-sin(d*x + c) + 1) + 2*(b*cos(d*x + c)^2 - 4*b)*sin(d*x + c))/d

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan (c + d x)) \sin ^3 (c + d x) d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c)),x)

[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**3, x)

Giac [B] time = 2.78391, size = 7223, normalized size = 104.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] -1/6*(3*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 3*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 4*a*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 12*b*tan(1/2*d*x)^6*tan(1/2*c)^5 + 9*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1

$$\begin{aligned}
&^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(\\
&1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) \\
&+ 1))*\tan(1/2*c)^2 + 60*b*\tan(1/2*d*x)*\tan(1/2*c)^2 + 40*b*\tan(1/2*c)^3 + 1 \\
&2*a*\tan(1/2*d*x)^2 + 12*a*\tan(1/2*c)^2 + 3*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(\\
&1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^3*\tan \\
&(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2*d*x) \\
&)^3 + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan \\
&(1/2*d*x) - 2*\tan(1/2*c) + 1)) - 3*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x) \\
&)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/2*c) \\
&^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2* \\
&\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) \\
&+ 2*\tan(1/2*c) + 1)) + 12*b*\tan(1/2*d*x) + 12*b*\tan(1/2*c) + 4*a)/(d*\tan(\\
&1/2*d*x)^6*\tan(1/2*c)^6 + 3*d*\tan(1/2*d*x)^6*\tan(1/2*c)^4 + 3*d*\tan(1/2*d*x) \\
&)^4*\tan(1/2*c)^6 + 3*d*\tan(1/2*d*x)^6*\tan(1/2*c)^2 + 9*d*\tan(1/2*d*x)^4*\tan \\
&(1/2*c)^4 + 3*d*\tan(1/2*d*x)^2*\tan(1/2*c)^6 + d*\tan(1/2*d*x)^6 + 9*d*\tan(1/ \\
&2*d*x)^4*\tan(1/2*c)^2 + 9*d*\tan(1/2*d*x)^2*\tan(1/2*c)^4 + d*\tan(1/2*c)^6 + \\
&3*d*\tan(1/2*d*x)^4 + 9*d*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 3*d*\tan(1/2*c)^4 + 3 \\
&*d*\tan(1/2*d*x)^2 + 3*d*\tan(1/2*c)^2 + d)
\end{aligned}$$

3.14 $\int \sin^2(c + dx)(a + b \tan(c + dx)) dx$

Optimal. Leaf size=49

$$-\frac{\sin(c + dx) \cos(c + dx)(a + b \tan(c + dx))}{2d} + \frac{ax}{2} - \frac{b \log(\cos(c + dx))}{d}$$

[Out] (a*x)/2 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c + d*x]))/(2*d)

Rubi [A] time = 0.0844788, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {819, 635, 203, 260}

$$-\frac{\sin(c + dx) \cos(c + dx)(a + b \tan(c + dx))}{2d} + \frac{ax}{2} - \frac{b \log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x]),x]

[Out] (a*x)/2 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c + d*x]))/(2*d)

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) || !ILtQ[m + 2*p + 3, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \sin^2(c + dx)(a + b \tan(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{x^{2(a+bx)}}{(1+x^2)^2} dx, x, \tan(c + dx)\right)}{d} \\
&= -\frac{\cos(c + dx) \sin(c + dx)(a + b \tan(c + dx))}{2d} + \frac{\text{Subst}\left(\int \frac{a+2bx}{1+x^2} dx, x, \tan(c + dx)\right)}{2d} \\
&= -\frac{\cos(c + dx) \sin(c + dx)(a + b \tan(c + dx))}{2d} + \frac{a \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \tan(c + dx)\right)}{2d} \\
&= \frac{ax}{2} - \frac{b \log(\cos(c + dx))}{d} - \frac{\cos(c + dx) \sin(c + dx)(a + b \tan(c + dx))}{2d}
\end{aligned}$$

Mathematica [A] time = 0.0480753, size = 56, normalized size = 1.14

$$\frac{a(c + dx)}{2d} - \frac{a \sin(2(c + dx))}{4d} - \frac{b \left(\log(\cos(c + dx)) - \frac{1}{2} \cos^2(c + dx) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x]),x]

[Out] (a*(c + d*x))/(2*d) - (b*(-Cos[c + d*x]^2/2 + Log[Cos[c + d*x]]))/d - (a*Sin[2*(c + d*x)])/(4*d)

Maple [A] time = 0.035, size = 58, normalized size = 1.2

$$\frac{b(\sin(dx + c))^2}{2d} - \frac{b \ln(\cos(dx + c))}{d} - \frac{a \sin(dx + c) \cos(dx + c)}{2d} + \frac{ax}{2} + \frac{ac}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c)),x)

[Out] -1/2/d*b*sin(d*x+c)^2-b*ln(cos(d*x+c))/d-1/2/d*a*sin(d*x+c)*cos(d*x+c)+1/2*a*x+1/2/d*a*c

Maxima [A] time = 2.51148, size = 70, normalized size = 1.43

$$\frac{(dx + c)a + b \log(\tan(dx + c)^2 + 1) - \frac{a \tan(dx + c) - b}{\tan(dx + c)^2 + 1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] 1/2*((d*x + c)*a + b*log(tan(d*x + c)^2 + 1) - (a*tan(d*x + c) - b)/(tan(d*x + c)^2 + 1))/d

Fricas [A] time = 2.25526, size = 120, normalized size = 2.45

$$\frac{adx + b \cos(dx + c)^2 - a \cos(dx + c) \sin(dx + c) - 2b \log(-\cos(dx + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(a*d*x + b*cos(d*x + c)^2 - a*cos(d*x + c)*sin(d*x + c) - 2*b*log(-cos(d*x + c)))/d

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx)) \sin^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c)),x)

[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**2, x)

Giac [B] time = 1.43679, size = 558, normalized size = 11.39

$$2 adx \tan(dx)^2 \tan(c)^2 - 2b \log\left(\frac{4(\tan(c)^2+1)}{\tan(dx)^4 \tan(c)^2 - 2 \tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1}\right) \tan(dx)^2 \tan(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] 1/4*(2*a*d*x*tan(d*x)^2*tan(c)^2 - 2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 2*a*d*x*tan(d*x)^2 + 2*a*d*x*tan(c)^2 + b*tan(d*x)^2*tan(c)^2 - 2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2 + 2*a*tan(d*x)^2*tan(c) - 2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(c)^2 + 2*a*tan(d*x)*tan(c)^2 + 2*a*d*x - b*tan(d*x)^2 - 4*b*tan(d*x)*tan(c) - b*tan(c)^2 - 2*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) - 2*a*tan(d*x) - 2*a*tan(c) + b)/(d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)

3.15 $\int \sin(c + dx)(a + b \tan(c + dx)) dx$

Optimal. Leaf size=37

$$-\frac{a \cos(c + dx)}{d} - \frac{b \sin(c + dx)}{d} + \frac{b \tanh^{-1}(\sin(c + dx))}{d}$$

[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d - (b*Sin[c + d*x])/d

Rubi [A] time = 0.0357625, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3517, 2638, 2592, 321, 206}

$$-\frac{a \cos(c + dx)}{d} - \frac{b \sin(c + dx)}{d} + \frac{b \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x]),x]

[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d - (b*Sin[c + d*x])/d

Rule 3517

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \sin(c + dx)(a + b \tan(c + dx)) dx &= \int (a \sin(c + dx) + b \sin(c + dx) \tan(c + dx)) dx \\
&= a \int \sin(c + dx) dx + b \int \sin(c + dx) \tan(c + dx) dx \\
&= -\frac{a \cos(c + dx)}{d} + \frac{b \operatorname{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, \sin(c + dx)\right)}{d} \\
&= -\frac{a \cos(c + dx)}{d} - \frac{b \sin(c + dx)}{d} + \frac{b \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sin(c + dx)\right)}{d} \\
&= \frac{b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a \cos(c + dx)}{d} - \frac{b \sin(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.019747, size = 48, normalized size = 1.3

$$\frac{a \sin(c) \sin(dx)}{d} - \frac{a \cos(c) \cos(dx)}{d} - \frac{b \sin(c + dx)}{d} + \frac{b \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x]),x]

[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c]*Cos[d*x])/d + (a*Sin[c]*Sin[d*x])/d - (b*Sin[c + d*x])/d

Maple [A] time = 0.027, size = 45, normalized size = 1.2

$$-\frac{a \cos(dx + c)}{d} - \frac{b \sin(dx + c)}{d} + \frac{b \ln(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)*(a+b*tan(d*x+c)),x)

[Out] -a*cos(d*x+c)/d-b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))

Maxima [A] time = 1.52106, size = 62, normalized size = 1.68

$$\frac{b(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1) - 2 \sin(dx + c)) - 2 a \cos(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 2*a*cos(d*x + c))/d

Fricas [A] time = 2.29999, size = 134, normalized size = 3.62

$$-\frac{2 a \cos(dx + c) - b \log(\sin(dx + c) + 1) + b \log(-\sin(dx + c) + 1) + 2 b \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cos(d*x + c) - b*log(sin(d*x + c) + 1) + b*log(-sin(d*x + c) + 1)
+ 2*b*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx)) \sin(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x), x)
```

Giac [B] time = 1.65799, size = 1669, normalized size = 45.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*tan(1/2*d*x)^2*tan(1/2*c)^2 + b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2 - b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2 - 4*b*tan(1/2*d*x)^2*tan(1/2*c) + b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*c)^2 - b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*c)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a*tan(1/2*d*x)^2 - 8*a*tan(1/2*d*x)*tan(1/2*c) - 2*a*tan(1/2*c)^2 + b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
```



```

*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)) - b*log(2*(
tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c
) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)) + 4*b*tan(1/2*d*x) +
4*b*tan(1/2*c) + 2*a)/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2 + d
*tan(1/2*c)^2 + d)

```

3.16 $\int \csc(c + dx)(a + b \tan(c + dx)) dx$

Optimal. Leaf size=26

$$\frac{b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a \tanh^{-1}(\cos(c + dx))}{d}$$

[Out] $-\left(\frac{a \operatorname{ArcTanh}[\cos[c + d*x]]}{d}\right) + \left(\frac{b \operatorname{ArcTanh}[\sin[c + d*x]]}{d}\right)$

Rubi [A] time = 0.029023, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3517, 3770}

$$\frac{b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a \tanh^{-1}(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c + d*x]*(a + b*\text{Tan}[c + d*x]),x]$

[Out] $-\left(\frac{a \operatorname{ArcTanh}[\cos[c + d*x]]}{d}\right) + \left(\frac{b \operatorname{ArcTanh}[\sin[c + d*x]]}{d}\right)$

Rule 3517

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{Expand}[\sin[e + f*x]^{m*(a + b*\text{Tan}[e + f*x])^n}, x], x] /;$ FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]

Rule 3770

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\cos[c + d*x]]/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \csc(c + dx)(a + b \tan(c + dx)) dx &= \int (a \csc(c + dx) + b \sec(c + dx)) dx \\ &= a \int \csc(c + dx) dx + b \int \sec(c + dx) dx \\ &= -\frac{a \tanh^{-1}(\cos(c + dx))}{d} + \frac{b \tanh^{-1}(\sin(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.0237081, size = 52, normalized size = 2.

$$\frac{a \log\left(\sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{a \log\left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} + \frac{b \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Csc}[c + d*x]*(a + b*\text{Tan}[c + d*x]),x]$

[Out] $\left(\frac{b \operatorname{ArcTanh}[\sin[c + d*x]]}{d}\right) - \left(\frac{a \operatorname{Log}[\cos[c/2 + (d*x)/2]]}{d}\right) + \left(\frac{a \operatorname{Log}[\sin[c/2 + (d*x)/2]]}{d}\right)$

Maple [A] time = 0.036, size = 42, normalized size = 1.6

$$\frac{a \ln(\csc(dx + c) - \cot(dx + c))}{d} + \frac{b \ln(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)*(a+b*tan(d*x+c)),x)

[Out] 1/d*a*ln(csc(d*x+c)-cot(d*x+c))+1/d*b*ln(sec(d*x+c)+tan(d*x+c))

Maxima [A] time = 1.53495, size = 62, normalized size = 2.38

$$\frac{b(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) - 2a \log(\cot(dx + c) + \csc(dx + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 2*a*log(cot(d*x + c) + csc(d*x + c)))/d

Fricas [B] time = 2.35074, size = 170, normalized size = 6.54

$$\frac{a \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) - a \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) - b \log(\sin(dx + c) + 1) + b \log(-\sin(dx + c) + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] -1/2*(a*log(1/2*cos(d*x + c) + 1/2) - a*log(-1/2*cos(d*x + c) + 1/2) - b*log(sin(d*x + c) + 1) + b*log(-sin(d*x + c) + 1))/d

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx)) \csc(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x)

[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x), x)

Giac [A] time = 1.41334, size = 66, normalized size = 2.54

$$\frac{b \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - b \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) + a \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + a*log(abs(tan(1/2*d*x + 1/2*c))))/d
```

3.17 $\int \csc^2(c + dx)(a + b \tan(c + dx)) dx$

Optimal. Leaf size=25

$$\frac{b \log(\tan(c + dx))}{d} - \frac{a \cot(c + dx)}{d}$$

[Out] $-\left(\frac{a \cot[c + d*x]}{d}\right) + \left(\frac{b \log[\tan[c + d*x]]}{d}\right)$

Rubi [A] time = 0.0783231, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {43}

$$\frac{b \log(\tan(c + dx))}{d} - \frac{a \cot(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x]), x]

[Out] $-\left(\frac{a \cot[c + d*x]}{d}\right) + \left(\frac{b \log[\tan[c + d*x]]}{d}\right)$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rubi steps

$$\begin{aligned} \int \csc^2(c + dx)(a + b \tan(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{a+bx}{x^2} dx, x, \tan(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^2} + \frac{b}{x}\right) dx, x, \tan(c + dx)\right)}{d} \\ &= -\frac{a \cot(c + dx)}{d} + \frac{b \log(\tan(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.0594411, size = 36, normalized size = 1.44

$$-\frac{a \cot(c + dx)}{d} - \frac{b(\log(\cos(c + dx)) - \log(\sin(c + dx)))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x]), x]

[Out] $-\left(\frac{a \cot[c + d*x]}{d}\right) - \left(\frac{b(\log[\cos[c + d*x]] - \log[\sin[c + d*x]])}{d}\right)$

Maple [A] time = 0.077, size = 26, normalized size = 1.

$$-\frac{\cot(dx + c) a}{d} + \frac{b \ln(\tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(d*x+c)^2*(a+b*tan(d*x+c)),x)`

[Out] `-a*cot(d*x+c)/d+b*ln(tan(d*x+c))/d`

Maxima [A] time = 1.38836, size = 34, normalized size = 1.36

$$\frac{b \log(\tan(dx + c)) - \frac{a}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out] `(b*log(tan(d*x + c)) - a/tan(d*x + c))/d`

Fricas [B] time = 2.49515, size = 171, normalized size = 6.84

$$\frac{b \log(\cos(dx + c)^2) \sin(dx + c) - b \log\left(-\frac{1}{4} \cos(dx + c)^2 + \frac{1}{4}\right) \sin(dx + c) + 2a \cos(dx + c)}{2d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out] `-1/2*(b*log(cos(d*x + c)^2)*sin(d*x + c) - b*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 2*a*cos(d*x + c))/(d*sin(d*x + c))`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx)) \csc^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(d*x+c)**2*(a+b*tan(d*x+c)),x)`

[Out] `Integral((a + b*tan(c + d*x))*csc(c + d*x)**2, x)`

Giac [A] time = 1.36389, size = 47, normalized size = 1.88

$$\frac{b \log(|\tan(dx + c)|) - \frac{b \tan(dx+c)+a}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")`

[Out] `(b*log(abs(tan(d*x + c))) - (b*tan(d*x + c) + a)/tan(d*x + c))/d`

3.18 $\int \csc^3(c + dx)(a + b \tan(c + dx)) dx$

Optimal. Leaf size=60

$$-\frac{a \tanh^{-1}(\cos(c + dx))}{2d} - \frac{a \cot(c + dx) \csc(c + dx)}{2d} - \frac{b \csc(c + dx)}{d} + \frac{b \tanh^{-1}(\sin(c + dx))}{d}$$

[Out] $-(a \operatorname{ArcTanh}[\cos[c + d*x]])/(2*d) + (b \operatorname{ArcTanh}[\sin[c + d*x]])/d - (b \operatorname{Csc}[c + d*x])/d - (a \operatorname{Cot}[c + d*x] * \operatorname{Csc}[c + d*x])/(2*d)$

Rubi [A] time = 0.0679689, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3517, 3768, 3770, 2621, 321, 207}

$$-\frac{a \tanh^{-1}(\cos(c + dx))}{2d} - \frac{a \cot(c + dx) \csc(c + dx)}{2d} - \frac{b \csc(c + dx)}{d} + \frac{b \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csc}[c + d*x]^3*(a + b*\operatorname{Tan}[c + d*x]), x]$

[Out] $-(a \operatorname{ArcTanh}[\cos[c + d*x]])/(2*d) + (b \operatorname{ArcTanh}[\sin[c + d*x]])/d - (b \operatorname{Csc}[c + d*x])/d - (a \operatorname{Cot}[c + d*x] * \operatorname{Csc}[c + d*x])/(2*d)$

Rule 3517

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{Expand}[\sin[e + f*x]^m*(a + b*\operatorname{Tan}[e + f*x])^n, x], x] /;$ $\operatorname{FreeQ}\{a, b, e, f\}, x\} \ \&\& \ \operatorname{IntegerQ}[(m - 1)/2] \ \&\& \ \operatorname{IGtQ}[n, 0]$

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x] * (b*\operatorname{Csc}[c + d*x])^{(n - 1)})/(d*(n - 1)), x] + \operatorname{Dist}[(b^2*(n - 2))/(n - 1), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n - 2)}, x], x] /;$ $\operatorname{FreeQ}\{b, c, d\}, x\} \ \&\& \ \operatorname{GtQ}[n, 1] \ \&\& \ \operatorname{IntegerQ}[2*n]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\cos[c + d*x]]/d, x] /;$ $\operatorname{FreeQ}\{c, d\}, x\}$

Rule 2621

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*\operatorname{sec}[(e_.) + (f_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[(f*a^n)^{-1}, \operatorname{Subst}[\operatorname{Int}[x^{(m + n - 1)} / (-1 + x^2/a^2)^{((n + 1)/2)}, x], x, a*\operatorname{Csc}[e + f*x]], x] /;$ $\operatorname{FreeQ}\{a, e, f, m\}, x\} \ \&\& \ \operatorname{IntegerQ}[(n + 1)/2] \ \&\& \ !(\operatorname{IntegerQ}[(m + 1)/2] \ \&\& \ \operatorname{LtQ}[0, m, n])$

Rule 321

$\operatorname{Int}[(c_.*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(c^{(n - 1)}*(c*x)^{(m - n + 1)}*(a + b*x^n)^{(p + 1)})/(b*(m + n*p + 1)), x] - \operatorname{Dist}[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), \operatorname{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, p\}, x\} \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{GtQ}[m, n - 1] \ \&\& \ \operatorname{NeQ}[m + n*p + 1, 0] \ \&\& \ \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \csc^3(c+dx)(a+b\tan(c+dx)) dx &= \int (a \csc^3(c+dx) + b \csc^2(c+dx) \sec(c+dx)) dx \\ &= a \int \csc^3(c+dx) dx + b \int \csc^2(c+dx) \sec(c+dx) dx \\ &= -\frac{a \cot(c+dx) \csc(c+dx)}{2d} + \frac{1}{2}a \int \csc(c+dx) dx - \frac{b \operatorname{Subst}\left(\int \frac{x^2}{-1+x^2} dx, x, \csc(c+dx)\right)}{d} \\ &= -\frac{a \tanh^{-1}(\cos(c+dx))}{2d} - \frac{b \csc(c+dx)}{d} - \frac{a \cot(c+dx) \csc(c+dx)}{2d} - \frac{b \operatorname{Subst}\left(\int \frac{x^2}{-1+x^2} dx, x, \csc(c+dx)\right)}{d} \\ &= -\frac{a \tanh^{-1}(\cos(c+dx))}{2d} + \frac{b \tanh^{-1}(\sin(c+dx))}{d} - \frac{b \csc(c+dx)}{d} - \frac{a \cot(c+dx) \csc(c+dx)}{2d} \end{aligned}$$

Mathematica [C] time = 0.0265519, size = 107, normalized size = 1.78

$$-\frac{a \csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a \sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{b \csc(c+dx) {}_2F_1\left(-\frac{1}{2}, 1; \frac{3}{2}; \frac{1}{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x]), x]

[Out] -(a*Csc[(c + d*x)/2]^2)/(8*d) - (b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[c + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/(2*d) + (a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)

Maple [A] time = 0.082, size = 75, normalized size = 1.3

$$-\frac{b}{d \sin(dx+c)} + \frac{b \ln(\sec(dx+c) + \tan(dx+c))}{d} - \frac{\cot(dx+c) a \csc(dx+c)}{2d} + \frac{a \ln(\csc(dx+c) - \cot(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c)), x)

[Out] -1/d*b/sin(d*x+c)+1/d*b*ln(sec(d*x+c)+tan(d*x+c))-1/2*a*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a*ln(csc(d*x+c)-cot(d*x+c))

Maxima [A] time = 1.14193, size = 112, normalized size = 1.87

$$\frac{a\left(\frac{2 \cos(dx+c)}{\cos(dx+c)^2-1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) - 2b\left(\frac{2}{\sin(dx+c)} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{4}*(a*(2*\cos(d*x + c)/(\cos(d*x + c)^2 - 1) - \log(\cos(d*x + c) + 1) + \log(\cos(d*x + c) - 1)) - 2*b*(2/\sin(d*x + c) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x + c) - 1)))/d$

Fricas [B] time = 2.89013, size = 367, normalized size = 6.12

$$\frac{2 a \cos (d x+c)-\left(a \cos (d x+c)^2-a\right) \log \left(\frac{1}{2} \cos (d x+c)+\frac{1}{2}\right)+\left(a \cos (d x+c)^2-a\right) \log \left(-\frac{1}{2} \cos (d x+c)+\frac{1}{2}\right)+2\left(b \cos (d x+c)-b\right) \log (\sin (d x+c)+1)-2\left(b \cos (d x+c)-b\right) \log (-\sin (d x+c)+1)+4 b \sin (d x+c)}{4\left(d \cos (d x+c)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{4}*(2*a*\cos(d*x + c) - (a*\cos(d*x + c)^2 - a)*\log(1/2*\cos(d*x + c) + 1/2) + (a*\cos(d*x + c)^2 - a)*\log(-1/2*\cos(d*x + c) + 1/2) + 2*(b*\cos(d*x + c) - b)*\log(\sin(d*x + c) + 1) - 2*(b*\cos(d*x + c) - b)*\log(-\sin(d*x + c) + 1) + 4*b*\sin(d*x + c))/(d*\cos(d*x + c)^2 - d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan (c + d x)) \csc ^3(c + d x) d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c)),x)

[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**3, x)

Giac [B] time = 1.51231, size = 159, normalized size = 2.65

$$\frac{a \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^2+8 b \log \left(\left|\tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)+1\right|\right)-8 b \log \left(\left|\tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)-1\right|\right)+4 a \log \left(\left|\tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)\right|\right)}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{8}*(a*\tan(1/2*d*x + 1/2*c)^2 + 8*b*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - 8*b*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) + 4*a*\log(\text{abs}(\tan(1/2*d*x + 1/2*c)))) - 4*b*\tan(1/2*d*x + 1/2*c) - (6*a*\tan(1/2*d*x + 1/2*c)^2 + 4*b*\tan(1/2*d*x + 1/2*c) + a)/\tan(1/2*d*x + 1/2*c)^2)/d$

3.19 $\int \csc^4(c + dx)(a + b \tan(c + dx)) dx$

Optimal. Leaf size=57

$$-\frac{a \cot^3(c + dx)}{3d} - \frac{a \cot(c + dx)}{d} - \frac{b \cot^2(c + dx)}{2d} + \frac{b \log(\tan(c + dx))}{d}$$

[Out] $-\frac{(a \cot[c + d*x])}{d} - \frac{(b \cot[c + d*x]^2)}{(2*d)} - \frac{(a \cot[c + d*x]^3)}{(3*d)} + \frac{(b \log[\tan[c + d*x]])}{d}$

Rubi [A] time = 0.0879247, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {766}

$$-\frac{a \cot^3(c + dx)}{3d} - \frac{a \cot(c + dx)}{d} - \frac{b \cot^2(c + dx)}{2d} + \frac{b \log(\tan(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x]),x]

[Out] $-\frac{(a \cot[c + d*x])}{d} - \frac{(b \cot[c + d*x]^2)}{(2*d)} - \frac{(a \cot[c + d*x]^3)}{(3*d)} + \frac{(b \log[\tan[c + d*x]])}{d}$

Rule 766

Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p_.), x_ Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \csc^4(c + dx)(a + b \tan(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)(1+x^2)}{x^4} dx, x, \tan(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^4} + \frac{b}{x^3} + \frac{a}{x^2} + \frac{b}{x}\right) dx, x, \tan(c + dx)\right)}{d} \\ &= -\frac{a \cot(c + dx)}{d} - \frac{b \cot^2(c + dx)}{2d} - \frac{a \cot^3(c + dx)}{3d} + \frac{b \log(\tan(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.244585, size = 72, normalized size = 1.26

$$-\frac{2a \cot(c + dx)}{3d} - \frac{a \cot(c + dx) \csc^2(c + dx)}{3d} - \frac{b (\csc^2(c + dx) - 2 \log(\sin(c + dx)) + 2 \log(\cos(c + dx)))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x]),x]

[Out] $\frac{(-2*a \cot[c + d*x])}{(3*d)} - \frac{(a \cot[c + d*x] * \csc[c + d*x]^2)}{(3*d)} - \frac{(b * (\csc[c + d*x]^2 + 2 * \log[\cos[c + d*x]] - 2 * \log[\sin[c + d*x]]))}{(2*d)}$

Maple [A] time = 0.081, size = 60, normalized size = 1.1

$$-\frac{b}{2d(\sin(dx+c))^2} + \frac{b \ln(\tan(dx+c))}{d} - \frac{2 \cot(dx+c)a}{3d} - \frac{\cot(dx+c)a(\csc(dx+c))^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c)), x)

[Out] -1/2/d*b/sin(d*x+c)^2+b*ln(tan(d*x+c))/d-2/3*a*cot(d*x+c)/d-1/3/d*a*cot(d*x+c)*csc(d*x+c)^2

Maxima [A] time = 1.12848, size = 68, normalized size = 1.19

$$\frac{6b \log(\tan(dx+c)) - \frac{6a \tan(dx+c)^2 + 3b \tan(dx+c) + 2a}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)), x, algorithm="maxima")

[Out] 1/6*(6*b*log(tan(d*x + c)) - (6*a*tan(d*x + c)^2 + 3*b*tan(d*x + c) + 2*a)/tan(d*x + c)^3)/d

Fricas [B] time = 2.79451, size = 316, normalized size = 5.54

$$\frac{4a \cos(dx+c)^3 + 3(b \cos(dx+c)^2 - b) \log(\cos(dx+c)^2) \sin(dx+c) - 3(b \cos(dx+c)^2 - b) \log\left(-\frac{1}{4} \cos(dx+c)\right)}{6(d \cos(dx+c)^2 - d) \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)), x, algorithm="fricas")

[Out] -1/6*(4*a*cos(d*x + c)^3 + 3*(b*cos(d*x + c)^2 - b)*log(cos(d*x + c)^2)*sin(d*x + c) - 3*(b*cos(d*x + c)^2 - b)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*a*cos(d*x + c) - 3*b*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx)) \csc^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c)), x)

[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**4, x)

Giac [A] time = 1.36171, size = 84, normalized size = 1.47

$$\frac{6b \log(|\tan(dx+c)|) - \frac{11b \tan(dx+c)^3 + 6a \tan(dx+c)^2 + 3b \tan(dx+c) + 2a}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] 1/6*(6*b*log(abs(tan(d*x + c))) - (11*b*tan(d*x + c)^3 + 6*a*tan(d*x + c)^2 + 3*b*tan(d*x + c) + 2*a)/tan(d*x + c)^3)/d

3.20 $\int \csc^5(c + dx)(a + b \tan(c + dx)) dx$

Optimal. Leaf size=98

$$\frac{3a \tanh^{-1}(\cos(c + dx))}{8d} - \frac{a \cot(c + dx) \csc^3(c + dx)}{4d} - \frac{3a \cot(c + dx) \csc(c + dx)}{8d} - \frac{b \csc^3(c + dx)}{3d} - \frac{b \csc(c + dx)}{d}$$

[Out] $(-3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[c + d*x])/d - (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (b*Csc[c + d*x]^3)/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)$

Rubi [A] time = 0.0922901, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3517, 3768, 3770, 2621, 302, 207}

$$\frac{3a \tanh^{-1}(\cos(c + dx))}{8d} - \frac{a \cot(c + dx) \csc^3(c + dx)}{4d} - \frac{3a \cot(c + dx) \csc(c + dx)}{8d} - \frac{b \csc^3(c + dx)}{3d} - \frac{b \csc(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[Csc[c + d*x]^5*(a + b*Tan[c + d*x]), x]$

[Out] $(-3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[c + d*x])/d - (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (b*Csc[c + d*x]^3)/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)$

Rule 3517

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{Expand}[\text{Sin}[e + f*x]^{m*(a + b*\text{Tan}[e + f*x])^n}, x], x] /; \text{FreeQ}\{a, b, e, f\}, x \ \&\& \ \text{IntegerQ}[(m - 1)/2] \ \&\& \ \text{IGtQ}[n, 0]$

Rule 3768

$\text{Int}[(\csc[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x] * (b*\csc[c + d*x])^{(n - 1)})/(d*(n - 1)), x] + \text{Dist}[(b^2*(n - 2))/(n - 1), \text{Int}[(b*\csc[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 3770

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2621

$\text{Int}[(\csc[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*\sec[(e_.) + (f_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[(f*a^n)^{-1}, \text{Subst}[\text{Int}[x^{(m + n - 1)}/(-1 + x^2/a^2)^{((n + 1)/2)}, x], x, a*\csc[e + f*x]], x] /; \text{FreeQ}\{a, e, f, m\}, x \ \&\& \ \text{IntegerQ}[(n + 1)/2] \ \&\& \ !(\text{IntegerQ}[(m + 1)/2] \ \&\& \ \text{LtQ}[0, m, n])$

Rule 302

$\text{Int}[(x_)^{(m_)}/((a_) + (b_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{PolynomialDivide}[x^m, a + b*x^n, x], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, 2*n - 1]$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \csc^5(c+dx)(a+b\tan(c+dx))dx &= \int (a \csc^5(c+dx) + b \csc^4(c+dx) \sec(c+dx)) dx \\
 &= a \int \csc^5(c+dx) dx + b \int \csc^4(c+dx) \sec(c+dx) dx \\
 &= -\frac{a \cot(c+dx) \csc^3(c+dx)}{4d} + \frac{1}{4}(3a) \int \csc^3(c+dx) dx - \frac{b \operatorname{Subst}\left(\int \frac{x^4}{-1+x^2} dx, x, \frac{\csc(c+dx)}{d}\right)}{d} \\
 &= -\frac{3a \cot(c+dx) \csc(c+dx)}{8d} - \frac{a \cot(c+dx) \csc^3(c+dx)}{4d} + \frac{1}{8}(3a) \int \csc(c+dx) dx \\
 &= -\frac{3a \tanh^{-1}(\cos(c+dx))}{8d} - \frac{b \csc(c+dx)}{d} - \frac{3a \cot(c+dx) \csc(c+dx)}{8d} - \frac{b \csc^3(c+dx)}{3d} \\
 &= -\frac{3a \tanh^{-1}(\cos(c+dx))}{8d} + \frac{b \tanh^{-1}(\sin(c+dx))}{d} - \frac{b \csc(c+dx)}{d} - \frac{3a \cot(c+dx) \csc(c+dx)}{8d}
 \end{aligned}$$

Mathematica [C] time = 0.0286861, size = 151, normalized size = 1.54

$$-\frac{a \csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{3a \csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a \sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a \sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3a \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{3a \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x]),x]

[Out] (-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (b*Csc[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d) - (3*a*Log[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) + (3*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)

Maple [A] time = 0.084, size = 109, normalized size = 1.1

$$-\frac{b}{3d(\sin(dx+c))^3} - \frac{b}{d\sin(dx+c)} + \frac{b \ln(\sec(dx+c) + \tan(dx+c))}{d} - \frac{\cot(dx+c) a (\csc(dx+c))^3}{4d} - \frac{3 \cot(dx+c) a}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c)),x)

[Out] -1/3/d*b/sin(d*x+c)^3-1/d*b/sin(d*x+c)+1/d*b*ln(sec(d*x+c)+tan(d*x+c))-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d-3/8*a*cot(d*x+c)*csc(d*x+c)/d+3/8/d*a*ln(csc(d*x+c))-cot(d*x+c)

Maxima [A] time = 1.26038, size = 166, normalized size = 1.69

$$\frac{3a \left(\frac{2(3 \cos(dx+c)^3 - 5 \cos(dx+c))}{\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1} - 3 \log(\cos(dx+c) + 1) + 3 \log(\cos(dx+c) - 1) \right) - 8b \left(\frac{2(3 \sin(dx+c)^2 + 1)}{\sin(dx+c)^3} - 3 \log(\sin(dx+c) + 1) + 3 \log(\sin(dx+c) - 1) \right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] 1/48*(3*a*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c)))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 8*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))/d

Fricas [B] time = 2.72285, size = 585, normalized size = 5.97

$$18a \cos(dx+c)^3 - 30a \cos(dx+c) - 9 \left(a \cos(dx+c)^4 - 2a \cos(dx+c)^2 + a \right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) + 9 \left(a \cos(dx+c)^4 - 2a \cos(dx+c)^2 + a \right) \log\left(\frac{1}{2} \cos(dx+c) - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] 1/48*(18*a*cos(d*x + c)^3 - 30*a*cos(d*x + c) - 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 24*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(sin(d*x + c) + 1) - 24*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-sin(d*x + c) + 1) + 16*(3*b*cos(d*x + c)^2 - 4*b)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c)),x)

[Out] Timed out

Giac [A] time = 1.42746, size = 239, normalized size = 2.44

$$3a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 8b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 24a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 192b \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 192b \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 - 8*b*tan(1/2*d*x + 1/2*c)^3 + 24*a*tan(1/2*d*x + 1/2*c)^2 + 192*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 192*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 72*a*log(abs(tan(1/2*d*x + 1/2*c)))) - 120*b*tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 + 120*b*tan(1/2*d*x + 1/2*c)^3 + 24*a*tan(1/2*d*x + 1/2*c)^2 + 8*b*tan(1/2*d*x + 1/2*c) + 3*a)/tan(1/2*d*x + 1/2*c)^4/d
```


3.21 $\int \csc^6(c + dx)(a + b \tan(c + dx)) dx$

Optimal. Leaf size=87

$$-\frac{a \cot^5(c + dx)}{5d} - \frac{2a \cot^3(c + dx)}{3d} - \frac{a \cot(c + dx)}{d} - \frac{b \cot^4(c + dx)}{4d} - \frac{b \cot^2(c + dx)}{d} + \frac{b \log(\tan(c + dx))}{d}$$

[Out] -((a*Cot[c + d*x])/d) - (b*Cot[c + d*x]^2)/d - (2*a*Cot[c + d*x]^3)/(3*d) - (b*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5*d) + (b*Log[Tan[c + d*x]])/d

Rubi [A] time = 0.107578, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {766}

$$-\frac{a \cot^5(c + dx)}{5d} - \frac{2a \cot^3(c + dx)}{3d} - \frac{a \cot(c + dx)}{d} - \frac{b \cot^4(c + dx)}{4d} - \frac{b \cot^2(c + dx)}{d} + \frac{b \log(\tan(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x]),x]

[Out] -((a*Cot[c + d*x])/d) - (b*Cot[c + d*x]^2)/d - (2*a*Cot[c + d*x]^3)/(3*d) - (b*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5*d) + (b*Log[Tan[c + d*x]])/d

Rule 766

Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_ Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \csc^6(c + dx)(a + b \tan(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)(1+x^2)^2}{x^6} dx, x, \tan(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^6} + \frac{b}{x^5} + \frac{2a}{x^4} + \frac{2b}{x^3} + \frac{a}{x^2} + \frac{b}{x}\right) dx, x, \tan(c + dx)\right)}{d} \\ &= -\frac{a \cot(c + dx)}{d} - \frac{b \cot^2(c + dx)}{d} - \frac{2a \cot^3(c + dx)}{3d} - \frac{b \cot^4(c + dx)}{4d} - \frac{a \cot^5(c + dx)}{5d} + \frac{b \log(\tan(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.545438, size = 104, normalized size = 1.2

$$-\frac{8a \cot(c + dx)}{15d} - \frac{a \cot(c + dx) \csc^4(c + dx)}{5d} - \frac{4a \cot(c + dx) \csc^2(c + dx)}{15d} - \frac{b(\csc^4(c + dx) + 2 \csc^2(c + dx) - 4 \log(\tan(c + dx)))}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x]),x]

[Out] (-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (b*(2*Csc[c + d*x]^2 + Csc[c + d*x]^4

$$+ 4*\text{Log}[\text{Cos}[c + d*x]] - 4*\text{Log}[\text{Sin}[c + d*x]])/(4*d)$$

Maple [A] time = 0.085, size = 94, normalized size = 1.1

$$-\frac{b}{4d(\sin(dx+c))^4} - \frac{b}{2d(\sin(dx+c))^2} + \frac{b \ln(\tan(dx+c))}{d} - \frac{8 \cot(dx+c)a}{15d} - \frac{\cot(dx+c)a(\csc(dx+c))^4}{5d} - \frac{4 \cot(dx+c)a}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c)),x)

[Out] -1/4/d*b/sin(d*x+c)^4-1/2/d*b/sin(d*x+c)^2+b*ln(tan(d*x+c))/d-8/15*a*cot(d*x+c)/d-1/5/d*a*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a*cot(d*x+c)*csc(d*x+c)^2

Maxima [A] time = 1.13162, size = 97, normalized size = 1.11

$$\frac{60 b \log(\tan(dx+c)) - \frac{60 a \tan(dx+c)^4 + 60 b \tan(dx+c)^3 + 40 a \tan(dx+c)^2 + 15 b \tan(dx+c) + 12 a}{\tan(dx+c)^5}}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] 1/60*(60*b*log(tan(d*x + c)) - (60*a*tan(d*x + c)^4 + 60*b*tan(d*x + c)^3 + 40*a*tan(d*x + c)^2 + 15*b*tan(d*x + c) + 12*a)/tan(d*x + c)^5)/d

Fricas [B] time = 2.10032, size = 473, normalized size = 5.44

$$\frac{32 a \cos(dx+c)^5 - 80 a \cos(dx+c)^3 + 30 (b \cos(dx+c)^4 - 2 b \cos(dx+c)^2 + b) \log(\cos(dx+c)^2) \sin(dx+c) - 30 (b \cos(dx+c)^4 - 2 b \cos(dx+c)^2 + b) \sin(dx+c)}{60 (d \cos(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] -1/60*(32*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 30*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 60*a*cos(d*x + c) - 15*(2*b*cos(d*x + c)^2 - 3*b)*sin(d*x + c))/((d*cos(d*x + c))^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c)),x)

[Out] Timed out

Giac [A] time = 1.38364, size = 113, normalized size = 1.3

$$\frac{60 b \log(|\tan(dx + c)|) - \frac{137 b \tan(dx+c)^5 + 60 a \tan(dx+c)^4 + 60 b \tan(dx+c)^3 + 40 a \tan(dx+c)^2 + 15 b \tan(dx+c) + 12 a}{\tan(dx+c)^5}}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] 1/60*(60*b*log(abs(tan(d*x + c))) - (137*b*tan(d*x + c)^5 + 60*a*tan(d*x + c)^4 + 60*b*tan(d*x + c)^3 + 40*a*tan(d*x + c)^2 + 15*b*tan(d*x + c) + 12*a)/tan(d*x + c)^5)/d

3.22 $\int \sin^4(c + dx)(a + b \tan(c + dx))^2 dx$

Optimal. Leaf size=113

$$\frac{3}{8}x(a^2 - 5b^2) + \frac{\cos^2(c + dx)(7b - 5a \tan(c + dx))(a + b \tan(c + dx))}{8d} - \frac{2ab \log(\cos(c + dx))}{d} + \frac{\sin(c + dx) \cos^3(c + dx)}{4d}$$

[Out] (3*(a^2 - 5*b^2)*x)/8 - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d + (Cos[c + d*x]^2*(7*b - 5*a*Tan[c + d*x])*(a + b*Tan[c + d*x]))/(8*d) + (Cos[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(4*d)

Rubi [A] time = 0.185813, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1645, 1810, 635, 203, 260}

$$\frac{3}{8}x(a^2 - 5b^2) + \frac{\cos^2(c + dx)(7b - 5a \tan(c + dx))(a + b \tan(c + dx))}{8d} - \frac{2ab \log(\cos(c + dx))}{d} + \frac{\sin(c + dx) \cos^3(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]

[Out] (3*(a^2 - 5*b^2)*x)/8 - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d + (Cos[c + d*x]^2*(7*b - 5*a*Tan[c + d*x])*(a + b*Tan[c + d*x]))/(8*d) + (Cos[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(4*d)

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :=> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 1645

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e*f*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

Rule 1810

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :=> Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :=> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \sin^4(c + dx)(a + b \tan(c + dx))^2 dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^4(a+x)^2}{(b^2+x^2)^3} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{\cos^3(c + dx) \sin(c + dx)(a + b \tan(c + dx))^2}{4d} - \frac{\operatorname{Subst}\left(\int \frac{(a+x)(ab^4+3b^4x-4ab^2x^2-(b^2+x^2)^2)}{(b^2+x^2)^2} dx, x, b \tan(c + dx)\right)}{4ba} \\ &= \frac{\cos^2(c + dx)(7b - 5a \tan(c + dx))(a + b \tan(c + dx))}{8d} + \frac{\cos^3(c + dx) \sin(c + dx)}{4d} \\ &= \frac{\cos^2(c + dx)(7b - 5a \tan(c + dx))(a + b \tan(c + dx))}{8d} + \frac{\cos^3(c + dx) \sin(c + dx)}{4d} \\ &= \frac{b^2 \tan(c + dx)}{d} + \frac{\cos^2(c + dx)(7b - 5a \tan(c + dx))(a + b \tan(c + dx))}{8d} + \frac{\cos^3(c + dx) \sin(c + dx)}{4d} \\ &= \frac{b^2 \tan(c + dx)}{d} + \frac{\cos^2(c + dx)(7b - 5a \tan(c + dx))(a + b \tan(c + dx))}{8d} + \frac{\cos^3(c + dx) \sin(c + dx)}{4d} \\ &= \frac{3}{8} (a^2 - 5b^2) x - \frac{2ab \log(\cos(c + dx))}{d} + \frac{b^2 \tan(c + dx)}{d} + \frac{\cos^2(c + dx)(7b - 5a \tan(c + dx))}{8d} \end{aligned}$$

Mathematica [B] time = 3.19675, size = 240, normalized size = 2.12

$$b \left(\frac{2(3b^2-2a^2) \sin(2(c+dx))}{b} + \frac{4(3b^2-2a^2) \tan^{-1}(\tan(c+dx))}{b} + 4 \left(\frac{a^2-3b^2}{\sqrt{-b^2}} + 2a \right) \log \left(\sqrt{-b^2} - b \tan(c + dx) \right) + 4 \left(\frac{3b^2-a^2}{\sqrt{-b^2}} + 2a \right) \log \left(\sqrt{-b^2} + b \tan(c + dx) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^2, x]

[Out] (b*((4*(-2*a^2 + 3*b^2)*ArcTan[Tan[c + d*x]])/b + 16*a*Cos[c + d*x]^2 - 4*a*Cos[c + d*x]^4 + 4*(2*a + (a^2 - 3*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 4*(2*a + (-a^2 + 3*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 - b^2)*Cos[c + d*x]^3*Sin[c + d*x])/b + (2*(-2*a^2 + 3*b^2)*Sin[2*(c + d*x)])/b + (3*(a^2 - b^2)*(2*ArcTan[Tan[c + d*x]] + Sin[2*(c + d*x)]))/(2*b) + 8*b*Tan[c + d*x]))/(8*d)

Maple [A] time = 0.04, size = 204, normalized size = 1.8

$$\frac{b^2 (\sin(dx + c))^7}{d \cos(dx + c)} + \frac{b^2 \cos(dx + c) (\sin(dx + c))^5}{d} + \frac{5 b^2 \cos(dx + c) (\sin(dx + c))^3}{4 d} + \frac{15 b^2 \cos(dx + c) \sin(dx + c)}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x)`

[Out] $1/d*b^2*\sin(d*x+c)^7/\cos(d*x+c)+1/d*b^2*\cos(d*x+c)*\sin(d*x+c)^5+5/4/d*b^2*\cos(d*x+c)*\sin(d*x+c)^3+15/8/d*b^2*\cos(d*x+c)*\sin(d*x+c)-15/8*b^2*x-15/8/d*b^2*c-1/2/d*a*b*\sin(d*x+c)^4-1/d*a*b*\sin(d*x+c)^2-2*a*b*\ln(\cos(d*x+c))/d-1/4/d*a^2*\cos(d*x+c)*\sin(d*x+c)^3-3/8/d*a^2*\cos(d*x+c)*\sin(d*x+c)+3/8*a^2*x+3/8/d*a^2*c$

Maxima [A] time = 1.47685, size = 173, normalized size = 1.53

$$\frac{8ab \log(\tan(dx+c)^2+1) + 8b^2 \tan(dx+c) + 3(a^2-5b^2)(dx+c) + \frac{16ab \tan(dx+c)^2 - (5a^2-9b^2)\tan(dx+c)^3 + 12ab - (3a^2-7b^2)\tan(dx+c)}{\tan(dx+c)^4 + 2\tan(dx+c)^2 + 1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")`

[Out] $1/8*(8*a*b*\log(\tan(d*x+c)^2+1) + 8*b^2*\tan(d*x+c) + 3*(a^2-5*b^2)*(d*x+c) + (16*a*b*\tan(d*x+c)^2 - (5*a^2-9*b^2)*\tan(d*x+c)^3 + 12*a*b - (3*a^2-7*b^2)*\tan(d*x+c))/(\tan(d*x+c)^4 + 2*\tan(d*x+c)^2 + 1))/d$

Fricas [A] time = 2.06341, size = 333, normalized size = 2.95

$$\frac{8ab \cos(dx+c)^5 - 32ab \cos(dx+c)^3 + 32ab \cos(dx+c) \log(-\cos(dx+c)) - (6(a^2-5b^2)dx - 13ab) \cos(dx+c)}{16d \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")`

[Out] $-1/16*(8*a*b*\cos(d*x+c)^5 - 32*a*b*\cos(d*x+c)^3 + 32*a*b*\cos(d*x+c)*\log(-\cos(d*x+c)) - (6*(a^2-5*b^2)*d*x - 13*a*b)*\cos(d*x+c) - 2*(2*(a^2-b^2)*\cos(d*x+c)^4 - (5*a^2-9*b^2)*\cos(d*x+c)^2 + 8*b^2)*\sin(d*x+c))/(d*\cos(d*x+c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)**4*(a+b*tan(d*x+c))**2,x)`

[Out] Timed out

Giac [B] time = 6.66882, size = 7808, normalized size = 69.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{64} \cdot (3\pi b^2 \operatorname{sgn}(2\tan(dx)^2 \tan(c)^2 - 2) \operatorname{sgn}(-2\tan(dx)^2 \tan(c)) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^5 \tan(c)^5 + 24a^2 dx \tan(dx)^5 \tan(c)^5 - 120b^2 dx \tan(dx)^5 \tan(c)^5 + 3\pi b^2 \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^5 \tan(c)^5 + 6\pi b^2 \operatorname{sgn}(2 \tan(dx)^2 \tan(c)^2 - 2) \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^5 \tan(c)^3 - 3\pi b^2 \operatorname{sgn}(2 \tan(dx)^2 \tan(c)^2 - 2) \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^4 \tan(c)^4 + 6\pi b^2 \operatorname{sgn}(2 \tan(dx)^2 \tan(c)^2 - 2) \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^3 \tan(c)^5 + 6b^2 \arctan((\tan(dx) + \tan(c))/(\tan(dx) \tan(c) - 1)) \tan(dx)^5 \tan(c)^5 - 6b^2 \arctan(-(\tan(dx) - \tan(c))/(\tan(dx) \tan(c) + 1)) \tan(dx)^5 \tan(c)^5 - 64ab \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2 \tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1)) \tan(dx)^5 \tan(c)^5 + 48a^2 dx \tan(dx)^5 \tan(c)^3 - 240b^2 dx \tan(dx)^5 \tan(c)^3 + 6\pi b^2 \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^5 \tan(c)^3 - 24a^2 dx \tan(dx)^4 \tan(c)^4 + 120b^2 dx \tan(dx)^4 \tan(c)^4 - 3\pi b^2 \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^4 \tan(c)^4 + 48a^2 dx \tan(dx)^3 \tan(c)^5 - 240b^2 dx \tan(dx)^3 \tan(c)^5 + 6\pi b^2 \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^3 \tan(c)^5 + 44ab \tan(dx)^5 \tan(c)^5 + 3\pi b^2 \operatorname{sgn}(2 \tan(dx)^2 \tan(c)^2 - 2) \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^5 \tan(c) - 6\pi b^2 \operatorname{sgn}(2 \tan(dx)^2 \tan(c)^2 - 2) \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^4 \tan(c)^2 + 12\pi b^2 \operatorname{sgn}(2 \tan(dx)^2 \tan(c)^2 - 2) \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^3 \tan(c)^3 + 12b^2 \arctan((\tan(dx) + \tan(c))/(\tan(dx) \tan(c) - 1)) \tan(dx)^5 \tan(c)^3 - 12b^2 \arctan(-(\tan(dx) - \tan(c))/(\tan(dx) \tan(c) + 1)) \tan(dx)^5 \tan(c)^3 - 128ab \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2 \tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1)) \tan(dx)^5 \tan(c)^3 - 6\pi b^2 \operatorname{sgn}(2 \tan(dx)^2 \tan(c)^2 - 2) \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^2 \tan(c)^4 - 6b^2 \arctan((\tan(dx) + \tan(c))/(\tan(dx) \tan(c) - 1)) \tan(dx)^4 \tan(c)^4 + 6b^2 \arctan(-(\tan(dx) - \tan(c))/(\tan(dx) \tan(c) + 1)) \tan(dx)^4 \tan(c)^4 + 64ab \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2 \tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1)) \tan(dx)^4 \tan(c)^4 + 24a^2 \tan(dx)^5 \tan(c)^4 - 120b^2 \tan(dx)^5 \tan(c)^4 + 3\pi b^2 \operatorname{sgn}(2 \tan(dx)^2 \tan(c)^2 - 2) \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx) \tan(c)^5 + 12b^2 \arctan((\tan(dx) + \tan(c))/(\tan(dx) \tan(c) - 1)) \tan(dx)^3 \tan(c)^5 - 12b^2 \arctan(-(\tan(dx) - \tan(c))/(\tan(dx) \tan(c) + 1)) \tan(dx)^3 \tan(c)^5 - 128ab \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2 \tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1)) \tan(dx)^3 \tan(c)^5 + 24a^2 \tan(dx)^4 \tan(c)^5 - 120b^2 \tan(dx)^4 \tan(c)^5 + 24a^2 dx \tan(dx)^5 \tan(c) - 120b^2 dx \tan(dx)^5 \tan(c) + 3\pi b^2 \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^5 \tan(c) - 48a^2 dx \tan(dx)^4 \tan(c)^2 + 240b^2 dx \tan(dx)^4 \tan(c)^2 - 6\pi b^2 \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^4 \tan(c)^2 + 96a^2 dx \tan(dx)^3 \tan(c)^3 - 480b^2 dx \tan(dx)^3 \tan(c)^3 + 12\pi b^2 \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^3 \tan(c)^3 + 24ab \tan(dx)^5 \tan(c)^3 - 48a^2 dx \tan(dx)^2 \tan(c)^4 + 240b^2 dx \tan(dx)^2 \tan(c)^4 - 6\pi b^2 \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx)^2 \tan(c)^4 - 172ab \tan(dx)^4 \tan(c)^4 + 24a^2 dx \tan(dx) \tan(c)^5 - 120b^2 dx \tan(dx) \tan(c)^5 + 3\pi b^2 \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx) \tan(c)^5 + 3\pi b^2 \operatorname{sgn}(-2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 + 2 \tan(dx) - 2 \tan(c)) \tan(dx) \tan(c)^5$

$d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(d*x)*\tan(c)^5 + 24*a*b*\tan(d*x)^3*\tan(c)^5 - 3*pi*b^2*sgn(2*\tan(d*x)^2*\tan(c)^2 - 2)*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(d*x)^4 + 6*pi*b^2*sgn(2*\tan(d*x)^2*\tan(c)^2 - 2)*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(d*x)^3*\tan(c) + 6*b^2*arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1))*\tan(d*x)^5*\tan(c) - 6*b^2*arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1))*\tan(d*x)^5*\tan(c) - 64*a*b*log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^5*\tan(c) - 12*pi*b^2*sgn(2*\tan(d*x)^2*\tan(c)^2 - 2)*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(d*x)^2*\tan(c)^2 - 12*b^2*arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1))*\tan(d*x)^4*\tan(c)^2 + 12*b^2*arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1))*\tan(d*x)^4*\tan(c)^2 + 128*a*b*log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^4*\tan(c)^2 + 40*a^2*\tan(d*x)^5*\tan(c)^2 - 200*b^2*\tan(d*x)^5*\tan(c)^2 + 6*pi*b^2*sgn(2*\tan(d*x)^2*\tan(c)^2 - 2)*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(d*x)*\tan(c)^3 + 24*b^2*arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1))*\tan(d*x)^3*\tan(c)^3 - 24*b^2*arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1))*\tan(d*x)^3*\tan(c)^3 - 256*a*b*log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^3*\tan(c)^3 + 24*a^2*\tan(d*x)^4*\tan(c)^3 - 120*b^2*\tan(d*x)^4*\tan(c)^3 - 3*pi*b^2*sgn(2*\tan(d*x)^2*\tan(c)^2 - 2)*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(c)^4 - 12*b^2*arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1))*\tan(d*x)^2*\tan(c)^4 + 12*b^2*arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(c)^4 + 128*a*b*log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(c)^4 + 24*a^2*\tan(d*x)^3*\tan(c)^4 - 120*b^2*\tan(d*x)^3*\tan(c)^4 + 6*b^2*arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1))*\tan(d*x)*\tan(c)^5 - 6*b^2*arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1))*\tan(d*x)*\tan(c)^5 - 64*a*b*log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)*\tan(c)^5 + 40*a^2*\tan(d*x)^2*\tan(c)^5 - 200*b^2*\tan(d*x)^2*\tan(c)^5 - 24*a^2*d*x*\tan(d*x)^4 + 120*b^2*d*x*\tan(d*x)^4 - 3*pi*b^2*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(d*x)^4 + 48*a^2*d*x*\tan(d*x)^3*\tan(c) - 240*b^2*d*x*\tan(d*x)^3*\tan(c) + 6*pi*b^2*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(d*x)^3*\tan(c) - 52*a*b*\tan(d*x)^5*\tan(c) - 96*a^2*d*x*\tan(d*x)^2*\tan(c)^2 + 480*b^2*d*x*\tan(d*x)^2*\tan(c)^2 - 12*pi*b^2*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(d*x)^2*\tan(c)^2 - 280*a*b*\tan(d*x)^4*\tan(c)^2 + 48*a^2*d*x*\tan(d*x)*\tan(c)^3 - 240*b^2*d*x*\tan(d*x)*\tan(c)^3 + 6*pi*b^2*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(d*x)*\tan(c)^3 - 16*a*b*\tan(d*x)^3*\tan(c)^3 - 24*a^2*d*x*\tan(c)^4 + 120*b^2*d*x*\tan(c)^4 - 3*pi*b^2*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(c)^4 - 280*a*b*\tan(d*x)^2*\tan(c)^4 - 52*a*b*\tan(d*x)*\tan(c)^5 - 6*pi*b^2*sgn(2*\tan(d*x)^2*\tan(c)^2 - 2)*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(d*x)^2 - 6*b^2*arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1))*\tan(d*x)^4 + 6*b^2*arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1))*\tan(d*x)^4 + 64*a*b*log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^4 - 64*b^2*\tan(d*x)^5 + 3*pi*b^2*sgn(2*\tan(d*x)^2*\tan(c)^2 - 2)*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(d*x)*\tan(c) + 12*b^2*arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1))*\tan(d*x)^3*\tan(c) - 12*b^2*arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1))*\tan(d*x)^3*\tan(c) - 128*a*b*log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^3*\tan(c) - 80*a^2*\tan(d*x)^4*\tan(c) + 80*b^2*\tan(d*x)^4*\tan(c) - 6*pi*b^2*sgn(2*\tan(d*x)^2*\tan(c)^2 - 2)*sgn(-2*\tan(d*x)^2*t$

$$\begin{aligned}
& \text{an}(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(c)^2 - 24*b^2*\arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1))*\tan(d*x)^2*\tan(c)^2 + 24*b^2*\arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(c)^2 + 256*a*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(c)^2 - 96*a^2*\tan(d*x)^3*\tan(c)^2 - 160*b^2*\tan(d*x)^3*\tan(c)^2 + 12*b^2*\arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1))*\tan(d*x)*\tan(c)^3 - 12*b^2*\arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1))*\tan(d*x)*\tan(c)^3 - 128*a*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)*\tan(c)^3 - 96*a^2*\tan(d*x)^2*\tan(c)^3 - 160*b^2*\tan(d*x)^2*\tan(c)^3 - 6*b^2*\arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1))*\tan(c)^4 + 6*b^2*\arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1))*\tan(c)^4 + 64*a*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(c)^4 - 80*a^2*\tan(d*x)*\tan(c)^4 + 80*b^2*\tan(d*x)*\tan(c)^4 - 64*b^2*\tan(c)^5 - 48*a^2*d*x*\tan(d*x)^2 + 240*b^2*d*x*\tan(d*x)^2 - 6*pi*b^2*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(d*x)^2 + 52*a*b*\tan(d*x)^4 + 24*a^2*d*x*\tan(d*x)*\tan(c) - 120*b^2*d*x*\tan(d*x)*\tan(c) + 3*pi*b^2*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(d*x)*\tan(c) + 280*a*b*\tan(d*x)^3*\tan(c) - 48*a^2*d*x*\tan(c)^2 + 240*b^2*d*x*\tan(c)^2 - 6*pi*b^2*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c))*\tan(c)^2 + 16*a*b*\tan(d*x)^2*\tan(c)^2 + 280*a*b*\tan(d*x)*\tan(c)^3 + 52*a*b*\tan(c)^4 - 3*pi*b^2*sgn(2*\tan(d*x)^2*\tan(c)^2 - 2)*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c)) - 12*b^2*\arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1))*\tan(d*x)^2 + 12*b^2*\arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2 + 128*a*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2 + 40*a^2*\tan(d*x)^3 - 200*b^2*\tan(d*x)^3 + 6*b^2*\arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1))*\tan(d*x)*\tan(c) - 6*b^2*\arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1))*\tan(d*x)*\tan(c) - 64*a*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)*\tan(c) + 24*a^2*\tan(d*x)^2*\tan(c) - 120*b^2*\tan(d*x)^2*\tan(c) - 12*b^2*\arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1))*\tan(c)^2 + 12*b^2*\arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1))*\tan(c)^2 + 128*a*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(c)^2 + 24*a^2*\tan(d*x)*\tan(c)^2 - 120*b^2*\tan(d*x)*\tan(c)^2 + 40*a^2*\tan(c)^3 - 200*b^2*\tan(c)^3 - 24*a^2*d*x + 120*b^2*d*x - 3*pi*b^2*sgn(-2*\tan(d*x)^2*\tan(c) + 2*\tan(d*x)*\tan(c)^2 + 2*\tan(d*x) - 2*\tan(c)) - 24*a*b*\tan(d*x)^2 + 172*a*b*\tan(d*x)*\tan(c) - 24*a*b*\tan(c)^2 - 6*b^2*\arctan((\tan(d*x) + \tan(c))/(\tan(d*x)*\tan(c) - 1)) + 6*b^2*\arctan(-(\tan(d*x) - \tan(c))/(\tan(d*x)*\tan(c) + 1)) + 64*a*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)) + 24*a^2*\tan(d*x) - 120*b^2*\tan(d*x) + 24*a^2*\tan(c) - 120*b^2*\tan(c) - 44*a*b)/(d*\tan(d*x)^5*\tan(c)^5 + 2*d*\tan(d*x)^5*\tan(c)^3 - d*\tan(d*x)^4*\tan(c)^4 + 2*d*\tan(d*x)^3*\tan(c)^5 + d*\tan(d*x)^5*\tan(c) - 2*d*\tan(d*x)^4*\tan(c)^2 + 4*d*\tan(d*x)^3*\tan(c)^3 - 2*d*\tan(d*x)^2*\tan(c)^4 + d*\tan(d*x)*\tan(c)^5 - d*\tan(d*x)^4 + 2*d*\tan(d*x)^3*\tan(c) - 4*d*\tan(d*x)^2*\tan(c)^2 + 2*d*\tan(d*x)*\tan(c)^3 - d*\tan(c)^4 - 2*d*\tan(d*x)^2 + d*\tan(d*x)*\tan(c) - 2*d*\tan(c)^2 - d)
\end{aligned}$$

3.23 $\int \sin^3(c + dx)(a + b \tan(c + dx))^2 dx$

Optimal. Leaf size=122

$$\frac{a^2 \cos^3(c + dx)}{3d} - \frac{a^2 \cos(c + dx)}{d} - \frac{2ab \sin^3(c + dx)}{3d} - \frac{2ab \sin(c + dx)}{d} + \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} - \frac{b^2 \cos^3(c + dx)}{3d} + \frac{2b^2 \cos(c + dx)}{d}$$

[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (2*b^2*Cos[c + d*x])/d + (a^2*Cos[c + d*x]^3)/(3*d) - (b^2*Cos[c + d*x]^3)/(3*d) + (b^2*Sec[c + d*x])/d - (2*a*b*Sin[c + d*x])/d - (2*a*b*Sin[c + d*x]^3)/(3*d)

Rubi [A] time = 0.130693, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3517, 2633, 2592, 302, 206, 2590, 270}

$$\frac{a^2 \cos^3(c + dx)}{3d} - \frac{a^2 \cos(c + dx)}{d} - \frac{2ab \sin^3(c + dx)}{3d} - \frac{2ab \sin(c + dx)}{d} + \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} - \frac{b^2 \cos^3(c + dx)}{3d} + \frac{2b^2 \cos(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]

[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (2*b^2*Cos[c + d*x])/d + (a^2*Cos[c + d*x]^3)/(3*d) - (b^2*Cos[c + d*x]^3)/(3*d) + (b^2*Sec[c + d*x])/d - (2*a*b*Sin[c + d*x])/d - (2*a*b*Sin[c + d*x]^3)/(3*d)

Rule 3517

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \sin^3(c + dx)(a + b \tan(c + dx))^2 dx &= \int (a^2 \sin^3(c + dx) + 2ab \sin^3(c + dx) \tan(c + dx) + b^2 \sin^3(c + dx) \tan^2(c + dx)) dx \\ &= a^2 \int \sin^3(c + dx) dx + (2ab) \int \sin^3(c + dx) \tan(c + dx) dx + b^2 \int \sin^3(c + dx) \tan^2(c + dx) dx \\ &= -\frac{a^2 \operatorname{Subst}\left(\int (1 - x^2) dx, x, \cos(c + dx)\right)}{d} + \frac{(2ab) \operatorname{Subst}\left(\int \frac{x^4}{1-x^2} dx, x, \sin(c + dx)\right)}{d} \\ &= -\frac{a^2 \cos(c + dx)}{d} + \frac{a^2 \cos^3(c + dx)}{3d} + \frac{(2ab) \operatorname{Subst}\left(\int \left(-1 - x^2 + \frac{1}{1-x^2}\right) dx, x, \sin(c + dx)\right)}{d} \\ &= -\frac{a^2 \cos(c + dx)}{d} + \frac{2b^2 \cos(c + dx)}{d} + \frac{a^2 \cos^3(c + dx)}{3d} - \frac{b^2 \cos^3(c + dx)}{3d} + \frac{b^2}{3d} \\ &= \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^2 \cos(c + dx)}{d} + \frac{2b^2 \cos(c + dx)}{d} + \frac{a^2 \cos^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 1.02334, size = 152, normalized size = 1.25

$$\sec(c + dx) \left((20b^2 - 8a^2) \cos(2(c + dx)) + (a^2 - b^2) \cos(4(c + dx)) - 9a^2 - 28ab \sin(2(c + dx)) + 2ab \sin(4(c + dx)) \right) -$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(-9*a^2 + 45*b^2 + (-8*a^2 + 20*b^2)*Cos[2*(c + d*x)] + (a^2 - b^2)*Cos[4*(c + d*x)] - 48*a*b*Cos[c + d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 48*a*b*Cos[c + d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 28*a*b*Sin[2*(c + d*x)] + 2*a*b*Sin[4*(c + d*x)]))/(24*d)
```

Maple [A] time = 0.041, size = 167, normalized size = 1.4

$$\frac{b^2 (\sin(dx + c))^6}{d \cos(dx + c)} + \frac{8b^2 \cos(dx + c)}{3d} + \frac{b^2 \cos(dx + c) (\sin(dx + c))^4}{d} + \frac{4b^2 \cos(dx + c) (\sin(dx + c))^2}{3d} - \frac{2ab (\sin(dx + c))^5}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*b^2*sin(d*x+c)^6/cos(d*x+c)+8/3*b^2*cos(d*x+c)/d+1/d*b^2*cos(d*x+c)*sin(d*x+c)^4+4/3/d*b^2*cos(d*x+c)*sin(d*x+c)^2-2/3*a*b*sin(d*x+c)^3/d-2*a*b*si
```

$n(dx+c)/d+2/d*a*b*\ln(\sec(dx+c)+\tan(dx+c))-1/3/d*\cos(dx+c)*\sin(dx+c)^2*$
 $a^2-2/3*a^2*\cos(dx+c)/d$

Maxima [A] time = 0.973981, size = 140, normalized size = 1.15

$$\frac{(\cos(dx+c)^3 - 3\cos(dx+c))a^2 - (2\sin(dx+c)^3 - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1) + 6\sin(dx+c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)^3*(a+b*tan(dx+c))^2,x, algorithm="maxima")

[Out] 1/3*((cos(dx+c)^3 - 3*cos(dx+c))*a^2 - (2*sin(dx+c)^3 - 3*log(sin(dx+c)+1) + 3*log(sin(dx+c)-1) + 6*sin(dx+c))*a*b - (cos(dx+c)^3 - 3/cos(dx+c) - 6*cos(dx+c))*b^2)/d

Fricas [A] time = 1.99257, size = 321, normalized size = 2.63

$$\frac{(a^2 - b^2)\cos(dx+c)^4 + 3ab\cos(dx+c)\log(\sin(dx+c)+1) - 3ab\cos(dx+c)\log(-\sin(dx+c)+1) - 3(a^2 - 2b^2)}{3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)^3*(a+b*tan(dx+c))^2,x, algorithm="fricas")

[Out] 1/3*((a^2 - b^2)*cos(dx+c)^4 + 3*a*b*cos(dx+c)*log(sin(dx+c)+1) - 3*a*b*cos(dx+c)*log(-sin(dx+c)+1) - 3*(a^2 - 2*b^2)*cos(dx+c)^2 + 3*b^2 + 2*(a*b*cos(dx+c)^3 - 4*a*b*cos(dx+c))*sin(dx+c))/(d*cos(dx+c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx))^2 \sin^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)**3*(a+b*tan(dx+c))**2,x)

[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**3, x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)^3*(a+b*tan(dx+c))^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

3.24 $\int \sin^2(c + dx)(a + b \tan(c + dx))^2 dx$

Optimal. Leaf size=76

$$\frac{1}{2}x(a^2 - 3b^2) - \frac{2ab \log(\cos(c + dx))}{d} - \frac{\sin(c + dx) \cos(c + dx)(a + b \tan(c + dx))^2}{2d} + \frac{3b^2 \tan(c + dx)}{2d}$$

[Out] $((a^2 - 3b^2)*x)/2 - (2*a*b*Log[Cos[c + d*x]])/d + (3*b^2*Tan[c + d*x])/(2*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(2*d)$

Rubi [A] time = 0.113079, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1645, 774, 635, 203, 260}

$$\frac{1}{2}x(a^2 - 3b^2) - \frac{2ab \log(\cos(c + dx))}{d} - \frac{\sin(c + dx) \cos(c + dx)(a + b \tan(c + dx))^2}{2d} + \frac{3b^2 \tan(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]

[Out] $((a^2 - 3b^2)*x)/2 - (2*a*b*Log[Cos[c + d*x]])/d + (3*b^2*Tan[c + d*x])/(2*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(2*d)$

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 1645

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e*f*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

Rule 774

Int[(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \sin^2(c + dx)(a + b \tan(c + dx))^2 dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^{2(a+x)^2}}{(b^2+x^2)^2} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{\cos(c + dx) \sin(c + dx)(a + b \tan(c + dx))^2}{2d} - \frac{\operatorname{Subst}\left(\int \frac{(a+x)(-ab^2-3b^2x)}{b^2+x^2} dx, x, b \tan(c + dx)\right)}{2bd} \\ &= \frac{3b^2 \tan(c + dx)}{2d} - \frac{\cos(c + dx) \sin(c + dx)(a + b \tan(c + dx))^2}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-a^2b^2}{b^2+x^2} dx, x, b \tan(c + dx)\right)}{2bd} \\ &= \frac{3b^2 \tan(c + dx)}{2d} - \frac{\cos(c + dx) \sin(c + dx)(a + b \tan(c + dx))^2}{2d} + \frac{(2ab) \operatorname{Subst}\left(\int \frac{1}{b^2+x^2} dx, x, b \tan(c + dx)\right)}{2bd} \\ &= \frac{1}{2} (a^2 - 3b^2)x - \frac{2ab \log(\cos(c + dx))}{d} + \frac{3b^2 \tan(c + dx)}{2d} - \frac{\cos(c + dx) \sin(c + dx)(a + b \tan(c + dx))^2}{2d} \end{aligned}$$

Mathematica [B] time = 2.36223, size = 162, normalized size = 2.13

$$\frac{b \left(\frac{(b^2-a^2) \sin(2(c+dx))}{2b} + \frac{(b^2-a^2) \tan^{-1}(\tan(c+dx))}{b} + \left(\frac{a^2-2b^2}{\sqrt{-b^2}} + 2a \right) \log\left(\sqrt{-b^2} - b \tan(c + dx)\right) + \left(\frac{2b^2-a^2}{\sqrt{-b^2}} + 2a \right) \log\left(\sqrt{-b^2} + b \tan(c + dx)\right) \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]

[Out] (b*(((−a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + 2*a*Cos[c + d*x]^2 + (2*a + (a^2 - 2*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + (2*a + (−a^2 + 2*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + ((−a^2 + b^2)*Sin[2*(c + d*x)])/(2*b) + 2*b*Tan[c + d*x]))/(2*d)

Maple [B] time = 0.039, size = 145, normalized size = 1.9

$$\frac{b^2 (\sin(dx + c))^5}{d \cos(dx + c)} + \frac{b^2 \cos(dx + c) (\sin(dx + c))^3}{d} + \frac{3 b^2 \cos(dx + c) \sin(dx + c)}{2 d} - \frac{3 b^2 x}{2} - \frac{3 b^2 c}{2 d} - \frac{ab (\sin(dx + c))^2}{d} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x)

[Out] 1/d*b^2*sin(d*x+c)^5/cos(d*x+c)+1/d*b^2*cos(d*x+c)*sin(d*x+c)^3+3/2/d*b^2*cos(d*x+c)*sin(d*x+c)-3/2*b^2*x-3/2/d*b^2*c-1/d*a*b*sin(d*x+c)^2-2*a*b*ln(cos(d*x+c))/d-1/2/d*a^2*cos(d*x+c)*sin(d*x+c)+1/2*a^2*x+1/2/d*a^2*c

Maxima [A] time = 1.47601, size = 111, normalized size = 1.46

$$\frac{2ab \log(\tan(dx+c)^2+1) + 2b^2 \tan(dx+c) + (a^2 - 3b^2)(dx+c) + \frac{2ab - (a^2 - b^2) \tan(dx+c)}{\tan(dx+c)^2 + 1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] 1/2*(2*a*b*log(tan(d*x + c)^2 + 1) + 2*b^2*tan(d*x + c) + (a^2 - 3*b^2)*(d*x + c) + (2*a*b - (a^2 - b^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d

Fricas [A] time = 2.01632, size = 240, normalized size = 3.16

$$\frac{2ab \cos(dx+c)^3 - 4ab \cos(dx+c) \log(-\cos(dx+c)) + ((a^2 - 3b^2)dx - ab) \cos(dx+c) - ((a^2 - b^2) \cos(dx+c))^2}{2d \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] 1/2*(2*a*b*cos(d*x + c)^3 - 4*a*b*cos(d*x + c)*log(-cos(d*x + c)) + ((a^2 - 3*b^2)*d*x - a*b)*cos(d*x + c) - ((a^2 - b^2)*cos(d*x + c)^2 - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx))^2 \sin^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**2,x)

[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**2, x)

Giac [B] time = 1.87871, size = 1432, normalized size = 18.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] 1/2*(a^2*d*x*tan(d*x)^3*tan(c)^3 - 3*b^2*d*x*tan(d*x)^3*tan(c)^3 - 2*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 + a^2*d*x*tan(d*x)^3*tan(c) - 3*b^2*d*x*tan(d*x)^3*tan(c) - a^2*d*x*tan(d*x)^2*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(c)^2 + a^2*d*x*tan(d*x)*tan(c)^3 - 3*b^2*d*x*tan(d*x)*tan(c)^3 + a*b*tan(d*x)^3*tan(c)^3 - 2*a*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)

$$\begin{aligned}
&)^2 - 2*\tan(dx)*\tan(c) + 1))*\tan(dx)^3*\tan(c) + 2*a*b*\log(4*(\tan(c)^2 + 1) \\
&)/(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx) \\
& x)^2 - 2*\tan(dx)*\tan(c) + 1))*\tan(dx)^2*\tan(c)^2 + a^2*\tan(dx)^3*\tan(c)^ \\
& 2 - 3*b^2*\tan(dx)^3*\tan(c)^2 - 2*a*b*\log(4*(\tan(c)^2 + 1)/(\tan(dx)^4*\tan(\\
& c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)* \\
& \tan(c) + 1))*\tan(dx)*\tan(c)^3 + a^2*\tan(dx)^2*\tan(c)^3 - 3*b^2*\tan(dx)^2 \\
& *\tan(c)^3 - a^2*d*x*\tan(dx)^2 + 3*b^2*d*x*\tan(dx)^2 + a^2*d*x*\tan(dx)*\tan \\
& (c) - 3*b^2*d*x*\tan(dx)*\tan(c) - a*b*\tan(dx)^3*\tan(c) - a^2*d*x*\tan(c)^2 \\
& + 3*b^2*d*x*\tan(c)^2 - 5*a*b*\tan(dx)^2*\tan(c)^2 - a*b*\tan(dx)*\tan(c)^3 + \\
& 2*a*b*\log(4*(\tan(c)^2 + 1)/(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan \\
& (dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1))*\tan(dx)^2 - 2*b^2 \\
& *\tan(dx)^3 - 2*a*b*\log(4*(\tan(c)^2 + 1)/(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3 \\
& *\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1))*\tan(dx) \\
& *\tan(c) - 2*a^2*\tan(dx)^2*\tan(c) + 2*a*b*\log(4*(\tan(c)^2 + 1)/(\tan(dx) \\
& ^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan(dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan \\
& (dx)*\tan(c) + 1))*\tan(c)^2 - 2*a^2*\tan(dx)*\tan(c)^2 - 2*b^2*\tan(c)^3 - a \\
& ^2*d*x + 3*b^2*d*x + a*b*\tan(dx)^2 + 5*a*b*\tan(dx)*\tan(c) + a*b*\tan(c)^2 \\
& + 2*a*b*\log(4*(\tan(c)^2 + 1)/(\tan(dx)^4*\tan(c)^2 - 2*\tan(dx)^3*\tan(c) + \tan \\
& (dx)^2*\tan(c)^2 + \tan(dx)^2 - 2*\tan(dx)*\tan(c) + 1)) + a^2*\tan(dx) - \\
& 3*b^2*\tan(dx) + a^2*\tan(c) - 3*b^2*\tan(c) - a*b)/(d*\tan(dx)^3*\tan(c)^3 + \\
& d*\tan(dx)^3*\tan(c) - d*\tan(dx)^2*\tan(c)^2 + d*\tan(dx)*\tan(c)^3 - d*\tan(dx) \\
& *x)^2 + d*\tan(dx)*\tan(c) - d*\tan(c)^2 - d)
\end{aligned}$$

3.25 $\int \sin(c + dx)(a + b \tan(c + dx))^2 dx$

Optimal. Leaf size=68

$$-\frac{a^2 \cos(c + dx)}{d} - \frac{2ab \sin(c + dx)}{d} + \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} + \frac{b^2 \cos(c + dx)}{d} + \frac{b^2 \sec(c + dx)}{d}$$

[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (b^2*Cos[c + d*x])/d + (b^2*Sec[c + d*x])/d - (2*a*b*Sin[c + d*x])/d

Rubi [A] time = 0.0736639, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3517, 2638, 2592, 321, 206, 2590, 14}

$$-\frac{a^2 \cos(c + dx)}{d} - \frac{2ab \sin(c + dx)}{d} + \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} + \frac{b^2 \cos(c + dx)}{d} + \frac{b^2 \sec(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^2,x]

[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (b^2*Cos[c + d*x])/d + (b^2*Sec[c + d*x])/d - (2*a*b*Sin[c + d*x])/d

Rule 3517

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \sin(c + dx)(a + b \tan(c + dx))^2 dx &= \int (a^2 \sin(c + dx) + 2ab \sin(c + dx) \tan(c + dx) + b^2 \sin(c + dx) \tan^2(c + dx)) dx \\ &= a^2 \int \sin(c + dx) dx + (2ab) \int \sin(c + dx) \tan(c + dx) dx + b^2 \int \sin(c + dx) \tan^2(c + dx) dx \\ &= -\frac{a^2 \cos(c + dx)}{d} + \frac{(2ab) \operatorname{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, \sin(c + dx)\right)}{d} - \frac{b^2 \operatorname{Subst}\left(\int \frac{1-x^2}{x^2} dx, x, \sin(c + dx)\right)}{d} \\ &= -\frac{a^2 \cos(c + dx)}{d} - \frac{2ab \sin(c + dx)}{d} + \frac{(2ab) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sin(c + dx)\right)}{d} - \frac{b^2 \sec(c + dx)}{d} \\ &= \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^2 \cos(c + dx)}{d} + \frac{b^2 \cos(c + dx)}{d} + \frac{b^2 \sec(c + dx)}{d} - \frac{2ab \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.439056, size = 111, normalized size = 1.63

$$\frac{\sec(c + dx) \left((a^2 - b^2) \cos(2(c + dx)) + a^2 + 2ab \sin(2(c + dx)) + 4ab \cos(c + dx) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right) - \sin\left(\frac{1}{2}(c + dx)\right) \right) \right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]*(a^2 - 3*b^2 + (a^2 - b^2)*Cos[2*(c + d*x)] + 4*a*b*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 2*a*b*Sin[2*(c + d*x)])/(2*d)
```

Maple [A] time = 0.036, size = 108, normalized size = 1.6

$$\frac{b^2 (\sin(dx + c))^4}{d \cos(dx + c)} + \frac{b^2 \cos(dx + c) (\sin(dx + c))^2}{d} + 2 \frac{b^2 \cos(dx + c)}{d} - 2 \frac{ab \sin(dx + c)}{d} + 2 \frac{ab \ln(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*b^2*sin(d*x+c)^4/cos(d*x+c)+1/d*b^2*cos(d*x+c)*sin(d*x+c)^2+2*b^2*cos(d*x+c)/d-2*a*b*sin(d*x+c)/d+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))-a^2*cos(d*x+c)/d
```

Maxima [A] time = 1.11944, size = 90, normalized size = 1.32

$$\frac{b^2 \left(\frac{1}{\cos(dx+c)} + \cos(dx+c) \right) + ab(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1) - 2 \sin(dx+c)) - a^2 \cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] (b^2*(1/cos(d*x + c) + cos(d*x + c)) + a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - a^2*cos(d*x + c))/d

Fricas [A] time = 2.32235, size = 230, normalized size = 3.38

$$\frac{ab \cos(dx+c) \log(\sin(dx+c)+1) - ab \cos(dx+c) \log(-\sin(dx+c)+1) - 2ab \cos(dx+c) \sin(dx+c) - (a^2 - b^2) \cos(dx+c)}{d \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] (a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - 2*a*b*cos(d*x + c)*sin(d*x + c) - (a^2 - b^2)*cos(d*x + c)^2 + b^2)/(d*cos(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx))^2 \sin(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**2,x)

[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x), x)

Giac [B] time = 4.53757, size = 3830, normalized size = 56.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] -(a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x

$$\begin{aligned} &) * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/ \\ & /2*c) + 1)) * \tan(1/2*d*x)^4 * \tan(1/2*c)^4 + a^2 * \tan(1/2*d*x)^4 * \tan(1/2*c)^4 - \\ & 2*b^2 * \tan(1/2*d*x)^4 * \tan(1/2*c)^4 - 4*a*b * \log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/ \\ & 2*d*x)^4 * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4 * \tan(1/2*c) + 2*\tan(1/2*d*x)^3 * \tan(\\ & 1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2 * \tan(1/2*c)^2 - 2*\tan(1/2*d*x)^ \\ & 3 + 2*\tan(1/2*d*x) * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1 \\ & /2*d*x) - 2*\tan(1/2*c) + 1)) * \tan(1/2*d*x)^3 * \tan(1/2*c)^3 + 4*a*b * \log(2*(\tan \\ & (1/2*c)^2 + 1)/(\tan(1/2*d*x)^4 * \tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4 * \tan(1/2*c) - \\ & 2*\tan(1/2*d*x)^3 * \tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2 * \tan(1/2* \\ & c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x) * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \\ & \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1)) * \tan(1/2*d*x)^3 * \tan(1/2*c \\ &)^3 - 4*a*b * \tan(1/2*d*x)^4 * \tan(1/2*c)^3 - 4*a*b * \tan(1/2*d*x)^3 * \tan(1/2*c)^4 \\ & - 2*a^2 * \tan(1/2*d*x)^4 * \tan(1/2*c)^2 - 8*a^2 * \tan(1/2*d*x)^3 * \tan(1/2*c)^3 + \\ & 8*b^2 * \tan(1/2*d*x)^3 * \tan(1/2*c)^3 - 2*a^2 * \tan(1/2*d*x)^2 * \tan(1/2*c)^4 - a*b \\ & * \log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4 * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4 * \tan \\ & (1/2*c) + 2*\tan(1/2*d*x)^3 * \tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x) \\ & ^2 * \tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x) * \tan(1/2*c)^2 + 2*\tan(1/ \\ & 2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1)) * \tan(1/2*d*x)^ \\ & 4 + a*b * \log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4 * \tan(1/2*c)^2 - 2*\tan(1/2*d \\ & *x)^4 * \tan(1/2*c) - 2*\tan(1/2*d*x)^3 * \tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1 \\ & /2*d*x)^2 * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x) * \tan(1/2*c)^2 + 2 \\ & * \tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1)) * \tan(1/ \\ & 2*d*x)^4 - 4*a*b * \log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4 * \tan(1/2*c)^2 + 2* \\ & \tan(1/2*d*x)^4 * \tan(1/2*c) + 2*\tan(1/2*d*x)^3 * \tan(1/2*c)^2 + \tan(1/2*d*x)^4 \\ & + 2*\tan(1/2*d*x)^2 * \tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 + 2*\tan(1/2*d*x) * \tan(1/2 \\ & *c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1 \\ &)) * \tan(1/2*d*x)^3 * \tan(1/2*c) + 4*a*b * \log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x) \\ & ^4 * \tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4 * \tan(1/2*c) - 2*\tan(1/2*d*x)^3 * \tan(1/2*c) \\ & ^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2 * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2* \\ & \tan(1/2*d*x) * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) \\ &) + 2*\tan(1/2*c) + 1)) * \tan(1/2*d*x)^3 * \tan(1/2*c) + 4*a*b * \tan(1/2*d*x)^4 * \tan \\ & (1/2*c) + 24*a*b * \tan(1/2*d*x)^3 * \tan(1/2*c)^2 - 4*a*b * \log(2*(\tan(1/2*c)^2 + \\ & 1)/(\tan(1/2*d*x)^4 * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4 * \tan(1/2*c) + 2*\tan(1/2*d \\ & *x)^3 * \tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2 * \tan(1/2*c)^2 - 2*\tan \\ & (1/2*d*x)^3 + 2*\tan(1/2*d*x) * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 \\ & - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1)) * \tan(1/2*d*x) * \tan(1/2*c)^3 + 4*a*b * \log \\ & (2*(\tan(1/2*c)^2 + 1)/(\tan(1/2*d*x)^4 * \tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4 * \tan(\\ & 1/2*c) - 2*\tan(1/2*d*x)^3 * \tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2 * \\ & \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x) * \tan(1/2*c)^2 + 2*\tan(1/2*d \\ & *x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1)) * \tan(1/2*d*x) * \tan \\ & (1/2*c)^3 + 24*a*b * \tan(1/2*d*x)^2 * \tan(1/2*c)^3 - a*b * \log(2*(\tan(1/2*c)^2 + \\ & 1)/(\tan(1/2*d*x)^4 * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4 * \tan(1/2*c) + 2*\tan(1/2*d \\ & *x)^3 * \tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2 * \tan(1/2*c)^2 - 2*\tan \\ & (1/2*d*x)^3 + 2*\tan(1/2*d*x) * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 \\ & - 2*\tan(1/2*d*x) - 2*\tan(1/2*c) + 1)) * \tan(1/2*c)^4 + a*b * \log(2*(\tan(1/2*c) \\ & ^2 + 1)/(\tan(1/2*d*x)^4 * \tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4 * \tan(1/2*c) - 2*\tan(\\ & 1/2*d*x)^3 * \tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2 * \tan(1/2*c)^2 + \\ & 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x) * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2 \\ & *c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1)) * \tan(1/2*c)^4 + 4*a*b * \tan(1/2*d* \\ & x) * \tan(1/2*c)^4 + a^2 * \tan(1/2*d*x)^4 - 2*b^2 * \tan(1/2*d*x)^4 + 8*a^2 * \tan(1/2 \\ & *d*x)^3 * \tan(1/2*c) - 8*b^2 * \tan(1/2*d*x)^3 * \tan(1/2*c) + 20*a^2 * \tan(1/2*d*x)^ \\ & 2 * \tan(1/2*c)^2 - 24*b^2 * \tan(1/2*d*x)^2 * \tan(1/2*c)^2 + 8*a^2 * \tan(1/2*d*x) * \tan \\ & (1/2*c)^3 - 8*b^2 * \tan(1/2*d*x) * \tan(1/2*c)^3 + a^2 * \tan(1/2*c)^4 - 2*b^2 * \tan \\ & (1/2*c)^4 - 4*a*b * \tan(1/2*d*x)^3 - 4*a*b * \log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2* \\ & d*x)^4 * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4 * \tan(1/2*c) + 2*\tan(1/2*d*x)^3 * \tan(1/ \\ & 2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2 * \tan(1/2*c)^2 - 2*\tan(1/2*d*x)^3 \\ & + 2*\tan(1/2*d*x) * \tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2*\tan(1/2 \\ & *d*x) - 2*\tan(1/2*c) + 1)) * \tan(1/2*d*x) * \tan(1/2*c) + 4*a*b * \log(2*(\tan(1/2*c) \\ &)^2 + 1)/(\tan(1/2*d*x)^4 * \tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4 * \tan(1/2*c) - 2*\tan \end{aligned}$$

$$\begin{aligned}
& (1/2*d*x)^3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + \\
& 2*\tan(1/2*d*x)^3 - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/ \\
& 2*c)^2 + 2*\tan(1/2*d*x) + 2*\tan(1/2*c) + 1))*\tan(1/2*d*x)*\tan(1/2*c) - 24*a \\
& *b*\tan(1/2*d*x)^2*\tan(1/2*c) - 24*a*b*\tan(1/2*d*x)*\tan(1/2*c)^2 - 4*a*b*\tan \\
& (1/2*c)^3 - 2*a^2*\tan(1/2*d*x)^2 - 8*a^2*\tan(1/2*d*x)*\tan(1/2*c) + 8*b^2*ta \\
& n(1/2*d*x)*\tan(1/2*c) - 2*a^2*\tan(1/2*c)^2 + a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\\
& \tan(1/2*d*x)^4*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^4*\tan(1/2*c) + 2*\tan(1/2*d*x)^ \\
& 3*\tan(1/2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 - 2*\tan(1/2 \\
& *d*x)^3 + 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 - 2 \\
& *\tan(1/2*d*x) - 2*\tan(1/2*c) + 1)) - a*b*\log(2*(\tan(1/2*c)^2 + 1)/(\tan(1/2* \\
& d*x)^4*\tan(1/2*c)^2 - 2*\tan(1/2*d*x)^4*\tan(1/2*c) - 2*\tan(1/2*d*x)^3*\tan(1/ \\
& 2*c)^2 + \tan(1/2*d*x)^4 + 2*\tan(1/2*d*x)^2*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^3 \\
& - 2*\tan(1/2*d*x)*\tan(1/2*c)^2 + 2*\tan(1/2*d*x)^2 + \tan(1/2*c)^2 + 2*\tan(1/2 \\
& *d*x) + 2*\tan(1/2*c) + 1)) + 4*a*b*\tan(1/2*d*x) + 4*a*b*\tan(1/2*c) + a^2 - \\
& 2*b^2)/(d*\tan(1/2*d*x)^4*\tan(1/2*c)^4 - 4*d*\tan(1/2*d*x)^3*\tan(1/2*c)^3 - d \\
& *\tan(1/2*d*x)^4 - 4*d*\tan(1/2*d*x)^3*\tan(1/2*c) - 4*d*\tan(1/2*d*x)*\tan(1/2* \\
& c)^3 - d*\tan(1/2*c)^4 - 4*d*\tan(1/2*d*x)*\tan(1/2*c) + d)
\end{aligned}$$

3.26 $\int \csc(c + dx)(a + b \tan(c + dx))^2 dx$

Optimal. Leaf size=43

$$-\frac{a^2 \tanh^{-1}(\cos(c + dx))}{d} + \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} + \frac{b^2 \sec(c + dx)}{d}$$

[Out] $-\frac{(a^2 \operatorname{ArcTanh}[\cos[c + d*x]])}{d} + \frac{(2*a*b*\operatorname{ArcTanh}[\sin[c + d*x]])}{d} + \frac{(b^2*\operatorname{Sec}[c + d*x])}{d}$

Rubi [A] time = 0.0563928, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {3517, 3770, 2606, 8}

$$-\frac{a^2 \tanh^{-1}(\cos(c + dx))}{d} + \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} + \frac{b^2 \sec(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csc}[c + d*x]*(a + b*\operatorname{Tan}[c + d*x])^2, x]$

[Out] $-\frac{(a^2 \operatorname{ArcTanh}[\cos[c + d*x]])}{d} + \frac{(2*a*b*\operatorname{ArcTanh}[\sin[c + d*x]])}{d} + \frac{(b^2*\operatorname{Sec}[c + d*x])}{d}$

Rule 3517

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{Expand}[\sin[e + f*x]^{m*(a + b*\operatorname{Tan}[e + f*x])^n}, x], x] /;$ $\operatorname{FreeQ}\{a, b, e, f, x\} \ \&\& \ \operatorname{IntegerQ}[(m - 1)/2] \ \&\& \ \operatorname{IGtQ}[n, 0]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\cos[c + d*x]]/d, x] /;$ $\operatorname{FreeQ}\{c, d, x\}$

Rule 2606

$\operatorname{Int}[(a_.)*\sec[(e_.) + (f_.)*(x_.)]^{(m_.)}*((b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \operatorname{Dist}[a/f, \operatorname{Subst}[\operatorname{Int}[(a*x)^{(m-1)}*(-1 + x^2)^{((n-1)/2)}, x], x, \operatorname{Sec}[e + f*x]], x] /;$ $\operatorname{FreeQ}\{a, e, f, m, x\} \ \&\& \ \operatorname{IntegerQ}[(n-1)/2] \ \&\& \ !(\operatorname{IntegerQ}[m/2] \ \&\& \ \operatorname{LtQ}[0, m, n + 1])$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /;$ $\operatorname{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned} \int \csc(c + dx)(a + b \tan(c + dx))^2 dx &= \int (a^2 \csc(c + dx) + 2ab \sec(c + dx) + b^2 \sec(c + dx) \tan(c + dx)) dx \\ &= a^2 \int \csc(c + dx) dx + (2ab) \int \sec(c + dx) dx + b^2 \int \sec(c + dx) \tan(c + dx) dx \\ &= -\frac{a^2 \tanh^{-1}(\cos(c + dx))}{d} + \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} + \frac{b^2 \operatorname{Subst}(\int 1 dx, x, \sec(c + dx))}{d} \\ &= -\frac{a^2 \tanh^{-1}(\cos(c + dx))}{d} + \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} + \frac{b^2 \sec(c + dx)}{d} \end{aligned}$$

Mathematica [B] time = 0.233457, size = 97, normalized size = 2.26

$$\frac{a \left(a \log \left(\sin \left(\frac{1}{2}(c + dx) \right) \right) - a \log \left(\cos \left(\frac{1}{2}(c + dx) \right) \right) - 2b \log \left(\cos \left(\frac{1}{2}(c + dx) \right) - \sin \left(\frac{1}{2}(c + dx) \right) \right) + 2b \log \left(\sin \left(\frac{1}{2}(c + dx) \right) + \cos \left(\frac{1}{2}(c + dx) \right) \right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^2,x]

[Out] (a*(-(a*Log[Cos[(c + d*x)/2]]) - 2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]) + a*Log[Sin[(c + d*x)/2]] + 2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + b^2*Sec[c + d*x])/d

Maple [A] time = 0.044, size = 61, normalized size = 1.4

$$\frac{b^2}{d \cos(dx + c)} + 2 \frac{ab \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{a^2 \ln(\csc(dx + c) - \cot(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^2,x)

[Out] 1/d*b^2/cos(d*x+c)+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))

Maxima [A] time = 1.06914, size = 81, normalized size = 1.88

$$\frac{ab(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) - a^2 \log(\cot(dx + c) + \csc(dx + c)) + \frac{b^2}{\cos(dx + c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] (a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - a^2*log(cot(d*x + c) + csc(d*x + c)) + b^2/cos(d*x + c))/d

Fricas [B] time = 2.4343, size = 288, normalized size = 6.7

$$\frac{a^2 \cos(dx + c) \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) - a^2 \cos(dx + c) \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) - 2ab \cos(dx + c) \log(\sin(dx + c) + 1) + 2ab \cos(dx + c) \log(-\sin(dx + c) + 1) - 2b^2}{2d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] -1/2*(a^2*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - a^2*cos(d*x + c)*log(-1/2*cos(d*x + c) + 1/2) - 2*a*b*cos(d*x + c)*log(sin(d*x + c) + 1) + 2*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - 2*b^2)/(d*cos(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx))^2 \csc(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**2,x)

[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x), x)

Giac [A] time = 1.5719, size = 100, normalized size = 2.33

$$\frac{2ab \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 2ab \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + a^2 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) - \frac{2b^2}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] (2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + a^2*log(abs(tan(1/2*d*x + 1/2*c)))) - 2*b^2/(tan(1/2*d*x + 1/2*c)^2 - 1))/d

3.27 $\int \csc^2(c + dx)(a + b \tan(c + dx))^2 dx$

Optimal. Leaf size=42

$$-\frac{a^2 \cot(c + dx)}{d} + \frac{2ab \log(\tan(c + dx))}{d} + \frac{b^2 \tan(c + dx)}{d}$$

[Out] $-\frac{(a^2 \cot[c + d*x])}{d} + \frac{(2*a*b*\log[\tan[c + d*x]])}{d} + \frac{(b^2*\tan[c + d*x])}{d}$

Rubi [A] time = 0.0474087, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 43}

$$-\frac{a^2 \cot(c + dx)}{d} + \frac{2ab \log(\tan(c + dx))}{d} + \frac{b^2 \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]

[Out] $-\frac{(a^2 \cot[c + d*x])}{d} + \frac{(2*a*b*\log[\tan[c + d*x]])}{d} + \frac{(b^2*\tan[c + d*x])}{d}$

Rule 3516

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 43

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \csc^2(c + dx)(a + b \tan(c + dx))^2 dx &= \frac{b \operatorname{Subst}\left(\int \frac{(a+x)^2}{x^2} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{b \operatorname{Subst}\left(\int \left(1 + \frac{a^2}{x^2} + \frac{2a}{x}\right) dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{a^2 \cot(c + dx)}{d} + \frac{2ab \log(\tan(c + dx))}{d} + \frac{b^2 \tan(c + dx)}{d} \end{aligned}$$

Mathematica [B] time = 0.544128, size = 91, normalized size = 2.17

$$\frac{\cos(c + dx)(a + b \tan(c + dx))^2 (a \cos(c + dx)(a \cot(c + dx) + 2b(\log(\cos(c + dx)) - \log(\sin(c + dx)))) - b^2 \sin(c + dx)}{d(a \cos(c + dx) + b \sin(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]

[Out] -((Cos[c + d*x]*(a*Cos[c + d*x]*(a*Cot[c + d*x] + 2*b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) - b^2*Sin[c + d*x])*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2))

Maple [A] time = 0.042, size = 43, normalized size = 1.

$$-\frac{a^2 \cot(dx + c)}{d} + 2 \frac{ab \ln(\tan(dx + c))}{d} + \frac{b^2 \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x)

[Out] -a^2*cot(d*x+c)/d+2*a*b*ln(tan(d*x+c))/d+b^2*tan(d*x+c)/d

Maxima [A] time = 1.10725, size = 53, normalized size = 1.26

$$\frac{2ab \log(\tan(dx + c)) + b^2 \tan(dx + c) - \frac{a^2}{\tan(dx + c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] (2*a*b*log(tan(d*x + c)) + b^2*tan(d*x + c) - a^2/tan(d*x + c))/d

Fricas [B] time = 2.28245, size = 246, normalized size = 5.86

$$\frac{ab \cos(dx + c) \log(\cos(dx + c)^2) \sin(dx + c) - ab \cos(dx + c) \log\left(-\frac{1}{4} \cos(dx + c)^2 + \frac{1}{4}\right) \sin(dx + c) + (a^2 + b^2) \cos(dx + c)}{d \cos(dx + c) \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] -(a*b*cos(d*x + c)*log(cos(d*x + c)^2)*sin(d*x + c) - a*b*cos(d*x + c)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + (a^2 + b^2)*cos(d*x + c)^2 - b^2)/(d*cos(d*x + c)*sin(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx))^2 \csc^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**2,x)

[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**2, x)

Giac [A] time = 1.53881, size = 69, normalized size = 1.64

$$\frac{2ab \log(|\tan(dx + c)|) + b^2 \tan(dx + c) - \frac{2ab \tan(dx+c)+a^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] (2*a*b*log(abs(tan(d*x + c))) + b^2*tan(d*x + c) - (2*a*b*tan(d*x + c) + a^2)/tan(d*x + c))/d

3.28 $\int \csc^3(c + dx)(a + b \tan(c + dx))^2 dx$

Optimal. Leaf size=95

$$-\frac{a^2 \tanh^{-1}(\cos(c + dx))}{2d} - \frac{a^2 \cot(c + dx) \csc(c + dx)}{2d} - \frac{2ab \csc(c + dx)}{d} + \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} + \frac{b^2 \sec(c + dx)}{d} - \frac{b^2 \csc(c + dx)}{d}$$

[Out] $-(a^2 \operatorname{ArcTanh}[\cos[c + d*x]])/(2*d) - (b^2 \operatorname{ArcTanh}[\cos[c + d*x]])/d + (2*a*b \operatorname{ArcTanh}[\sin[c + d*x]])/d - (2*a*b*\csc[c + d*x])/d - (a^2*\cot[c + d*x]*\csc[c + d*x])/(2*d) + (b^2*\sec[c + d*x])/d$

Rubi [A] time = 0.117606, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3517, 3768, 3770, 2621, 321, 207, 2622}

$$-\frac{a^2 \tanh^{-1}(\cos(c + dx))}{2d} - \frac{a^2 \cot(c + dx) \csc(c + dx)}{2d} - \frac{2ab \csc(c + dx)}{d} + \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} + \frac{b^2 \sec(c + dx)}{d} - \frac{b^2 \csc(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\csc[c + d*x]^3*(a + b*\tan[c + d*x])^2, x]$

[Out] $-(a^2 \operatorname{ArcTanh}[\cos[c + d*x]])/(2*d) - (b^2 \operatorname{ArcTanh}[\cos[c + d*x]])/d + (2*a*b \operatorname{ArcTanh}[\sin[c + d*x]])/d - (2*a*b*\csc[c + d*x])/d - (a^2*\cot[c + d*x]*\csc[c + d*x])/(2*d) + (b^2*\sec[c + d*x])/d$

Rule 3517

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{Expand}[\sin[e + f*x]^{m*(a + b*\tan[e + f*x])^n}, x], x] /; \text{FreeQ}\{a, b, e, f\}, x \ \&\& \ \text{IntegerQ}[(m - 1)/2] \ \&\& \ \text{IGtQ}[n, 0]$

Rule 3768

$\operatorname{Int}[(\csc[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\cos[c + d*x] * (b*\csc[c + d*x])^{(n - 1)})/(d*(n - 1)), x] + \operatorname{Dist}[(b^2*(n - 2))/(n - 1), \operatorname{Int}[(b*\csc[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 3770

$\operatorname{Int}[\csc[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\cos[c + d*x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2621

$\operatorname{Int}[(\csc[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*\sec[(e_.) + (f_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[(f*a^n)^{-1}, \operatorname{Subst}[\operatorname{Int}[x^{(m + n - 1)} / (-1 + x^2/a^2)^{((n + 1)/2)}, x], x, a*\csc[e + f*x]], x] /; \text{FreeQ}\{a, e, f, m\}, x \ \&\& \ \text{IntegerQ}[(n + 1)/2] \ \&\& \ !(\text{IntegerQ}[(m + 1)/2] \ \&\& \ \text{LtQ}[0, m, n])$

Rule 321

$\operatorname{Int}[(c_.)*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(c^{(n - 1)}*(c*x)^{(m - n + 1)}*(a + b*x^n)^{(p + 1)})/(b*(m + n*p + 1)), x] - \operatorname{Dist}[(a*c^{(n*(m - n + 1))})/(b*(m + n*p + 1)), \operatorname{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1] \ \&\& \ \text{NeQ}[m + n*p]$

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2622

Int[csc[(e_) + (f_)*(x_)]^(n_)*((a_)*sec[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rubi steps

$$\begin{aligned} \int \csc^3(c + dx)(a + b \tan(c + dx))^2 dx &= \int (a^2 \csc^3(c + dx) + 2ab \csc^2(c + dx) \sec(c + dx) + b^2 \csc(c + dx) \sec^2(c + dx)) dx \\ &= a^2 \int \csc^3(c + dx) dx + (2ab) \int \csc^2(c + dx) \sec(c + dx) dx + b^2 \int \csc(c + dx) \sec^2(c + dx) dx \\ &= -\frac{a^2 \cot(c + dx) \csc(c + dx)}{2d} + \frac{1}{2} a^2 \int \csc(c + dx) dx - \frac{(2ab) \operatorname{Subst}\left(\int \frac{x^2}{-1+x^2} dx\right)}{d} \\ &= -\frac{a^2 \tanh^{-1}(\cos(c + dx))}{2d} - \frac{2ab \csc(c + dx)}{d} - \frac{a^2 \cot(c + dx) \csc(c + dx)}{2d} + \frac{b^2 \sec^2(c + dx)}{d} \\ &= -\frac{a^2 \tanh^{-1}(\cos(c + dx))}{2d} - \frac{b^2 \tanh^{-1}(\cos(c + dx))}{d} + \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} \end{aligned}$$

Mathematica [B] time = 1.95109, size = 250, normalized size = 2.63

$$4(a^2 + 2b^2) \log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) - 4(a^2 + 2b^2) \log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right) - a^2 \csc^2\left(\frac{1}{2}(c + dx)\right) + a^2 \sec^2\left(\frac{1}{2}(c + dx)\right) - 8ab \cot\left(\frac{1}{2}(c + dx)\right) \csc\left(\frac{1}{2}(c + dx)\right) \sec\left(\frac{1}{2}(c + dx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^2, x]

[Out] (8*b^2 - 8*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 - 4*(a^2 + 2*b^2)*Log[Cos[(c + d*x)/2]] - 16*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*(a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]] + 16*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 + (8*b^2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (8*b^2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 8*a*b*Tan[(c + d*x)/2])/(8*d)

Maple [A] time = 0.051, size = 120, normalized size = 1.3

$$\frac{b^2}{d \cos(dx + c)} + \frac{b^2 \ln(\csc(dx + c) - \cot(dx + c))}{d} - 2 \frac{ab}{d \sin(dx + c)} + 2 \frac{ab \ln(\sec(dx + c) + \tan(dx + c))}{d} - \frac{a^2 \cot(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^2, x)

[Out] $1/d*b^2/\cos(d*x+c)+1/d*b^2*\ln(\csc(d*x+c)-\cot(d*x+c))-2/d*a*b/\sin(d*x+c)+2/d$
 $*a*b*\ln(\sec(d*x+c)+\tan(d*x+c))-1/2*a^2*\cot(d*x+c)*\csc(d*x+c)/d+1/2/d*a^2*\ln$
 $(\csc(d*x+c)-\cot(d*x+c))$

Maxima [A] time = 1.10054, size = 165, normalized size = 1.74

$$\frac{a^2 \left(\frac{2 \cos(dx+c)}{\cos(dx+c)^2-1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1) \right) + 2 b^2 \left(\frac{2}{\cos(dx+c)} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1) \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] $1/4*(a^2*(2*\cos(d*x+c)/(\cos(d*x+c)^2-1) - \log(\cos(d*x+c)+1) + \log(\cos(d*x+c)-1)) + 2*b^2*(2/\cos(d*x+c) - \log(\cos(d*x+c)+1) + \log(\cos(d*x+c)-1)) - 4*a*b*(2/\sin(d*x+c) - \log(\sin(d*x+c)+1) + \log(\sin(d*x+c)-1)))/d$

Fricas [B] time = 2.77791, size = 581, normalized size = 6.12

$$8ab \cos(dx+c) \sin(dx+c) + 2(a^2 + 2b^2) \cos(dx+c)^2 - 4b^2 - ((a^2 + 2b^2) \cos(dx+c)^3 - (a^2 + 2b^2) \cos(dx+c)) \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] $1/4*(8*a*b*\cos(d*x+c)*\sin(d*x+c) + 2*(a^2 + 2*b^2)*\cos(d*x+c)^2 - 4*b^2 - ((a^2 + 2*b^2)*\cos(d*x+c)^3 - (a^2 + 2*b^2)*\cos(d*x+c))*\log(1/2*\cos(d*x+c) + 1/2) + ((a^2 + 2*b^2)*\cos(d*x+c)^3 - (a^2 + 2*b^2)*\cos(d*x+c))*\log(-1/2*\cos(d*x+c) + 1/2) + 4*(a*b*\cos(d*x+c)^3 - a*b*\cos(d*x+c))*\log(\sin(d*x+c) + 1) - 4*(a*b*\cos(d*x+c)^3 - a*b*\cos(d*x+c))*\log(-\sin(d*x+c) + 1))/(d*\cos(d*x+c)^3 - d*\cos(d*x+c))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx))^2 \csc^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**2,x)

[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**3, x)

Giac [A] time = 1.63598, size = 232, normalized size = 2.44

$$a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 16 ab \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 16 ab \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) - 8 ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 4(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 16*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 8*a*b*tan(1/2*d*x + 1/2*c) +
4*(a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - 16*b^2/(tan(1/2*d*x + 1/2
*c)^2 - 1) - (6*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2
+ 8*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.29 $\int \csc^4(c + dx)(a + b \tan(c + dx))^2 dx$

Optimal. Leaf size=79

$$-\frac{(a^2 + b^2) \cot(c + dx)}{d} - \frac{a^2 \cot^3(c + dx)}{3d} - \frac{ab \cot^2(c + dx)}{d} + \frac{2ab \log(\tan(c + dx))}{d} + \frac{b^2 \tan(c + dx)}{d}$$

[Out] -(((a^2 + b^2)*Cot[c + d*x])/d) - (a*b*Cot[c + d*x]^2)/d - (a^2*Cot[c + d*x]^3)/(3*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d

Rubi [A] time = 0.0700101, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 894}

$$-\frac{(a^2 + b^2) \cot(c + dx)}{d} - \frac{a^2 \cot^3(c + dx)}{3d} - \frac{ab \cot^2(c + dx)}{d} + \frac{2ab \log(\tan(c + dx))}{d} + \frac{b^2 \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]

[Out] -(((a^2 + b^2)*Cot[c + d*x])/d) - (a*b*Cot[c + d*x]^2)/d - (a^2*Cot[c + d*x]^3)/(3*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 894

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \csc^4(c + dx)(a + b \tan(c + dx))^2 dx &= \frac{b \operatorname{Subst}\left(\int \frac{(a+x)^2(b^2+x^2)}{x^4} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{b \operatorname{Subst}\left(\int \left(1 + \frac{a^2 b^2}{x^4} + \frac{2ab^2}{x^3} + \frac{a^2 + b^2}{x^2} + \frac{2a}{x}\right) dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{(a^2 + b^2) \cot(c + dx)}{d} - \frac{ab \cot^2(c + dx)}{d} - \frac{a^2 \cot^3(c + dx)}{3d} + \frac{2ab \log(\tan(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 1.40886, size = 127, normalized size = 1.61

$$\frac{(a + b \tan(c + dx))^2 \left(\cos^2(c + dx) \left((2a^2 + 3b^2) \cot(c + dx) + 6ab(\log(\cos(c + dx)) - \log(\sin(c + dx))) \right) + a^2 \cot^3(c + dx) \right)}{3d(a \cos(c + dx) + b \sin(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]

[Out] $-\left(\left(3ab\cot^2[c + dx] + a^2\cot^3[c + dx] + \cos^2[c + dx]\left(2a^2 + 3b^2\right)\cot[c + dx] + 6ab\left(\log[\cos[c + dx]] - \log[\sin[c + dx]]\right)\right) - \left(3b^2\sin^2[2(c + dx)]\right)/2\right)\left(a + b\tan[c + dx]\right)^2 / \left(3d\left(a\cos[c + dx] + b\sin[c + dx]\right)^2\right)$

Maple [A] time = 0.053, size = 104, normalized size = 1.3

$$\frac{b^2}{d \sin(dx + c) \cos(dx + c)} - 2 \frac{b^2 \cot(dx + c)}{d} - \frac{ab}{d (\sin(dx + c))^2} + 2 \frac{ab \ln(\tan(dx + c))}{d} - \frac{2a^2 \cot(dx + c)}{3d} - \frac{a^2 \cot(dx + c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x)

[Out] $1/d*b^2/\sin(d*x+c)/\cos(d*x+c)-2/d*b^2*\cot(d*x+c)-1/d*a*b/\sin(d*x+c)^2+2*a*b*\ln(\tan(d*x+c))/d-2/3*a^2*\cot(d*x+c)/d-1/3/d*a^2*\cot(d*x+c)*\csc(d*x+c)^2$

Maxima [A] time = 1.08606, size = 93, normalized size = 1.18

$$\frac{6ab \log(\tan(dx + c)) + 3b^2 \tan(dx + c) - \frac{3ab \tan(dx + c) + 3(a^2 + b^2) \tan(dx + c)^2 + a^2}{\tan(dx + c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] $1/3*(6*a*b*\log(\tan(d*x + c)) + 3*b^2*\tan(d*x + c) - (3*a*b*\tan(d*x + c) + 3*(a^2 + b^2)*\tan(d*x + c)^2 + a^2)/\tan(d*x + c)^3)/d$

Fricas [B] time = 2.39439, size = 446, normalized size = 5.65

$$\frac{2(a^2 + 3b^2) \cos(dx + c)^4 - 3ab \cos(dx + c) \sin(dx + c) - 3(a^2 + 3b^2) \cos(dx + c)^2 + 3(ab \cos(dx + c)^3 - ab \cos(dx + c))}{3(d \cos(dx + c))^3 - 3d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] $-1/3*(2*(a^2 + 3*b^2)*\cos(d*x + c)^4 - 3*a*b*\cos(d*x + c)*\sin(d*x + c) - 3*(a^2 + 3*b^2)*\cos(d*x + c)^2 + 3*(a*b*\cos(d*x + c)^3 - a*b*\cos(d*x + c))*\log(\cos(d*x + c)^2*\sin(d*x + c) - 3*(a*b*\cos(d*x + c)^3 - a*b*\cos(d*x + c))*\log(-1/4*\cos(d*x + c)^2 + 1/4*\sin(d*x + c) + 3*b^2)/((d*\cos(d*x + c))^3 - d*\cos(d*x + c))*\sin(d*x + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**2,x)

[Out] Timed out

Giac [A] time = 1.61879, size = 123, normalized size = 1.56

$$\frac{6ab \log(|\tan(dx+c)|) + 3b^2 \tan(dx+c) - \frac{11ab \tan(dx+c)^3 + 3a^2 \tan(dx+c)^2 + 3b^2 \tan(dx+c)^2 + 3ab \tan(dx+c) + a^2}{\tan(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] 1/3*(6*a*b*log(abs(tan(d*x + c))) + 3*b^2*tan(d*x + c) - (11*a*b*tan(d*x + c)^3 + 3*a^2*tan(d*x + c)^2 + 3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2)/tan(d*x + c)^3)/d

3.30 $\int \csc^5(c + dx)(a + b \tan(c + dx))^2 dx$

Optimal. Leaf size=165

$$\frac{3a^2 \tanh^{-1}(\cos(c + dx))}{8d} - \frac{a^2 \cot(c + dx) \csc^3(c + dx)}{4d} - \frac{3a^2 \cot(c + dx) \csc(c + dx)}{8d} - \frac{2ab \csc^3(c + dx)}{3d} - \frac{2ab \csc(c + dx)}{d}$$

```
[Out] (-3*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (3*b^2*ArcTanh[Cos[c + d*x]])/(2*d)
+ (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (3*a^2*Cot[c +
d*x]*Csc[c + d*x])/(8*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x
]*Csc[c + d*x]^3)/(4*d) + (3*b^2*Sec[c + d*x])/(2*d) - (b^2*Csc[c + d*x]^2*
Sec[c + d*x])/(2*d)
```

Rubi [A] time = 0.160082, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3517, 3768, 3770, 2621, 302, 207, 2622, 288, 321}

$$\frac{3a^2 \tanh^{-1}(\cos(c + dx))}{8d} - \frac{a^2 \cot(c + dx) \csc^3(c + dx)}{4d} - \frac{3a^2 \cot(c + dx) \csc(c + dx)}{8d} - \frac{2ab \csc^3(c + dx)}{3d} - \frac{2ab \csc(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-3*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (3*b^2*ArcTanh[Cos[c + d*x]])/(2*d)
+ (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (3*a^2*Cot[c +
d*x]*Csc[c + d*x])/(8*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x
]*Csc[c + d*x]^3)/(4*d) + (3*b^2*Sec[c + d*x])/(2*d) - (b^2*Csc[c + d*x]^2*
Sec[c + d*x])/(2*d)
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] := Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] := -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n +
1)/2], x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_) + (f_)*(x_)]^(n_)*((a_)*sec[(e_) + (f_)*(x_)])^(m_), x_S
ymbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \csc^5(c + dx)(a + b \tan(c + dx))^2 dx &= \int (a^2 \csc^5(c + dx) + 2ab \csc^4(c + dx) \sec(c + dx) + b^2 \csc^3(c + dx) \sec^2(c + dx)) dx \\
&= a^2 \int \csc^5(c + dx) dx + (2ab) \int \csc^4(c + dx) \sec(c + dx) dx + b^2 \int \csc^3(c + dx) dx \\
&= -\frac{a^2 \cot(c + dx) \csc^3(c + dx)}{4d} + \frac{1}{4} (3a^2) \int \csc^3(c + dx) dx - \frac{(2ab) \text{Subst}\left(\int \frac{x^4}{-1+x^2} dx\right)}{4d} \\
&= -\frac{3a^2 \cot(c + dx) \csc(c + dx)}{8d} - \frac{a^2 \cot(c + dx) \csc^3(c + dx)}{4d} - \frac{b^2 \csc^2(c + dx) \sec(c + dx)}{2d} \\
&= -\frac{3a^2 \tanh^{-1}(\cos(c + dx))}{8d} - \frac{2ab \csc(c + dx)}{d} - \frac{3a^2 \cot(c + dx) \csc(c + dx)}{8d} - \frac{2ab \tanh^{-1}(\sin(c + dx))}{d} \\
&= -\frac{3a^2 \tanh^{-1}(\cos(c + dx))}{8d} - \frac{3b^2 \tanh^{-1}(\cos(c + dx))}{2d} + \frac{2ab \tanh^{-1}(\sin(c + dx))}{d}
\end{aligned}$$

Mathematica [B] time = 6.19963, size = 994, normalized size = 6.02

$$-\frac{a^2 \cos^2(c + dx)(a + b \tan(c + dx))^2 \csc^4\left(\frac{1}{2}(c + dx)\right)}{64d(a \cos(c + dx) + b \sin(c + dx))^2} + \frac{(-3a^2 - 4b^2) \cos^2(c + dx)(a + b \tan(c + dx))^2 \csc^2\left(\frac{1}{2}(c + dx)\right)}{32d(a \cos(c + dx) + b \sin(c + dx))^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]

[Out] $(b^2 \cos[c + d*x]^2 (a + b \tan[c + d*x])^2) / (d (a \cos[c + d*x] + b \sin[c + d*x])^2) - (7 a b \cos[c + d*x]^2 \cot[(c + d*x)/2] (a + b \tan[c + d*x])^2) / (6 d (a \cos[c + d*x] + b \sin[c + d*x])^2) + ((-3 a^2 - 4 b^2) \cos[c + d*x]^2 \csc[(c + d*x)/2]^2 (a + b \tan[c + d*x])^2) / (32 d (a \cos[c + d*x] + b \sin[c + d*x])^2) - (a b \cos[c + d*x]^2 \cot[(c + d*x)/2] \csc[(c + d*x)/2]^2 (a + b \tan[c + d*x])^2) / (12 d (a \cos[c + d*x] + b \sin[c + d*x])^2) - (a^2 \cos[c + d*x]^2 \csc[(c + d*x)/2]^4 (a + b \tan[c + d*x])^2) / (64 d (a \cos[c + d*x] + b \sin[c + d*x])^2) - (3 (a^2 + 4 b^2) \cos[c + d*x]^2 \log[\cos[(c + d*x)/2]] (a + b \tan[c + d*x])^2) / (8 d (a \cos[c + d*x] + b \sin[c + d*x])^2) - (2 a b \cos[c + d*x]^2 \log[\cos[(c + d*x)/2] - \sin[(c + d*x)/2]] (a + b \tan[c + d*x])^2) / (d (a \cos[c + d*x] + b \sin[c + d*x])^2) + (3 (a^2 + 4 b^2) \cos[c + d*x]^2 \log[\sin[(c + d*x)/2]] (a + b \tan[c + d*x])^2) / (8 d (a \cos[c + d*x] + b \sin[c + d*x])^2) + (2 a b \cos[c + d*x]^2 \log[\cos[(c + d*x)/2] + \sin[(c + d*x)/2]] (a + b \tan[c + d*x])^2) / (d (a \cos[c + d*x] + b \sin[c + d*x])^2) + ((3 a^2 + 4 b^2) \cos[c + d*x]^2 \sec[(c + d*x)/2]^2 (a + b \tan[c + d*x])^2) / (32 d (a \cos[c + d*x] + b \sin[c + d*x])^2) + (a^2 \cos[c + d*x]^2 \sec[(c + d*x)/2]^4 (a + b \tan[c + d*x])^2) / (64 d (a \cos[c + d*x] + b \sin[c + d*x])^2) + (b^2 \cos[c + d*x]^2 \sin[(c + d*x)/2] (a + b \tan[c + d*x])^2) / (d (\cos[(c + d*x)/2] - \sin[(c + d*x)/2]) (a \cos[c + d*x] + b \sin[c + d*x])^2) - (b^2 \cos[c + d*x]^2 \sin[(c + d*x)/2] (a + b \tan[c + d*x])^2) / (d (\cos[(c + d*x)/2] + \sin[(c + d*x)/2]) (a \cos[c + d*x] + b \sin[c + d*x])^2) - (7 a b \cos[c + d*x]^2 \tan[(c + d*x)/2] (a + b \tan[c + d*x])^2) / (6 d (a \cos[c + d*x] + b \sin[c + d*x])^2) - (a b \cos[c + d*x]^2 \sec[(c + d*x)/2]^2 \tan[(c + d*x)/2] (a + b \tan[c + d*x])^2) / (12 d (a \cos[c + d*x] + b \sin[c + d*x])^2)$

Maple [A] time = 0.054, size = 183, normalized size = 1.1

$$-\frac{b^2}{2d(\sin(dx+c))^2 \cos(dx+c)} + \frac{3b^2}{2d \cos(dx+c)} + \frac{3b^2 \ln(\csc(dx+c) - \cot(dx+c))}{2d} - \frac{2ab}{3d(\sin(dx+c))^3} - 2 \frac{ab}{d \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x)

[Out] $-1/2/d*b^2/\sin(d*x+c)^2/\cos(d*x+c)+3/2/d*b^2/\cos(d*x+c)+3/2/d*b^2*\ln(\csc(d*x+c)-\cot(d*x+c))-2/3/d*a*b/\sin(d*x+c)^3-2/d*a*b/\sin(d*x+c)+2/d*a*b*\ln(\sec(d*x+c)+\tan(d*x+c))-1/4*a^2*\cot(d*x+c)*\csc(d*x+c)^3/d-3/8*a^2*\cot(d*x+c)*\csc(d*x+c)/d+3/8/d*a^2*\ln(\csc(d*x+c)-\cot(d*x+c))$

Maxima [A] time = 1.07451, size = 252, normalized size = 1.53

$$3a^2 \left(\frac{2(3 \cos(dx+c)^3 - 5 \cos(dx+c))}{\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1} - 3 \log(\cos(dx+c) + 1) + 3 \log(\cos(dx+c) - 1) \right) + 12b^2 \left(\frac{2(3 \cos(dx+c)^2 - 2)}{\cos(dx+c)^3 - \cos(dx+c)} - 3 \log(\cos(dx+c) + 1) + 3 \log(\cos(dx+c) - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] $1/48*(3*a^2*(2*(3*\cos(d*x + c)^3 - 5*\cos(d*x + c)))/(\cos(d*x + c)^4 - 2*\cos(d*x + c)^2 + 1) - 3*\log(\cos(d*x + c) + 1) + 3*\log(\cos(d*x + c) - 1)) + 12*b^2*(2*(3*\cos(d*x + c)^2 - 2)/(\cos(d*x + c)^3 - \cos(d*x + c)) - 3*\log(\cos(d*x + c) + 1) + 3*\log(\cos(d*x + c) - 1))$

$x + c) + 1) + 3 \log(\cos(dx + c) - 1) - 16ab(2(3 \sin(dx + c)^2 + 1) / \sin(dx + c)^3 - 3 \log(\sin(dx + c) + 1) + 3 \log(\sin(dx + c) - 1)) / d$

Fricas [B] time = 2.97415, size = 856, normalized size = 5.19

$18(a^2 + 4b^2) \cos(dx + c)^4 - 30(a^2 + 4b^2) \cos(dx + c)^2 + 48b^2 - 9((a^2 + 4b^2) \cos(dx + c)^5 - 2(a^2 + 4b^2) \cos(dx + c)) / d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)^5*(a+b*tan(dx+c))^2,x, algorithm="fricas")

[Out] $\frac{1}{48} (18(a^2 + 4b^2) \cos(dx + c)^4 - 30(a^2 + 4b^2) \cos(dx + c)^2 + 48b^2 - 9((a^2 + 4b^2) \cos(dx + c)^5 - 2(a^2 + 4b^2) \cos(dx + c)) \log(\frac{1}{2} \cos(dx + c) + \frac{1}{2}) + 9((a^2 + 4b^2) \cos(dx + c)^5 - 2(a^2 + 4b^2) \cos(dx + c)^3 + (a^2 + 4b^2) \cos(dx + c)) \log(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}) + 48(a b \cos(dx + c)^5 - 2a b \cos(dx + c)^3 + a b \cos(dx + c)) \log(\sin(dx + c) + 1) - 48(a b \cos(dx + c)^5 - 2a b \cos(dx + c)^3 + a b \cos(dx + c)) \log(-\sin(dx + c) + 1) + 32(3a b \cos(dx + c)^3 - 4a b \cos(dx + c)) \sin(dx + c)) / (d \cos(dx + c)^5 - 2d \cos(dx + c)^3 + d \cos(dx + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)**5*(a+b*tan(dx+c))**2,x)

[Out] Timed out

Giac [A] time = 1.6524, size = 363, normalized size = 2.2

$3a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 16ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 24a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 24b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 384ab \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)^5*(a+b*tan(dx+c))^2,x, algorithm="giac")

[Out] $\frac{1}{192} (3a^2 \tan(1/2 dx + 1/2 c)^4 - 16ab \tan(1/2 dx + 1/2 c)^3 + 24a^2 \tan(1/2 dx + 1/2 c)^2 + 24b^2 \tan(1/2 dx + 1/2 c)^2 + 384ab \log(\abs{\tan(1/2 dx + 1/2 c) + 1}) - 384ab \log(\abs{\tan(1/2 dx + 1/2 c) - 1}) - 240ab \tan(1/2 dx + 1/2 c) + 72(a^2 + 4b^2) \log(\abs{\tan(1/2 dx + 1/2 c)}) - 384b^2 / (\tan(1/2 dx + 1/2 c)^2 - 1) - (150a^2 \tan(1/2 dx + 1/2 c)^4 + 600b^2 \tan(1/2 dx + 1/2 c)^4 + 240ab \tan(1/2 dx + 1/2 c)^3 + 24a^2 \tan(1/2 dx + 1/2 c)^2 + 24b^2 \tan(1/2 dx + 1/2 c)^2 + 16ab \tan(1/2 dx + 1/2 c) + 3a^2) / \tan(1/2 dx + 1/2 c)^4) / d$

3.31 $\int \csc^6(c + dx)(a + b \tan(c + dx))^2 dx$

Optimal. Leaf size=122

$$\frac{(2a^2 + b^2) \cot^3(c + dx)}{3d} - \frac{(a^2 + 2b^2) \cot(c + dx)}{d} - \frac{a^2 \cot^5(c + dx)}{5d} - \frac{ab \cot^4(c + dx)}{2d} - \frac{2ab \cot^2(c + dx)}{d} + \frac{2ab \log[\tan(c + dx)]}{d}$$

[Out] -(((a^2 + 2*b^2)*Cot[c + d*x])/d) - (2*a*b*Cot[c + d*x]^2)/d - ((2*a^2 + b^2)*Cot[c + d*x]^3)/(3*d) - (a*b*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5)/(5*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d

Rubi [A] time = 0.101281, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 948}

$$\frac{(2a^2 + b^2) \cot^3(c + dx)}{3d} - \frac{(a^2 + 2b^2) \cot(c + dx)}{d} - \frac{a^2 \cot^5(c + dx)}{5d} - \frac{ab \cot^4(c + dx)}{2d} - \frac{2ab \cot^2(c + dx)}{d} + \frac{2ab \log[\tan(c + dx)]}{d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]

[Out] -(((a^2 + 2*b^2)*Cot[c + d*x])/d) - (2*a*b*Cot[c + d*x]^2)/d - ((2*a^2 + b^2)*Cot[c + d*x]^3)/(3*d) - (a*b*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5)/(5*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d

Rule 3516

Int[sin[(e_.) + (f_.)*(x_.)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_), x_Symbol] := Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 948

Int[((d_.) + (e_.)*(x_.))^(m_)*((f_.) + (g_.)*(x_.))^(n_)*((a_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[d, 0]))

Rubi steps

$$\begin{aligned} \int \csc^6(c + dx)(a + b \tan(c + dx))^2 dx &= \frac{b \operatorname{Subst}\left(\int \frac{(a+x)^2(b^2+x^2)^2}{x^6} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{b \operatorname{Subst}\left(\int \left(1 + \frac{a^2 b^4}{x^6} + \frac{2ab^4}{x^5} + \frac{2a^2 b^2 + b^4}{x^4} + \frac{4ab^2}{x^3} + \frac{a^2 + 2b^2}{x^2} + \frac{2a}{x}\right) dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{(a^2 + 2b^2) \cot(c + dx)}{d} - \frac{2ab \cot^2(c + dx)}{d} - \frac{(2a^2 + b^2) \cot^3(c + dx)}{3d} - \frac{ab \cot^4(c + dx)}{2d} - \frac{2ab \cot^2(c + dx)}{d} + \frac{2ab \log[\tan(c + dx)]}{d} \end{aligned}$$

Mathematica [A] time = 1.5103, size = 114, normalized size = 0.93

$$\frac{2 \cot(c + dx) \left((4a^2 + 5b^2) \csc^2(c + dx) + 3a^2 \csc^4(c + dx) + 8a^2 + 25b^2 \right) + 15b \left(a \csc^4(c + dx) + 2a \csc^2(c + dx) - 4 \right)}{30d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]

[Out] $-(2*\cot[c + d*x]*(8*a^2 + 25*b^2 + (4*a^2 + 5*b^2)*\csc[c + d*x]^2 + 3*a^2*\csc[c + d*x]^4) + 15*b*(2*a*\csc[c + d*x]^2 + a*\csc[c + d*x]^4 + 4*a*\log[\cos[c + d*x]] - 4*a*\log[\sin[c + d*x]] - 2*b*\tan[c + d*x]))/(30*d)$

Maple [A] time = 0.055, size = 166, normalized size = 1.4

$$-\frac{b^2}{3d(\sin(dx+c))^3\cos(dx+c)} + \frac{4b^2}{3d\sin(dx+c)\cos(dx+c)} - \frac{8b^2\cot(dx+c)}{3d} - \frac{ab}{2d(\sin(dx+c))^4} - \frac{ab}{d(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x)

[Out] $-1/3/d*b^2/\sin(d*x+c)^3/\cos(d*x+c)+4/3/d*b^2/\sin(d*x+c)/\cos(d*x+c)-8/3/d*b^2*\cot(d*x+c)-1/2/d*a*b/\sin(d*x+c)^4-1/d*a*b/\sin(d*x+c)^2+2*a*b*\ln(\tan(d*x+c))/d-8/15*a^2*\cot(d*x+c)/d-1/5/d*a^2*\cot(d*x+c)*\csc(d*x+c)^4-4/15/d*a^2*\cot(d*x+c)*\csc(d*x+c)^2$

Maxima [A] time = 1.03974, size = 140, normalized size = 1.15

$$\frac{60ab\log(\tan(dx+c)) + 30b^2\tan(dx+c) - \frac{60ab\tan(dx+c)^3 + 30(a^2+2b^2)\tan(dx+c)^4 + 15ab\tan(dx+c) + 10(2a^2+b^2)\tan(dx+c)^2 + 6a^2}{\tan(dx+c)^5}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] $1/30*(60*a*b*\log(\tan(d*x + c)) + 30*b^2*\tan(d*x + c) - (60*a*b*\tan(d*x + c)^3 + 30*(a^2 + 2*b^2)*\tan(d*x + c)^4 + 15*a*b*\tan(d*x + c) + 10*(2*a^2 + b^2)*\tan(d*x + c)^2 + 6*a^2)/\tan(d*x + c)^5)/d$

Fricas [B] time = 2.42819, size = 628, normalized size = 5.15

$$\frac{16(a^2 + 5b^2)\cos(dx+c)^6 - 40(a^2 + 5b^2)\cos(dx+c)^4 + 30(a^2 + 5b^2)\cos(dx+c)^2 + 30(ab\cos(dx+c)^5 - 2ab\cos(dx+c)^3 + a^2\cos(dx+c))\log(\cos(dx+c)^2)\sin(dx+c) - 30(a*b*\cos(d*x + c)^5 - 2*a*b*\cos(d*x + c)^3 + a*b*\cos(d*x + c))*\log(-1/4*\cos(d*x + c)^2 + 1/4)*\sin(d*x + c) - 30*b^2 - 15*(2*a*b*\cos(d*x + c)^3 - 3*a*b*\cos(d*x + c))*\sin(d*x + c)}{((d*\cos(d*x + c))^5 - 2*d*\cos(d*x + c)^3 + d*\cos(d*x + c))*\sin(d*x + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] $-1/30*(16*(a^2 + 5*b^2)*\cos(d*x + c)^6 - 40*(a^2 + 5*b^2)*\cos(d*x + c)^4 + 30*(a^2 + 5*b^2)*\cos(d*x + c)^2 + 30*(a*b*\cos(d*x + c)^5 - 2*a*b*\cos(d*x + c)^3 + a*b*\cos(d*x + c))*\log(\cos(d*x + c)^2)*\sin(d*x + c) - 30*(a*b*\cos(d*x + c)^5 - 2*a*b*\cos(d*x + c)^3 + a*b*\cos(d*x + c))*\log(-1/4*\cos(d*x + c)^2 + 1/4)*\sin(d*x + c) - 30*b^2 - 15*(2*a*b*\cos(d*x + c)^3 - 3*a*b*\cos(d*x + c))*\sin(d*x + c))/((d*\cos(d*x + c))^5 - 2*d*\cos(d*x + c)^3 + d*\cos(d*x + c))*\sin(d*x + c)$

$\sin(d*x + c)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**2,x)

[Out] Timed out

Giac [A] time = 1.40844, size = 177, normalized size = 1.45

$$60 ab \log(|\tan(dx + c)|) + 30 b^2 \tan(dx + c) - \frac{137 ab \tan(dx+c)^5 + 30 a^2 \tan(dx+c)^4 + 60 b^2 \tan(dx+c)^4 + 60 ab \tan(dx+c)^3 + 20 a^2 \tan(dx+c)^2}{\tan(dx+c)^5}$$

$$30 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] 1/30*(60*a*b*log(abs(tan(d*x + c))) + 30*b^2*tan(d*x + c) - (137*a*b*tan(d*x + c)^5 + 30*a^2*tan(d*x + c)^4 + 60*b^2*tan(d*x + c)^4 + 60*a*b*tan(d*x + c)^3 + 20*a^2*tan(d*x + c)^2 + 10*b^2*tan(d*x + c)^2 + 15*a*b*tan(d*x + c) + 6*a^2)/tan(d*x + c)^5)/d

3.32 $\int \sin^3(c + dx)(a + b \tan(c + dx))^3 dx$

Optimal. Leaf size=205

$$-\frac{a^2 b \sin^3(c + dx)}{d} - \frac{3a^2 b \sin(c + dx)}{d} + \frac{3a^2 b \tanh^{-1}(\sin(c + dx))}{d} + \frac{a^3 \cos^3(c + dx)}{3d} - \frac{a^3 \cos(c + dx)}{d} - \frac{ab^2 \cos^3(c + dx)}{d}$$

[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (a^3*Cos[c + d*x])/d + (6*a*b^2*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) - (a*b^2*Cos[c + d*x]^3)/d + (3*a*b^2*Sec[c + d*x])/d - (3*a^2*b*Sin[c + d*x])/d + (5*b^3*Sin[c + d*x])/(2*d) - (a^2*b*Sin[c + d*x]^3)/d + (5*b^3*Sin[c + d*x]^3)/(6*d) + (b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/(2*d)

Rubi [A] time = 0.188873, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3517, 2633, 2592, 302, 206, 2590, 270, 288}

$$-\frac{a^2 b \sin^3(c + dx)}{d} - \frac{3a^2 b \sin(c + dx)}{d} + \frac{3a^2 b \tanh^{-1}(\sin(c + dx))}{d} + \frac{a^3 \cos^3(c + dx)}{3d} - \frac{a^3 \cos(c + dx)}{d} - \frac{ab^2 \cos^3(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]

[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (a^3*Cos[c + d*x])/d + (6*a*b^2*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) - (a*b^2*Cos[c + d*x]^3)/d + (3*a*b^2*Sec[c + d*x])/d - (3*a^2*b*Sin[c + d*x])/d + (5*b^3*Sin[c + d*x])/(2*d) - (a^2*b*Sin[c + d*x]^3)/d + (5*b^3*Sin[c + d*x]^3)/(6*d) + (b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/(2*d)

Rule 3517

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_) + (f_)*(x_)]^(m_)*tan[(e_) + (f_)*(x_)]^(n_), x_Symbol]
:= -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \sin^3(c + dx)(a + b \tan(c + dx))^3 dx &= \int (a^3 \sin^3(c + dx) + 3a^2b \sin^3(c + dx) \tan(c + dx) + 3ab^2 \sin^3(c + dx) \tan^2(c + dx) + b^3 \sin^3(c + dx) \tan^3(c + dx)) dx \\
&= a^3 \int \sin^3(c + dx) dx + (3a^2b) \int \sin^3(c + dx) \tan(c + dx) dx + (3ab^2) \int \sin^3(c + dx) \tan^2(c + dx) dx + b^3 \int \sin^3(c + dx) \tan^3(c + dx) dx \\
&= -\frac{a^3 \operatorname{Subst}\left(\int (1 - x^2) dx, x, \cos(c + dx)\right)}{d} + \frac{(3a^2b) \operatorname{Subst}\left(\int \frac{x^4}{1 - x^2} dx, x, \sin(c + dx)\right)}{d} \\
&= -\frac{a^3 \cos(c + dx)}{d} + \frac{a^3 \cos^3(c + dx)}{3d} + \frac{b^3 \sin^3(c + dx) \tan^2(c + dx)}{2d} + \frac{(3a^2b) \operatorname{Subst}\left(\int \frac{x^4}{1 - x^2} dx, x, \sin(c + dx)\right)}{d} \\
&= -\frac{a^3 \cos(c + dx)}{d} + \frac{6ab^2 \cos(c + dx)}{d} + \frac{a^3 \cos^3(c + dx)}{3d} - \frac{ab^2 \cos^3(c + dx)}{d} + \frac{3a^2b \operatorname{tanh}^{-1}(\sin(c + dx))}{d} \\
&= \frac{3a^2b \operatorname{tanh}^{-1}(\sin(c + dx))}{d} - \frac{a^3 \cos(c + dx)}{d} + \frac{6ab^2 \cos(c + dx)}{d} + \frac{a^3 \cos^3(c + dx)}{3d} \\
&= \frac{3a^2b \operatorname{tanh}^{-1}(\sin(c + dx))}{d} - \frac{5b^3 \operatorname{tanh}^{-1}(\sin(c + dx))}{2d} - \frac{a^3 \cos(c + dx)}{d} + \frac{6ab^2 \cos(c + dx)}{d} + \frac{a^3 \cos^3(c + dx)}{3d}
\end{aligned}$$

Mathematica [B] time = 6.26557, size = 771, normalized size = 3.76

$$\frac{3a(a^2 - 7b^2) \cos^4(c + dx)(a + b \tan(c + dx))^3}{4d(a \cos(c + dx) + b \sin(c + dx))^3} + \frac{b(3a^2 - b^2) \sin(3(c + dx)) \cos^3(c + dx)(a + b \tan(c + dx))^3}{12d(a \cos(c + dx) + b \sin(c + dx))^3} + \frac{a(a^2 - 7b^2) \cos^4(c + dx)(a + b \tan(c + dx))^3}{4d(a \cos(c + dx) + b \sin(c + dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^3, x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (3*a*(a^2 - 7*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3)/(4
```

$$\begin{aligned} & *d*(a*\cos[c + d*x] + b*\sin[c + d*x])^3 + (a*(a^2 - 3*b^2)*\cos[c + d*x]^3*\cos[3*(c + d*x)]*(a + b*\tan[c + d*x])^3)/(12*d*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) + ((-6*a^2*b + 5*b^3)*\cos[c + d*x]^3*\log[\cos[(c + d*x)/2] - \sin[(c + d*x)/2]]*(a + b*\tan[c + d*x])^3)/(2*d*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) + ((6*a^2*b - 5*b^3)*\cos[c + d*x]^3*\log[\cos[(c + d*x)/2] + \sin[(c + d*x)/2]]*(a + b*\tan[c + d*x])^3)/(2*d*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) + (b^3*\cos[c + d*x]^3*(a + b*\tan[c + d*x])^3)/(4*d*(\cos[(c + d*x)/2] - \sin[(c + d*x)/2])^2*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) + (3*a*b^2*\cos[c + d*x]^3*\sin[(c + d*x)/2]*(a + b*\tan[c + d*x])^3)/(d*(\cos[(c + d*x)/2] - \sin[(c + d*x)/2])*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) - (b^3*\cos[c + d*x]^3*(a + b*\tan[c + d*x])^3)/(4*d*(\cos[(c + d*x)/2] + \sin[(c + d*x)/2])^2*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) - (3*a*b^2*\cos[c + d*x]^3*\sin[(c + d*x)/2]*(a + b*\tan[c + d*x])^3)/(d*(\cos[(c + d*x)/2] + \sin[(c + d*x)/2])*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) - (3*b*(5*a^2 - 3*b^2)*\cos[c + d*x]^3*\sin[c + d*x]*(a + b*\tan[c + d*x])^3)/(4*d*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) + (b*(3*a^2 - b^2)*\cos[c + d*x]^3*\sin[3*(c + d*x)]*(a + b*\tan[c + d*x])^3)/(12*d*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) \end{aligned}$$

Maple [A] time = 0.053, size = 271, normalized size = 1.3

$$\frac{b^3 (\sin(dx + c))^7}{2d (\cos(dx + c))^2} + \frac{b^3 (\sin(dx + c))^5}{2d} + \frac{5b^3 (\sin(dx + c))^3}{6d} + \frac{5b^3 \sin(dx + c)}{2d} - \frac{5b^3 \ln(\sec(dx + c) + \tan(dx + c))}{2d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x)

[Out] 1/2/d*b^3*sin(d*x+c)^7/cos(d*x+c)^2+1/2/d*b^3*sin(d*x+c)^5+5/6*b^3*sin(d*x+c)^3/d+5/2*b^3*sin(d*x+c)/d-5/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*a*b^2*sin(d*x+c)^6/cos(d*x+c)+8*a*b^2*cos(d*x+c)/d+3/d*a*b^2*cos(d*x+c)*sin(d*x+c)^4+4/d*cos(d*x+c)*sin(d*x+c)^2*a*b^2-a^2*b*sin(d*x+c)^3/d-3*a^2*b*sin(d*x+c)/d+3/d*b*a^2*ln(sec(d*x+c)+tan(d*x+c))-1/3/d*cos(d*x+c)*sin(d*x+c)^2*a^3-2/3*a^3*cos(d*x+c)/d

Maxima [A] time = 1.03738, size = 234, normalized size = 1.14

$$4(\cos(dx + c)^3 - 3\cos(dx + c))a^3 - 6(2\sin(dx + c)^3 - 3\log(\sin(dx + c) + 1) + 3\log(\sin(dx + c) - 1) + 6\sin(dx + c) - 6\cos(dx + c))a^2b - 12(\cos(dx + c)^3 - 3/\cos(dx + c) - 6\cos(dx + c))a*b^2 + (4\sin(dx + c)^3 - 6\sin(dx + c)/(\sin(dx + c)^2 - 1) - 15\log(\sin(dx + c) + 1) + 15\log(\sin(dx + c) - 1) + 24\sin(dx + c))*b^3/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] 1/12*(4*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3 - 6*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^2*b - 12*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a*b^2 + (4*sin(d*x + c)^3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 24*sin(d*x + c))*b^3)/d

Fricas [A] time = 2.39564, size = 455, normalized size = 2.22

$$4(a^3 - 3ab^2)\cos(dx + c)^5 + 36ab^2\cos(dx + c) - 12(a^3 - 6ab^2)\cos(dx + c)^3 + 3(6a^2b - 5b^3)\cos(dx + c)^2\log(\sin(dx + c) + \tan(dx + c)) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(a^3 - 3*a*b^2)*cos(d*x + c)^5 + 36*a*b^2*cos(d*x + c) - 12*(a^3 -
6*a*b^2)*cos(d*x + c)^3 + 3*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(sin(d*x +
c) + 1) - 3*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
(3*a^2*b - b^3)*cos(d*x + c)^4 + 3*b^3 - 2*(12*a^2*b - 7*b^3)*cos(d*x + c)^
2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.33 $\int \sin^2(c + dx)(a + b \tan(c + dx))^3 dx$

Optimal. Leaf size=103

$$-\frac{b(3a^2 - 2b^2)\log(\cos(c + dx))}{d} + \frac{1}{2}ax(a^2 - 9b^2) + \frac{9ab^2 \tan(c + dx)}{2d} - \frac{\sin(c + dx)\cos(c + dx)(a + b \tan(c + dx))^3}{2d} + \frac{b^3}{2d}$$

[Out] (a*(a^2 - 9*b^2)*x)/2 - (b*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]])/d + (9*a*b^2*Tan[c + d*x])/(2*d) + (b^3*Tan[c + d*x]^2)/d - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(2*d)

Rubi [A] time = 0.144593, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1645, 801, 635, 203, 260}

$$-\frac{b(3a^2 - 2b^2)\log(\cos(c + dx))}{d} + \frac{1}{2}ax(a^2 - 9b^2) + \frac{9ab^2 \tan(c + dx)}{2d} - \frac{\sin(c + dx)\cos(c + dx)(a + b \tan(c + dx))^3}{2d} + \frac{b^3}{2d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]

[Out] (a*(a^2 - 9*b^2)*x)/2 - (b*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]])/d + (9*a*b^2*Tan[c + d*x])/(2*d) + (b^3*Tan[c + d*x]^2)/d - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(2*d)

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 1645

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e*f*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \sin^2(c + dx)(a + b \tan(c + dx))^3 dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^{2(a+x)^3}}{(b^2+x^2)^2} dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{\cos(c + dx) \sin(c + dx)(a + b \tan(c + dx))^3}{2d} - \frac{\operatorname{Subst}\left(\int \frac{(a+x)^2(-ab^2-4b^2x)}{b^2+x^2} dx, x, b \tan(c + dx)\right)}{2bd} \\ &= -\frac{\cos(c + dx) \sin(c + dx)(a + b \tan(c + dx))^3}{2d} - \frac{\operatorname{Subst}\left(\int \left(-9ab^2 - 4b^2x - \frac{ab^2}{x}\right) dx, x, b \tan(c + dx)\right)}{2d} \\ &= \frac{9ab^2 \tan(c + dx)}{2d} + \frac{b^3 \tan^2(c + dx)}{d} - \frac{\cos(c + dx) \sin(c + dx)(a + b \tan(c + dx))^3}{2d} \\ &= \frac{9ab^2 \tan(c + dx)}{2d} + \frac{b^3 \tan^2(c + dx)}{d} - \frac{\cos(c + dx) \sin(c + dx)(a + b \tan(c + dx))^3}{2d} \\ &= \frac{1}{2}a(a^2 - 9b^2)x - \frac{b(3a^2 - 2b^2) \log(\cos(c + dx))}{d} + \frac{9ab^2 \tan(c + dx)}{2d} + \frac{b^3 \tan^2(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 4.22673, size = 203, normalized size = 1.97

$$b \left(-\frac{a(a^2-3b^2)\sin(2(c+dx))}{2b} + (3a^2 - b^2) \cos^2(c + dx) - \frac{a(a^2-3b^2)\tan^{-1}(\tan(c+dx))}{b} + \left(\frac{a^3-6ab^2}{\sqrt{-b^2}} + 3a^2 - 2b^2 \right) \log\left(\sqrt{-b^2} - b \tan(c + dx)\right) \right) / (2d)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^3, x]

[Out] (b*(-((a*(a^2 - 3*b^2)*ArcTan[Tan[c + d*x]])/b) + (3*a^2 - b^2)*Cos[c + d*x]^2 + (3*a^2 - 2*b^2 + (a^3 - 6*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + (3*a^2 - 2*b^2 + (-a^3 + 6*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - (a*(a^2 - 3*b^2)*Sin[2*(c + d*x)]/(2*b) + 6*a*b*Tan[c + d*x] + b^2*Tan[c + d*x]^2))/(2*d)

Maple [B] time = 0.052, size = 226, normalized size = 2.2

$$\frac{b^3 (\sin(dx + c))^6}{2d (\cos(dx + c))^2} + \frac{b^3 (\sin(dx + c))^4}{2d} + \frac{(\sin(dx + c))^2 b^3}{d} + 2 \frac{b^3 \ln(\cos(dx + c))}{d} + 3 \frac{ab^2 (\sin(dx + c))^5}{d \cos(dx + c)} + 3 \frac{ab^2 \cos(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x)

[Out] $\frac{1}{2}db^3\sin(d*x+c)^6/\cos(d*x+c)^2 + \frac{1}{2}db^3\sin(d*x+c)^4 + \frac{1}{d}\sin(d*x+c)^2 * b^3 + 2*b^3*\ln(\cos(d*x+c))/d + 3/d*a*b^2*\sin(d*x+c)^5/\cos(d*x+c) + 3/d*a*b^2*\cos(d*x+c)*\sin(d*x+c)^3 + 9/2/d*\sin(d*x+c)*\cos(d*x+c)*a*b^2 - 9/2*a*b^2*x - 9/2/d*a*b^2*c - 3/2/d*b*a^2*\sin(d*x+c)^2 - 3/d*b*a^2*\ln(\cos(d*x+c)) - 1/2/d*a^3*\sin(d*x+c)*\cos(d*x+c) + 1/2*a^3*x + 1/2/d*a^3*c$

Maxima [A] time = 1.51541, size = 153, normalized size = 1.49

$$\frac{b^3 \tan(dx+c)^2 + 6ab^2 \tan(dx+c) + (a^3 - 9ab^2)(dx+c) + (3a^2b - 2b^3) \log(\tan(dx+c)^2 + 1) + \frac{3a^2b - b^3 - (a^3 - 3ab^2) \tan(dx+c)}{\tan(dx+c)^2 + 1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] $\frac{1}{2}*(b^3*\tan(d*x+c)^2 + 6*a*b^2*\tan(d*x+c) + (a^3 - 9*a*b^2)*(d*x+c) + (3*a^2*b - 2*b^3)*\log(\tan(d*x+c)^2 + 1) + (3*a^2*b - b^3 - (a^3 - 3*a*b^2)^2)*\tan(d*x+c))/(\tan(d*x+c)^2 + 1)/d$

Fricas [A] time = 2.16828, size = 342, normalized size = 3.32

$$\frac{2(3a^2b - b^3) \cos(dx+c)^4 - 4(3a^2b - 2b^3) \cos(dx+c)^2 \log(-\cos(dx+c)) + 2b^3 - (3a^2b - b^3 - 2(a^3 - 9ab^2)dx) \cos(dx+c)}{4d \cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] $\frac{1}{4}*(2*(3*a^2*b - b^3)*\cos(d*x+c)^4 - 4*(3*a^2*b - 2*b^3)*\cos(d*x+c)^2*\log(-\cos(d*x+c)) + 2*b^3 - (3*a^2*b - b^3 - 2*(a^3 - 9*a*b^2)*d*x)*\cos(d*x+c)^2 + 2*(6*a*b^2*\cos(d*x+c) - (a^3 - 3*a*b^2)*\cos(d*x+c)^3)*\sin(d*x+c))/(d*\cos(d*x+c)^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx))^3 \sin^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**3,x)

[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**2, x)

Giac [B] time = 3.22714, size = 3497, normalized size = 33.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] $\frac{1}{4} \cdot (2a^3 dx \tan(dx)^4 \tan(c)^4 - 18ab^2 dx \tan(dx)^4 \tan(c)^4 - 6a^2 b \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^4 \tan(c)^4 + 4b^3 \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^4 \tan(c)^4 + 2a^3 dx \tan(dx)^4 \tan(c)^2 - 18ab^2 dx \tan(dx)^4 \tan(c)^2 - 4a^3 dx \tan(dx)^3 \tan(c)^3 + 36ab^2 dx \tan(dx)^3 \tan(c)^3 + 2a^3 dx \tan(dx)^2 \tan(c)^4 - 18ab^2 dx \tan(dx)^2 \tan(c)^4 + 3a^2 b \tan(dx)^4 \tan(c)^4 + b^3 \tan(dx)^4 \tan(c)^4 - 6a^2 b \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^4 \tan(c)^2 + 4b^3 \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^4 \tan(c)^2 + 12a^2 b \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^3 \tan(c)^3 - 8b^3 \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^3 \tan(c)^3 + 2a^3 \tan(dx)^4 \tan(c)^3 - 18ab^2 \tan(dx)^4 \tan(c)^3 - 6a^2 b \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^2 \tan(c)^4 + 4b^3 \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^2 \tan(c)^4 + 2a^3 \tan(dx)^3 \tan(c)^4 - 18ab^2 \tan(dx)^3 \tan(c)^4 - 4a^3 dx \tan(dx)^3 \tan(c) + 36ab^2 dx \tan(dx)^3 \tan(c) + 4a^3 dx \tan(dx)^2 \tan(c)^2 - 36ab^2 dx \tan(dx)^2 \tan(c)^2 - 3a^2 b \tan(dx)^4 \tan(c)^2 + 5b^3 \tan(dx)^4 \tan(c)^2 - 4a^3 dx \tan(dx) \tan(c)^3 + 36ab^2 dx \tan(dx) \tan(c)^3 - 18a^2 b \tan(dx)^3 \tan(c)^3 + 6b^3 \tan(dx)^3 \tan(c)^3 - 3a^2 b \tan(dx)^2 \tan(c)^4 + 5b^3 \tan(dx)^2 \tan(c)^4 + 12a^2 b \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^3 \tan(c) - 8b^3 \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^3 \tan(c) - 12ab^2 \tan(dx)^4 \tan(c) - 12a^2 b \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^2 \tan(c)^2 + 8b^3 \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^2 \tan(c)^2 - 6a^3 \tan(dx)^3 \tan(c)^2 + 18ab^2 \tan(dx)^3 \tan(c)^2 + 12a^2 b \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx) \tan(c)^3 - 8b^3 \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx) \tan(c)^3 - 6a^3 \tan(dx)^2 \tan(c)^3 + 18ab^2 \tan(dx)^2 \tan(c)^3 - 12ab^2 \tan(dx) \tan(c)^4 + 2a^3 dx \tan(dx)^2 - 18ab^2 dx \tan(dx)^2 + 2b^3 \tan(dx)^4 - 4a^3 dx \tan(dx) \tan(c) + 36ab^2 dx \tan(dx) \tan(c) + 6a^2 b \tan(dx)^3 \tan(c) - 2b^3 \tan(dx)^3 \tan(c) + 2a^3 dx \tan(c)^2 - 18ab^2 dx \tan(c)^2 + 30a^2 b \tan(dx)^2 \tan(c)^2 - 2b^3 \tan(dx)^2 \tan(c)^2 + 6a^2 b \tan(dx) \tan(c)^3 - 2b^3 \tan(dx) \tan(c)^3 + 2b^3 \tan(c)^4 - 6a^2 b \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^2 + 4b^3 \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx)^2 + 12ab^2 \tan(dx)^3 + 12a^2 b \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx) \tan(c) - 8b^3 \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2\tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2\tan(dx) \tan(c) + 1)) \tan(dx) \tan(c) + 6a^3 \tan(dx)$

$$\begin{aligned}
&)^2 \tan(c) - 18ab^2 \tan(dx)^2 \tan(c) - 6a^2 b \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2 \tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1)) \tan(c)^2 + 4b^3 \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2 \tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1)) \tan(c)^2 + 6a^3 \tan(dx) \tan(c)^2 - 18ab^2 \tan(dx) \tan(c)^2 + 12ab^2 \tan(c)^3 + 2a^3 dx - 18ab^2 dx - 3a^2 b \tan(dx)^2 + 5b^3 \tan(dx)^2 - 18a^2 b \tan(dx) \tan(c) + 6b^3 \tan(dx) \tan(c) - 3a^2 b \tan(c)^2 + 5b^3 \tan(c)^2 - 6a^2 b \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2 \tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1)) + 4b^3 \log(4(\tan(c)^2 + 1)/(\tan(dx)^4 \tan(c)^2 - 2 \tan(dx)^3 \tan(c) + \tan(dx)^2 \tan(c)^2 + \tan(dx)^2 - 2 \tan(dx) \tan(c) + 1)) - 2a^3 \tan(dx) + 18ab^2 \tan(dx) - 2a^3 \tan(c) + 18ab^2 \tan(c) + 3a^2 b + b^3)/(d \tan(dx)^4 \tan(c)^4 + d \tan(dx)^4 \tan(c)^2 - 2d \tan(dx)^3 \tan(c)^3 + d \tan(dx)^2 \tan(c)^4 - 2d \tan(dx)^3 \tan(c) + 2d \tan(dx)^2 \tan(c)^2 - 2d \tan(dx) \tan(c)^3 + d \tan(dx)^2 - 2d \tan(dx) \tan(c) + d \tan(c)^2 + d)
\end{aligned}$$

3.34 $\int \sin(c + dx)(a + b \tan(c + dx))^3 dx$

Optimal. Leaf size=133

$$-\frac{3a^2b \sin(c + dx)}{d} + \frac{3a^2b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^3 \cos(c + dx)}{d} + \frac{3ab^2 \cos(c + dx)}{d} + \frac{3ab^2 \sec(c + dx)}{d} + \frac{3b^3 \sin(c + dx)}{2d}$$

[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (a^3*Cos[c + d*x])/d + (3*a*b^2*Cos[c + d*x])/d + (3*a*b^2*Sec[c + d*x])/d - (3*a^2*b*Sin[c + d*x])/d + (3*b^3*Sin[c + d*x])/(2*d) + (b^3*Sin[c + d*x]*Tan[c + d*x]^2)/(2*d)

Rubi [A] time = 0.116482, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3517, 2638, 2592, 321, 206, 2590, 14, 288}

$$-\frac{3a^2b \sin(c + dx)}{d} + \frac{3a^2b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^3 \cos(c + dx)}{d} + \frac{3ab^2 \cos(c + dx)}{d} + \frac{3ab^2 \sec(c + dx)}{d} + \frac{3b^3 \sin(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^3,x]

[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (a^3*Cos[c + d*x])/d + (3*a*b^2*Cos[c + d*x])/d + (3*a*b^2*Sec[c + d*x])/d - (3*a^2*b*Sin[c + d*x])/d + (3*b^3*Sin[c + d*x])/(2*d) + (b^3*Sin[c + d*x]*Tan[c + d*x]^2)/(2*d)

Rule 3517

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:= -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \sin(c + dx)(a + b \tan(c + dx))^3 dx &= \int (a^3 \sin(c + dx) + 3a^2b \sin(c + dx) \tan(c + dx) + 3ab^2 \sin(c + dx) \tan^2(c + dx) \\
&= a^3 \int \sin(c + dx) dx + (3a^2b) \int \sin(c + dx) \tan(c + dx) dx + (3ab^2) \int \sin(c + dx) \tan^2(c + dx) dx \\
&= -\frac{a^3 \cos(c + dx)}{d} + \frac{(3a^2b) \operatorname{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, \sin(c + dx)\right)}{d} - \frac{(3ab^2) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sin(c + dx)\right)}{d} \\
&= -\frac{a^3 \cos(c + dx)}{d} - \frac{3a^2b \sin(c + dx)}{d} + \frac{b^3 \sin(c + dx) \tan^2(c + dx)}{2d} + \frac{(3a^2b) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sin(c + dx)\right)}{d} \\
&= \frac{3a^2b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^3 \cos(c + dx)}{d} + \frac{3ab^2 \cos(c + dx)}{d} + \frac{3ab^2 \sec(c + dx)}{d} \\
&= \frac{3a^2b \tanh^{-1}(\sin(c + dx))}{d} - \frac{3b^3 \tanh^{-1}(\sin(c + dx))}{2d} - \frac{a^3 \cos(c + dx)}{d} + \frac{3ab^2 \cos(c + dx)}{d}
\end{aligned}$$

Mathematica [B] time = 6.1559, size = 637, normalized size = 4.79

$$\frac{a(a^2 - 3b^2) \cos^4(c + dx)(a + b \tan(c + dx))^3}{d(a \cos(c + dx) + b \sin(c + dx))^3} - \frac{b(3a^2 - b^2) \sin(c + dx) \cos^3(c + dx)(a + b \tan(c + dx))^3}{d(a \cos(c + dx) + b \sin(c + dx))^3} - \frac{3(2a^2b - b^3)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^3, x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (a*(a^2 - 3*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3)/(d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*(2*a^2*b - b^3)*Cos[c + d*x]^3*Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c
```

$$\begin{aligned}
& + d*x] + b*\sin[c + d*x])^3) + (3*(2*a^2*b - b^3)*\cos[c + d*x]^3*\log[\cos[(c \\
& + d*x)/2] + \sin[(c + d*x)/2]]*(a + b*\tan[c + d*x])^3)/(2*d*(a*\cos[c + d*x] \\
& + b*\sin[c + d*x])^3) + (b^3*\cos[c + d*x]^3*(a + b*\tan[c + d*x])^3)/(4*d*(\cos \\
& \cos[(c + d*x)/2] - \sin[(c + d*x)/2])^2*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) \\
& + (3*a*b^2*\cos[c + d*x]^3*\sin[(c + d*x)/2]*(a + b*\tan[c + d*x])^3)/(d*(\cos[\\
& (c + d*x)/2] - \sin[(c + d*x)/2])*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) - (b^ \\
& 3*\cos[c + d*x]^3*(a + b*\tan[c + d*x])^3)/(4*d*(\cos[(c + d*x)/2] + \sin[(c + \\
& d*x)/2])^2*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) - (3*a*b^2*\cos[c + d*x]^3*S \\
& \sin[(c + d*x)/2]*(a + b*\tan[c + d*x])^3)/(d*(\cos[(c + d*x)/2] + \sin[(c + d*x] \\
&)/2))*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) - (b*(3*a^2 - b^2)*\cos[c + d*x]^ \\
& 3*\sin[c + d*x]*(a + b*\tan[c + d*x])^3)/(d*(a*\cos[c + d*x] + b*\sin[c + d*x]) \\
& ^3)
\end{aligned}$$

Maple [A] time = 0.043, size = 193, normalized size = 1.5

$$\frac{b^3 (\sin(dx + c))^5}{2d (\cos(dx + c))^2} + \frac{b^3 (\sin(dx + c))^3}{2d} + \frac{3b^3 \sin(dx + c)}{2d} - \frac{3b^3 \ln(\sec(dx + c) + \tan(dx + c))}{2d} + 3 \frac{ab^2 (\sin(dx + c))}{d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^3,x)

[Out] 1/2/d*b^3*sin(d*x+c)^5/cos(d*x+c)^2+1/2*b^3*sin(d*x+c)^3/d+3/2*b^3*sin(d*x+c)/d-3/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*a*b^2*sin(d*x+c)^4/cos(d*x+c)+3/d*cos(d*x+c)*sin(d*x+c)^2*a*b^2+6*a*b^2*cos(d*x+c)/d-3*a^2*b*sin(d*x+c)/d+3/d*b*a^2*ln(sec(d*x+c)+tan(d*x+c))-a^3*cos(d*x+c)/d

Maxima [A] time = 1.11825, size = 173, normalized size = 1.3

$$\frac{b^3 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} + 3 \log(\sin(dx+c)+1) - 3 \log(\sin(dx+c)-1) - 4 \sin(dx+c) \right) - 12 ab^2 \left(\frac{1}{\cos(dx+c)} + \cos(dx+c) \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] -1/4*(b^3*(2*sin(d*x+c)/(sin(d*x+c)^2-1)+3*log(sin(d*x+c)+1)-3*log(sin(d*x+c)-1)-4*sin(d*x+c))-12*a*b^2*(1/cos(d*x+c)+cos(d*x+c))-6*a^2*b*(log(sin(d*x+c)+1)-log(sin(d*x+c)-1)-2*sin(d*x+c))+4*a^3*cos(d*x+c))/d

Fricas [A] time = 2.12391, size = 346, normalized size = 2.6

$$\frac{12 ab^2 \cos(dx + c) - 4(a^3 - 3 ab^2) \cos(dx + c)^3 + 3(2 a^2 b - b^3) \cos(dx + c)^2 \log(\sin(dx + c) + 1) - 3(2 a^2 b - b^3) \cos(dx + c)}{4d \cos(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/4*(12*a*b^2*cos(d*x+c)-4*(a^3-3*a*b^2)*cos(d*x+c)^3+3*(2*a^2*b-b^3)*cos(d*x+c)^2*log(sin(d*x+c)+1)-3*(2*a^2*b-b^3)*cos(d*x+c)

)²*log(-sin(d*x + c) + 1) + 2*(b³ - 2*(3*a²*b - b³)*cos(d*x + c)²)*sin(d*x + c)/(d*cos(d*x + c)²)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx))^3 \sin(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**3,x)

[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] Timed out

3.35 $\int \csc(c + dx)(a + b \tan(c + dx))^3 dx$

Optimal. Leaf size=86

$$\frac{3a^2b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^3 \tanh^{-1}(\cos(c + dx))}{d} + \frac{3ab^2 \sec(c + dx)}{d} - \frac{b^3 \tanh^{-1}(\sin(c + dx))}{2d} + \frac{b^3 \tan(c + dx) \sec(c + dx)}{2d}$$

```
[Out] -((a^3*ArcTanh[Cos[c + d*x]])/d) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (b^3*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0856801, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3517, 3770, 2606, 8, 2611}

$$\frac{3a^2b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^3 \tanh^{-1}(\cos(c + dx))}{d} + \frac{3ab^2 \sec(c + dx)}{d} - \frac{b^3 \tanh^{-1}(\sin(c + dx))}{2d} + \frac{b^3 \tan(c + dx) \sec(c + dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a^3*ArcTanh[Cos[c + d*x]])/d) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (b^3*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\int \csc(c+dx)(a+b\tan(c+dx))^3 dx &= \int (a^3 \csc(c+dx) + 3a^2b \sec(c+dx) + 3ab^2 \sec(c+dx) \tan(c+dx) + b^3 \sec(c+dx) \tan^2(c+dx)) dx \\
&= a^3 \int \csc(c+dx) dx + (3a^2b) \int \sec(c+dx) dx + (3ab^2) \int \sec(c+dx) \tan(c+dx) dx + b^3 \int \sec(c+dx) \tan^2(c+dx) dx \\
&= -\frac{a^3 \tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^2b \tanh^{-1}(\sin(c+dx))}{d} + \frac{b^3 \sec(c+dx) \tan(c+dx)}{2d} \\
&= -\frac{a^3 \tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^2b \tanh^{-1}(\sin(c+dx))}{d} - \frac{b^3 \tanh^{-1}(\sin(c+dx))}{2d}
\end{aligned}$$

Mathematica [B] time = 2.25956, size = 241, normalized size = 2.8

$$-12a^2b \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 12a^2b \log\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right) + 4a^3 \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^3,x]

[Out] (12*a*b^2 - 4*a^3*Log[Cos[(c + d*x)/2]] - 12*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*a^3*Log[Sin[(c + d*x)/2]] + 12*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 24*a*b^2*Sec[c + d*x]*Sin[(c + d*x)/2]^2 - b^3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(4*d)

Maple [A] time = 0.061, size = 125, normalized size = 1.5

$$\frac{b^3 (\sin(dx+c))^3}{2d (\cos(dx+c))^2} + \frac{b^3 \sin(dx+c)}{2d} - \frac{b^3 \ln(\sec(dx+c) + \tan(dx+c))}{2d} + 3 \frac{ab^2}{d \cos(dx+c)} + 3 \frac{ba^2 \ln(\sec(dx+c) + \tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^3,x)

[Out] 1/2/d*b^3*sin(d*x+c)^3/cos(d*x+c)^2+1/2*b^3*sin(d*x+c)/d-1/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*a*b^2/cos(d*x+c)+3/d*b*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))

Maxima [A] time = 1.08484, size = 150, normalized size = 1.74

$$\frac{b^3 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1) \right) - 6a^2b(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 4a^3 \log(\sin(dx+c))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] -1/4*(b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*a^3*log(sin(d*x + c)))/4d

$$1)) + 4*a^3*\log(\cot(d*x + c) + \csc(d*x + c)) - 12*a*b^2/\cos(d*x + c))/d$$

Fricas [A] time = 2.49657, size = 383, normalized size = 4.45

$$\frac{2 a^3 \cos(dx + c)^2 \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) - 2 a^3 \cos(dx + c)^2 \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) - 12 a b^2 \cos(dx + c) - (6 a^2 b^2 \cos(dx + c) - (6 a^2 b^2 \cos(dx + c) - b^3) \cos(dx + c)^2 \log(\sin(dx + c) + 1) + (6 a^2 b^2 \cos(dx + c) - b^3) \cos(dx + c)^2 \log(-\sin(dx + c) + 1) - 2 b^3 \sin(dx + c)) / (d \cos(dx + c)^2)}{4 d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] -1/4*(2*a^3*cos(d*x + c)^2*log(1/2*cos(d*x + c) + 1/2) - 2*a^3*cos(d*x + c)^2*log(-1/2*cos(d*x + c) + 1/2) - 12*a*b^2*cos(d*x + c) - (6*a^2*b - b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (6*a^2*b - b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*b^3*sin(d*x + c))/(d*cos(d*x + c)^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx))^3 \csc(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**3,x)

[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x), x)

Giac [A] time = 2.06041, size = 194, normalized size = 2.26

$$\frac{2 a^3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) + (6 a^2 b - b^3) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - (6 a^2 b - b^3) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) + \frac{2 (6 a^2 b \cos(dx + c) - b^3) \cos(dx + c)^2 \log(\sin(dx + c) + 1) - 2 (6 a^2 b \cos(dx + c) - b^3) \cos(dx + c)^2 \log(-\sin(dx + c) + 1) - 2 b^3 \sin(dx + c)}{d \cos(dx + c)^2}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] 1/2*(2*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + b^3*tan(1/2*d*x + 1/2*c) + 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2/d

3.36 $\int \csc^2(c + dx)(a + b \tan(c + dx))^3 dx$

Optimal. Leaf size=64

$$\frac{3a^2b \log(\tan(c + dx))}{d} - \frac{a^3 \cot(c + dx)}{d} + \frac{3ab^2 \tan(c + dx)}{d} + \frac{b^3 \tan^2(c + dx)}{2d}$$

[Out] $-\frac{(a^3 \cot[c + d*x])}{d} + \frac{(3*a^2*b*\log[\tan[c + d*x]])}{d} + \frac{(3*a*b^2*\tan[c + d*x])}{d} + \frac{(b^3*\tan[c + d*x]^2)}{(2*d)}$

Rubi [A] time = 0.0525311, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 43}

$$\frac{3a^2b \log(\tan(c + dx))}{d} - \frac{a^3 \cot(c + dx)}{d} + \frac{3ab^2 \tan(c + dx)}{d} + \frac{b^3 \tan^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]

[Out] $-\frac{(a^3 \cot[c + d*x])}{d} + \frac{(3*a^2*b*\log[\tan[c + d*x]])}{d} + \frac{(3*a*b^2*\tan[c + d*x])}{d} + \frac{(b^3*\tan[c + d*x]^2)}{(2*d)}$

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \csc^2(c + dx)(a + b \tan(c + dx))^3 dx &= \frac{b \operatorname{Subst}\left(\int \frac{(a+x)^3}{x^2} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{b \operatorname{Subst}\left(\int \left(3a + \frac{a^3}{x^2} + \frac{3a^2}{x} + x\right) dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{a^3 \cot(c + dx)}{d} + \frac{3a^2b \log(\tan(c + dx))}{d} + \frac{3ab^2 \tan(c + dx)}{d} + \frac{b^3 \tan^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.975801, size = 126, normalized size = 1.97

$$\frac{\csc(c + dx) \sec^2(c + dx) \left(3a(a^2 - b^2) \cos(c + dx) + (a^3 + 3ab^2) \cos(3(c + dx)) - 2b \sin(c + dx) (3a^2 \log(\sin(c + dx)) - \dots)\right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]

[Out] $-(\text{Csc}[c + d*x] * \text{Sec}[c + d*x]^2 * (3*a*(a^2 - b^2) * \text{Cos}[c + d*x] + (a^3 + 3*a*b^2) * \text{Cos}[3*(c + d*x)] - 2*b*(b^2 - 3*a^2 * \text{Log}[\text{Cos}[c + d*x]] - 3*a^2 * \text{Cos}[2*(c + d*x)]) * (\text{Log}[\text{Cos}[c + d*x]] - \text{Log}[\text{Sin}[c + d*x]]) + 3*a^2 * \text{Log}[\text{Sin}[c + d*x]]) * \text{Sin}[c + d*x])) / (4*d)$

Maple [A] time = 0.056, size = 63, normalized size = 1.

$$\frac{b^3}{2d(\cos(dx+c))^2} + 3\frac{ab^2 \tan(dx+c)}{d} + 3\frac{ba^2 \ln(\tan(dx+c))}{d} - \frac{a^3 \cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x)

[Out] $1/2/d*b^3/\cos(d*x+c)^2+3*a*b^2*\tan(d*x+c)/d+3*a^2*b*\ln(\tan(d*x+c))/d-a^3*\cot(d*x+c)/d$

Maxima [A] time = 1.10142, size = 76, normalized size = 1.19

$$\frac{b^3 \tan(dx+c)^2 + 6a^2b \log(\tan(dx+c)) + 6ab^2 \tan(dx+c) - \frac{2a^3}{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] $1/2*(b^3*\tan(d*x + c)^2 + 6*a^2*b*\log(\tan(d*x + c)) + 6*a*b^2*\tan(d*x + c) - 2*a^3/\tan(d*x + c))/d$

Fricas [B] time = 2.04725, size = 327, normalized size = 5.11

$$\frac{3a^2b \cos(dx+c)^2 \log(\cos(dx+c)^2) \sin(dx+c) - 3a^2b \cos(dx+c)^2 \log\left(-\frac{1}{4} \cos(dx+c)^2 + \frac{1}{4}\right) \sin(dx+c) - 6ab^2 \cos(dx+c) \log(\cos(dx+c)) \sin(dx+c) + 6ab^2 \cos(dx+c) \log\left(-\frac{1}{4} \cos(dx+c)^2 + \frac{1}{4}\right) \sin(dx+c) - 6a^3 \cos(dx+c) \log(\cos(dx+c)) \sin(dx+c) + 6a^3 \cos(dx+c) \log\left(-\frac{1}{4} \cos(dx+c)^2 + \frac{1}{4}\right) \sin(dx+c)}{2d \cos(dx+c)^2 \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] $-1/2*(3*a^2*b*\cos(d*x + c)^2*\log(\cos(d*x + c)^2)*\sin(d*x + c) - 3*a^2*b*\cos(d*x + c)^2*\log(-1/4*\cos(d*x + c)^2 + 1/4)*\sin(d*x + c) - 6*a*b^2*\cos(d*x + c) + 2*(a^3 + 3*a*b^2)*\cos(d*x + c)^3 - b^3*\sin(d*x + c))/(d*\cos(d*x + c)^2*\sin(d*x + c))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx))^3 \csc^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**3,x)

[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**2, x)

Giac [A] time = 2.06722, size = 95, normalized size = 1.48

$$\frac{b^3 \tan(dx + c)^2 + 6a^2b \log(|\tan(dx + c)|) + 6ab^2 \tan(dx + c) - \frac{2(3a^2b \tan(dx+c) + a^3)}{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a^2*b*log(abs(tan(d*x + c))) + 6*a*b^2*tan(d*x + c) - 2*(3*a^2*b*tan(d*x + c) + a^3)/tan(d*x + c))/d

3.37 $\int \csc^3(c + dx)(a + b \tan(c + dx))^3 dx$

Optimal. Leaf size=141

$$\frac{3a^2b \csc(c + dx)}{d} + \frac{3a^2b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^3 \tanh^{-1}(\cos(c + dx))}{2d} - \frac{a^3 \cot(c + dx) \csc(c + dx)}{2d} + \frac{3ab^2 \sec(c + dx)}{d}$$

[Out] $-(a^3 \operatorname{ArcTanh}[\cos[c + dx]])/(2d) - (3a^2b \operatorname{ArcTanh}[\sin[c + dx]])/d + (3a^2b \operatorname{ArcTanh}[\sin[c + dx]])/d + (b^3 \operatorname{ArcTanh}[\sin[c + dx]])/(2d) - (3a^2b \operatorname{Csc}[c + dx])/d - (a^3 \operatorname{Cot}[c + dx] \operatorname{Csc}[c + dx])/(2d) + (3a^2b \operatorname{Sec}[c + dx])/d + (b^3 \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx])/(2d)$

Rubi [A] time = 0.134904, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3517, 3768, 3770, 2621, 321, 207, 2622}

$$\frac{3a^2b \csc(c + dx)}{d} + \frac{3a^2b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^3 \tanh^{-1}(\cos(c + dx))}{2d} - \frac{a^3 \cot(c + dx) \csc(c + dx)}{2d} + \frac{3ab^2 \sec(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csc}[c + dx]^3(a + b \operatorname{Tan}[c + dx])^3, x]$

[Out] $-(a^3 \operatorname{ArcTanh}[\cos[c + dx]])/(2d) - (3a^2b \operatorname{ArcTanh}[\sin[c + dx]])/d + (3a^2b \operatorname{ArcTanh}[\sin[c + dx]])/d + (b^3 \operatorname{ArcTanh}[\sin[c + dx]])/(2d) - (3a^2b \operatorname{Csc}[c + dx])/d - (a^3 \operatorname{Cot}[c + dx] \operatorname{Csc}[c + dx])/(2d) + (3a^2b \operatorname{Sec}[c + dx])/d + (b^3 \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx])/(2d)$

Rule 3517

$\operatorname{Int}[\sin[(e_.) + (f_.) \cdot (x_)]^{(m_.)} \cdot ((a_.) + (b_.) \cdot \tan[(e_.) + (f_.) \cdot (x_)])]^{(n_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{Expand}[\sin[e + f \cdot x]^{m \cdot (a + b \cdot \tan[e + f \cdot x])^n}, x], x] /;$ FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]

Rule 3768

$\operatorname{Int}[(\csc[(c_.) + (d_.) \cdot (x_)] \cdot (b_.)^{(n_.)}), x_Symbol] \rightarrow -\operatorname{Simp}[(b \cdot \cos[c + dx] \cdot (b \cdot \csc[c + dx])^{(n - 1)}) / (d \cdot (n - 1)), x] + \operatorname{Dist}[(b^2 \cdot (n - 2)) / (n - 1), \operatorname{Int}[(b \cdot \csc[c + dx])^{(n - 2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2 * n]

Rule 3770

$\operatorname{Int}[\csc[(c_.) + (d_.) \cdot (x_)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\cos[c + dx]] / d, x] /;$ FreeQ[{c, d}, x]

Rule 2621

$\operatorname{Int}[(\csc[(e_.) + (f_.) \cdot (x_)] \cdot (a_.)^{(m_.)}) \cdot \sec[(e_.) + (f_.) \cdot (x_)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[(f \cdot a^n)^{-1}, \operatorname{Subst}[\operatorname{Int}[x^{(m + n - 1)} / (-1 + x^2/a^2)^{((n + 1)/2)}, x], x, a \cdot \csc[e + f \cdot x]], x] /;$ FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 321

$\operatorname{Int}[(c_.) \cdot (x_)]^{(m_.)} \cdot ((a_.) + (b_.) \cdot (x_)]^{(n_.)} \cdot (p_.), x_Symbol] \rightarrow \operatorname{Simp}[(c^{(n - 1)} \cdot (c \cdot x)^{(m - n + 1)} \cdot (a + b \cdot x^n)^{(p + 1)}) / (b \cdot (m + n \cdot p + 1)), x] - \operatorname{Dist}[\operatorname{Int}[(c \cdot x)^{(m - n + 1)} \cdot (a + b \cdot x^n)^{(p + 1)} / (b \cdot (m + n \cdot p + 1)), x], x]$

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[Rt[b, 2]*x]/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_) + (f_)*(x_)]^(n_)*((a_)*sec[(e_) + (f_)*(x_)])^(m_), x_S
ymbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2
], x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned} \int \csc^3(c + dx)(a + b \tan(c + dx))^3 dx &= \int (a^3 \csc^3(c + dx) + 3a^2b \csc^2(c + dx) \sec(c + dx) + 3ab^2 \csc(c + dx) \sec^2(c + dx) + b^3 \sec^3(c + dx)) dx \\ &= a^3 \int \csc^3(c + dx) dx + (3a^2b) \int \csc^2(c + dx) \sec(c + dx) dx + (3ab^2) \int \csc(c + dx) \sec^2(c + dx) dx + b^3 \int \sec^3(c + dx) dx \\ &= -\frac{a^3 \cot(c + dx) \csc(c + dx)}{2d} + \frac{b^3 \sec(c + dx) \tan(c + dx)}{2d} + \frac{1}{2}a^3 \int \csc(c + dx) a dx \\ &= -\frac{a^3 \tanh^{-1}(\cos(c + dx))}{2d} + \frac{b^3 \tanh^{-1}(\sin(c + dx))}{2d} - \frac{3a^2b \csc(c + dx)}{d} - \frac{a^3 \cot(c + dx)}{d} \\ &= -\frac{a^3 \tanh^{-1}(\cos(c + dx))}{2d} - \frac{3ab^2 \tanh^{-1}(\cos(c + dx))}{d} + \frac{3a^2b \tanh^{-1}(\sin(c + dx))}{d} \end{aligned}$$

Mathematica [B] time = 6.18345, size = 897, normalized size = 6.36

$$\frac{3a^2b \cos^3(c + dx) \tan\left(\frac{1}{2}(c + dx)\right) (a + b \tan(c + dx))^3}{2d(a \cos(c + dx) + b \sin(c + dx))^3} + \frac{3ab^2 \cos^3(c + dx)(a + b \tan(c + dx))^3}{d(a \cos(c + dx) + b \sin(c + dx))^3} - \frac{a^3 \cos^3(c + dx) \csc^2\left(\frac{1}{2}(c + dx)\right)}{8d(a \cos(c + dx) + b \sin(c + dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (3*a^2*b*Cos[c + d*x]^3*Cot[(c + d*x)/2]*(a + b*Tan[c + d*x]
)^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^3*Cos[c + d*x]^3*Csc[(c
+ d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x]
)^3) + ((-a^3 - 6*a*b^2)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Tan[c +
d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + ((-6*a^2*b - b^3)*Cos
[c + d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3
)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + ((a^3 + 6*a*b^2)*Cos[c + d*x]
^3*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) + ((6*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d
*x])^3) + (a^3*Cos[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8*
d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c +
d*x])^3)/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*
Sin[c + d*x])^3) + (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c +
```

$$d*x])^3)/(d*(\cos[(c + d*x)/2] - \sin[(c + d*x)/2])*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) - (b^3*\cos[c + d*x]^3*(a + b*\tan[c + d*x])^3)/(4*d*(\cos[(c + d*x)/2] + \sin[(c + d*x)/2])^2*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) - (3*a*b^2*\cos[c + d*x]^3*\sin[(c + d*x)/2]*(a + b*\tan[c + d*x])^3)/(d*(\cos[(c + d*x)/2] + \sin[(c + d*x)/2])*(a*\cos[c + d*x] + b*\sin[c + d*x])^3) - (3*a^2*b*\cos[c + d*x]^3*\tan[(c + d*x)/2]*(a + b*\tan[c + d*x])^3)/(2*d*(a*\cos[c + d*x] + b*\sin[c + d*x])^3)$$

Maple [A] time = 0.065, size = 170, normalized size = 1.2

$$\frac{b^3 \sec(dx + c) \tan(dx + c)}{2d} + \frac{b^3 \ln(\sec(dx + c) + \tan(dx + c))}{2d} + 3 \frac{ab^2}{d \cos(dx + c)} + 3 \frac{ab^2 \ln(\csc(dx + c) - \cot(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x)

[Out] 1/2*b^3*sec(d*x+c)*tan(d*x+c)/d+1/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*a*b^2/cos(d*x+c)+3/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))-3/d*b*a^2/sin(d*x+c)+3/d*b*a^2*ln(sec(d*x+c)+tan(d*x+c))-1/2*a^3*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))

Maxima [A] time = 1.15234, size = 231, normalized size = 1.64

$$a^3 \left(\frac{2 \cos(dx+c)}{\cos(dx+c)^2-1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1) \right) - b^3 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1) \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] 1/4*(a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a*b^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 6*a^2*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d

Fricas [B] time = 2.73967, size = 720, normalized size = 5.11

$$12 ab^2 \cos(dx + c) - 2(a^3 + 6 ab^2) \cos(dx + c)^3 + ((a^3 + 6 ab^2) \cos(dx + c)^4 - (a^3 + 6 ab^2) \cos(dx + c)^2) \log\left(\frac{1}{2} \cos(dx + c) + 1\right) - ((a^3 + 6 ab^2) \cos(dx + c)^4 - (a^3 + 6 ab^2) \cos(dx + c)^2) \log\left(\frac{1}{2} \cos(dx + c) - 1\right) + (6 a^2 b + b^3) \cos(dx + c)^4 - (6 a^2 b + b^3) \cos(dx + c)^2 \log(\sin(dx + c) + 1) + (6 a^2 b + b^3) \cos(dx + c)^2 \log(\sin(dx + c) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] -1/4*(12*a*b^2*cos(d*x + c) - 2*(a^3 + 6*a*b^2)*cos(d*x + c)^3 + ((a^3 + 6*a*b^2)*cos(d*x + c)^4 - (a^3 + 6*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) - ((a^3 + 6*a*b^2)*cos(d*x + c)^4 - (a^3 + 6*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - ((6*a^2*b + b^3)*cos(d*x + c)^4 - (6*a^2*b + b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*a^2*b + b^3)*cos(d*x + c)^2)*log(sin(d*x + c) - 1)

$$c)^4 - (6a^2b + b^3)\cos(dx + c)^2 \log(-\sin(dx + c) + 1) + 2(b^3 - (6a^2b + b^3)\cos(dx + c)^2)\sin(dx + c) / (d\cos(dx + c)^4 - d\cos(dx + c)^2)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)**3*(a+b*tan(dx+c))**3,x)

[Out] Timed out

Giac [B] time = 1.98513, size = 410, normalized size = 2.91

$$a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 12a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 4(6a^2b + b^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 4(6a^2b + b^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)^3*(a+b*tan(dx+c))^3,x, algorithm="giac")

[Out] $\frac{1}{8}(a^3 \tan(1/2dx + 1/2c)^2 - 12a^2b \tan(1/2dx + 1/2c) + 4(6a^2b + b^3) \log(\tan(1/2dx + 1/2c) + 1) - 4(6a^2b + b^3) \log(\tan(1/2dx + 1/2c) - 1) + 4(a^3 + 6ab^2) \log(\tan(1/2dx + 1/2c))) - (2a^3 \tan(1/2dx + 1/2c)^6 + 12ab^2 \tan(1/2dx + 1/2c)^6 + 12a^2b \tan(1/2dx + 1/2c)^5 - 8b^3 \tan(1/2dx + 1/2c)^5 - 3a^3 \tan(1/2dx + 1/2c)^4 + 24ab^2 \tan(1/2dx + 1/2c)^4 - 24a^2b \tan(1/2dx + 1/2c)^3 - 8b^3 \tan(1/2dx + 1/2c)^3 - 36ab^2 \tan(1/2dx + 1/2c)^2 + 12a^2b \tan(1/2dx + 1/2c) + a^3) / (\tan(1/2dx + 1/2c)^3 - \tan(1/2dx + 1/2c))^2 / d$

3.38 $\int \csc^4(c + dx)(a + b \tan(c + dx))^3 dx$

Optimal. Leaf size=113

$$-\frac{a(a^2 + 3b^2) \cot(c + dx)}{d} + \frac{b(3a^2 + b^2) \log(\tan(c + dx))}{d} - \frac{3a^2b \cot^2(c + dx)}{2d} - \frac{a^3 \cot^3(c + dx)}{3d} + \frac{3ab^2 \tan(c + dx)}{d}$$

[Out] $-\frac{((a*(a^2 + 3*b^2)*\text{Cot}[c + d*x]))}{d} - \frac{(3*a^2*b*\text{Cot}[c + d*x]^2)}{(2*d)} - \frac{(a^3*\text{Cot}[c + d*x]^3)}{(3*d)} + \frac{(b*(3*a^2 + b^2)*\text{Log}[\text{Tan}[c + d*x]])}{d} + \frac{(3*a*b^2*\text{Tan}[c + d*x])}{d} + \frac{(b^3*\text{Tan}[c + d*x]^2)}{(2*d)}$

Rubi [A] time = 0.0860254, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 894}

$$-\frac{a(a^2 + 3b^2) \cot(c + dx)}{d} + \frac{b(3a^2 + b^2) \log(\tan(c + dx))}{d} - \frac{3a^2b \cot^2(c + dx)}{2d} - \frac{a^3 \cot^3(c + dx)}{3d} + \frac{3ab^2 \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c + d*x]^4*(a + b*\text{Tan}[c + d*x])^3, x]$

[Out] $-\frac{((a*(a^2 + 3*b^2)*\text{Cot}[c + d*x]))}{d} - \frac{(3*a^2*b*\text{Cot}[c + d*x]^2)}{(2*d)} - \frac{(a^3*\text{Cot}[c + d*x]^3)}{(3*d)} + \frac{(b*(3*a^2 + b^2)*\text{Log}[\text{Tan}[c + d*x]])}{d} + \frac{(3*a*b^2*\text{Tan}[c + d*x])}{d} + \frac{(b^3*\text{Tan}[c + d*x]^2)}{(2*d)}$

Rule 3516

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[b/f, \text{Subst}[\text{Int}[(x^m*(a + x)^n)/(b^2 + x^2)^{(m/2 + 1)}], x], x, b*\text{Tan}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, n\}, x \ \&\& \ \text{IntegerQ}[m/2]$

Rule 894

$\text{Int}[(d_.) + (e_.)*(x_.)]^{(m_.)}*((f_.) + (g_.)*(x_.))^{(n_.)}*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, f, g\}, x \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ ((\text{EqQ}[p, 1] \ \&\& \ \text{IntegersQ}[m, n]) \ || \ (\text{ILtQ}[m, 0] \ \&\& \ \text{ILtQ}[n, 0]))$

Rubi steps

$$\begin{aligned} \int \csc^4(c + dx)(a + b \tan(c + dx))^3 dx &= \frac{b \text{Subst}\left(\int \frac{(a+x)^3(b^2+x^2)}{x^4} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{b \text{Subst}\left(\int \left(3a + \frac{a^3b^2}{x^4} + \frac{3a^2b^2}{x^3} + \frac{a^3+3ab^2}{x^2} + \frac{3a^2+b^2}{x} + x\right) dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{a(a^2 + 3b^2) \cot(c + dx)}{d} - \frac{3a^2b \cot^2(c + dx)}{2d} - \frac{a^3 \cot^3(c + dx)}{3d} + \frac{b(3a^2 + b^2) \tan(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 2.20723, size = 212, normalized size = 1.88

$\frac{\sec^2(c + dx)(a \cot(c + dx) + b)^3 (-2 \sin(c + dx) (6 (3a^2b + b^3) \cos(2(c + dx)) - 3b(3a^2 + b^2) \cos(4(c + dx)))(\log(\cos(c + dx)))}{d}$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]

[Out] $((b + a \cot[c + d*x])^3 \sec[c + d*x]^2 (-16a^3 \cos[c + d*x] - 2 \sin[c + d*x]) (18a^2 b - 6b^3 + 6(3a^2 b + b^3) \cos[2(c + d*x)] + 9a^2 b \log[\cos[c + d*x]] + 3b^3 \log[\cos[c + d*x]] - 3b(3a^2 + b^2) \cos[4(c + d*x)] * (\log[\cos[c + d*x]] - \log[\sin[c + d*x]]) - 9a^2 b \log[\sin[c + d*x]] - 3b^3 \log[\sin[c + d*x]] + 2a^3 \sin[4(c + d*x)] + 18a b^2 \sin[4(c + d*x)])) / (48d(a \cos[c + d*x] + b \sin[c + d*x])^3)$

Maple [A] time = 0.068, size = 141, normalized size = 1.3

$$\frac{b^3}{2d(\cos(dx+c))^2} + \frac{b^3 \ln(\tan(dx+c))}{d} + 3 \frac{ab^2}{d \sin(dx+c) \cos(dx+c)} - 6 \frac{ab^2 \cot(dx+c)}{d} - \frac{3ba^2}{2d(\sin(dx+c))^2} + 3 \frac{ba^2}{2d(\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x)

[Out] $1/2/d*b^3/\cos(d*x+c)^2 + 1/d*b^3*\ln(\tan(d*x+c)) + 3/d*a*b^2/\sin(d*x+c)/\cos(d*x+c) - 6/d*a*b^2*\cot(d*x+c) - 3/2/d*b*a^2/\sin(d*x+c)^2 + 3*a^2*b*\ln(\tan(d*x+c))/d - 2/3*a^3*\cot(d*x+c)/d - 1/3/d*a^3*\cot(d*x+c)*\csc(d*x+c)^2$

Maxima [A] time = 1.0644, size = 132, normalized size = 1.17

$$\frac{3b^3 \tan(dx+c)^2 + 18ab^2 \tan(dx+c) + 6(3a^2b + b^3) \log(\tan(dx+c)) - \frac{9a^2b \tan(dx+c) + 2a^3 + 6(a^3 + 3ab^2) \tan(dx+c)^2}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] $1/6*(3b^3*\tan(d*x+c)^2 + 18a*b^2*\tan(d*x+c) + 6*(3a^2*b + b^3)*\log(\tan(d*x+c)) - (9a^2*b*\tan(d*x+c) + 2a^3 + 6*(a^3 + 3a*b^2)*\tan(d*x+c)^2)/\tan(d*x+c)^3)/d$

Fricas [B] time = 2.01836, size = 575, normalized size = 5.09

$$4(a^3 + 9ab^2) \cos(dx+c)^5 + 18ab^2 \cos(dx+c) - 6(a^3 + 9ab^2) \cos(dx+c)^3 + 3((3a^2b + b^3) \cos(dx+c)^4 - (3a^2b + b^3) \cos(dx+c)^2) \log(\cos(dx+c)^2) \sin(dx+c) - 3((3a^2b + b^3) \cos(dx+c)^4 - (3a^2b + b^3) \cos(dx+c)^2) \log(-1/4 \cos(dx+c)^2 + 1/4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] $-1/6*(4*(a^3 + 9a*b^2)*\cos(d*x+c)^5 + 18*a*b^2*\cos(d*x+c) - 6*(a^3 + 9a*b^2)*\cos(d*x+c)^3 + 3*((3*a^2*b + b^3)*\cos(d*x+c)^4 - (3*a^2*b + b^3)*\cos(d*x+c)^2)*\log(\cos(d*x+c)^2)*\sin(d*x+c) - 3*((3*a^2*b + b^3)*\cos(d*x+c)^4 - (3*a^2*b + b^3)*\cos(d*x+c)^2)*\log(-1/4*\cos(d*x+c)^2 + 1/4)$

) $\sin(dx + c) + 3(b^3 - (3a^2b + b^3)\cos(dx + c)^2)\sin(dx + c)/((d$
 $\cos(dx + c)^4 - d\cos(dx + c)^2)\sin(dx + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)**4*(a+b*tan(dx+c))**3,x)

[Out] Timed out

Giac [A] time = 2.04811, size = 180, normalized size = 1.59

$$\frac{3b^3 \tan(dx + c)^2 + 18ab^2 \tan(dx + c) + 6(3a^2b + b^3) \log(|\tan(dx + c)|) - \frac{33a^2b \tan(dx+c)^3 + 11b^3 \tan(dx+c)^3 + 6a^3 \tan(dx+c)^3}{\tan(dx+c)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)^4*(a+b*tan(dx+c))^3,x, algorithm="giac")

[Out] $\frac{1}{6} * (3b^3 \tan(dx + c)^2 + 18a * b^2 \tan(dx + c) + 6 * (3a^2 * b + b^3) * \log(a$
 $b \tan(dx + c))) - (33a^2 * b * \tan(dx + c)^3 + 11 * b^3 * \tan(dx + c)^3 + 6a^3$
 $* \tan(dx + c)^2 + 18a * b^2 * \tan(dx + c)^2 + 9a^2 * b * \tan(dx + c) + 2a^3) /$
 $\tan(dx + c)^3) / d$

3.39 $\int \csc^5(c + dx)(a + b \tan(c + dx))^3 dx$

Optimal. Leaf size=229

$$-\frac{a^2 b \csc^3(c + dx)}{d} - \frac{3a^2 b \csc(c + dx)}{d} + \frac{3a^2 b \tanh^{-1}(\sin(c + dx))}{d} - \frac{3a^3 \tanh^{-1}(\cos(c + dx))}{8d} - \frac{a^3 \cot(c + dx) \csc^3(c + dx)}{4d}$$

[Out] $(-3a^3 \text{ArcTanh}[\text{Cos}[c + dx]])/(8d) - (9ab^2 \text{ArcTanh}[\text{Cos}[c + dx]])/(2d) + (3a^2 b \text{ArcTanh}[\text{Sin}[c + dx]])/d + (3b^3 \text{ArcTanh}[\text{Sin}[c + dx]])/(2d) - (3a^2 b \text{Csc}[c + dx])/d - (3b^3 \text{Csc}[c + dx])/(2d) - (3a^3 \text{Cot}[c + dx] \text{Csc}[c + dx])/(8d) - (a^2 b \text{Csc}[c + dx]^3)/d - (a^3 \text{Cot}[c + dx] \text{Csc}[c + dx]^3)/(4d) + (9ab^2 \text{Sec}[c + dx])/(2d) - (3ab^2 \text{Csc}[c + dx]^2 \text{Sec}[c + dx])/(2d) + (b^3 \text{Csc}[c + dx] \text{Sec}[c + dx]^2)/(2d)$

Rubi [A] time = 0.208038, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3517, 3768, 3770, 2621, 302, 207, 2622, 288, 321}

$$-\frac{a^2 b \csc^3(c + dx)}{d} - \frac{3a^2 b \csc(c + dx)}{d} + \frac{3a^2 b \tanh^{-1}(\sin(c + dx))}{d} - \frac{3a^3 \tanh^{-1}(\cos(c + dx))}{8d} - \frac{a^3 \cot(c + dx) \csc^3(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c + dx]^5(a + b \text{Tan}[c + dx])^3, x]$

[Out] $(-3a^3 \text{ArcTanh}[\text{Cos}[c + dx]])/(8d) - (9ab^2 \text{ArcTanh}[\text{Cos}[c + dx]])/(2d) + (3a^2 b \text{ArcTanh}[\text{Sin}[c + dx]])/d + (3b^3 \text{ArcTanh}[\text{Sin}[c + dx]])/(2d) - (3a^2 b \text{Csc}[c + dx])/d - (3b^3 \text{Csc}[c + dx])/(2d) - (3a^3 \text{Cot}[c + dx] \text{Csc}[c + dx])/(8d) - (a^2 b \text{Csc}[c + dx]^3)/d - (a^3 \text{Cot}[c + dx] \text{Csc}[c + dx]^3)/(4d) + (9ab^2 \text{Sec}[c + dx])/(2d) - (3ab^2 \text{Csc}[c + dx]^2 \text{Sec}[c + dx])/(2d) + (b^3 \text{Csc}[c + dx] \text{Sec}[c + dx]^2)/(2d)$

Rule 3517

$\text{Int}[\sin[(e_.) + (f_.)x]^{(m_.)}((a_.) + (b_.)\tan[(e_.) + (f_.)x])^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{Expand}[\text{Sin}[e + fx]^{m_.}(a + b \text{Tan}[e + fx])^{n_.}, x], x] /; \text{FreeQ}\{a, b, e, f, x\} \ \&\& \ \text{IntegerQ}[(m - 1)/2] \ \&\& \ \text{IGtQ}[n, 0]$

Rule 3768

$\text{Int}[(\csc[(c_.) + (d_.)x] \cdot (b_.)^n), x_Symbol] \rightarrow -\text{Simp}[(b \text{Cos}[c + dx] \cdot (b \text{Csc}[c + dx])^{(n - 1)})/(d(n - 1)), x] + \text{Dist}[(b^2(n - 2))/(n - 1), \text{Int}[(b \text{Csc}[c + dx])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d, x\} \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2n]$

Rule 3770

$\text{Int}[\csc[(c_.) + (d_.)x], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + dx]]/d, x] /; \text{FreeQ}\{c, d, x\}$

Rule 2621

$\text{Int}[(\csc[(e_.) + (f_.)x] \cdot (a_.)^m) \cdot \sec[(e_.) + (f_.)x]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[(f \cdot a^n)^{-1}, \text{Subst}[\text{Int}[x^{(m + n - 1)} / (-1 + x^2/a^2)^{((n + 1)/2)}, x], x, a \text{Csc}[e + fx]], x] /; \text{FreeQ}\{a, e, f, m, x\} \ \&\& \ \text{IntegerQ}[(n + 1)/2] \ \&\& \ !(\text{IntegerQ}[(m + 1)/2] \ \&\& \ \text{LtQ}[0, m, n])$

Rule 302

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2622

Int[csc[(e_) + (f_)*(x_)]^(n_)*((a_)*sec[(e_) + (f_)*(x_)]^(m_)), x_Symbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 288

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned}
 \int \csc^5(c + dx)(a + b \tan(c + dx))^3 dx &= \int (a^3 \csc^5(c + dx) + 3a^2b \csc^4(c + dx) \sec(c + dx) + 3ab^2 \csc^3(c + dx) \sec^2(c + dx) + b^3 \csc^2(c + dx) \sec^3(c + dx)) dx \\
 &= a^3 \int \csc^5(c + dx) dx + (3a^2b) \int \csc^4(c + dx) \sec(c + dx) dx + (3ab^2) \int \csc^3(c + dx) \sec^2(c + dx) dx + b^3 \int \csc^2(c + dx) \sec^3(c + dx) dx \\
 &= -\frac{a^3 \cot(c + dx) \csc^3(c + dx)}{4d} + \frac{1}{4} (3a^3) \int \csc^3(c + dx) dx - \frac{(3a^2b) \text{Subst}\left(\int \csc^3(u) du, u = \frac{a + b \tan(c + dx)}{a}\right)}{4d} \\
 &= -\frac{3a^3 \cot(c + dx) \csc(c + dx)}{8d} - \frac{a^3 \cot(c + dx) \csc^3(c + dx)}{4d} - \frac{3ab^2 \csc^2(c + dx) \sec(c + dx)}{2d} + \frac{b^3 \csc^2(c + dx) \sec^3(c + dx)}{2d} \\
 &= -\frac{3a^3 \tanh^{-1}(\cos(c + dx))}{8d} - \frac{3a^2b \csc(c + dx)}{d} - \frac{3b^3 \csc(c + dx)}{2d} - \frac{3a^3 \cot(c + dx) \csc^3(c + dx)}{4d} \\
 &= -\frac{3a^3 \tanh^{-1}(\cos(c + dx))}{8d} - \frac{9ab^2 \tanh^{-1}(\cos(c + dx))}{2d} + \frac{3a^2b \tanh^{-1}(\sin(c + dx))}{d}
 \end{aligned}$$

Mathematica [B] time = 6.20045, size = 1229, normalized size = 5.37

$$\frac{a^3 \cos^3(c + dx)(a + b \tan(c + dx))^3 \csc^4\left(\frac{1}{2}(c + dx)\right)}{64d(a \cos(c + dx) + b \sin(c + dx))^3} - \frac{3(a^3 + 4b^2a) \cos^3(c + dx)(a + b \tan(c + dx))^3 \csc^2\left(\frac{1}{2}(c + dx)\right)}{32d(a \cos(c + dx) + b \sin(c + dx))^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]

[Out] (3*a*b^2*cos[c + d*x]^3*(a + b*tan[c + d*x])^3)/(d*(a*cos[c + d*x] + b*sin[c + d*x])^3) + ((-7*a^2*b*cos[(c + d*x)/2] - 2*b^3*cos[(c + d*x)/2])*cos[c + d*x]^3*csc[(c + d*x)/2]*(a + b*tan[c + d*x])^3)/(4*d*(a*cos[c + d*x] + b*sin[c + d*x])^3) - (3*(a^3 + 4*a*b^2)*cos[c + d*x]^3*csc[(c + d*x)/2]^2*(a + b*tan[c + d*x])^3)/(32*d*(a*cos[c + d*x] + b*sin[c + d*x])^3) - (a^2*b*cos[c + d*x]^3*cot[(c + d*x)/2]*csc[(c + d*x)/2]^2*(a + b*tan[c + d*x])^3)/(8*d*(a*cos[c + d*x] + b*sin[c + d*x])^3) - (a^3*cos[c + d*x]^3*csc[(c + d*x)/2]^4*(a + b*tan[c + d*x])^3)/(64*d*(a*cos[c + d*x] + b*sin[c + d*x])^3) - (3*(a^3 + 12*a*b^2)*cos[c + d*x]^3*log[cos[(c + d*x)/2]]*(a + b*tan[c + d*x])^3)/(8*d*(a*cos[c + d*x] + b*sin[c + d*x])^3) - (3*(2*a^2*b + b^3)*cos[c + d*x]^3*log[cos[(c + d*x)/2] - sin[(c + d*x)/2]]*(a + b*tan[c + d*x])^3)/(2*d*(a*cos[c + d*x] + b*sin[c + d*x])^3) + (3*(a^3 + 12*a*b^2)*cos[c + d*x]^3*log[sin[(c + d*x)/2]]*(a + b*tan[c + d*x])^3)/(8*d*(a*cos[c + d*x] + b*sin[c + d*x])^3) + (3*(2*a^2*b + b^3)*cos[c + d*x]^3*log[cos[(c + d*x)/2] + sin[(c + d*x)/2]]*(a + b*tan[c + d*x])^3)/(2*d*(a*cos[c + d*x] + b*sin[c + d*x])^3) + (3*(a^3 + 4*a*b^2)*cos[c + d*x]^3*sec[(c + d*x)/2]^2*(a + b*tan[c + d*x])^3)/(32*d*(a*cos[c + d*x] + b*sin[c + d*x])^3) + (a^3*cos[c + d*x]^3*sec[(c + d*x)/2]^4*(a + b*tan[c + d*x])^3)/(64*d*(a*cos[c + d*x] + b*sin[c + d*x])^3) + (b^3*cos[c + d*x]^3*(a + b*tan[c + d*x])^3)/(4*d*(cos[(c + d*x)/2] - sin[(c + d*x)/2])^2*(a*cos[c + d*x] + b*sin[c + d*x])^3) + (3*a*b^2*cos[c + d*x]^3*sin[(c + d*x)/2]*(a + b*tan[c + d*x])^3)/(d*(cos[(c + d*x)/2] - sin[(c + d*x)/2])*(a*cos[c + d*x] + b*sin[c + d*x])^3) - (b^3*cos[c + d*x]^3*(a + b*tan[c + d*x])^3)/(4*d*(cos[(c + d*x)/2] + sin[(c + d*x)/2])^2*(a*cos[c + d*x] + b*sin[c + d*x])^3) - (3*a*b^2*cos[c + d*x]^3*sin[(c + d*x)/2]*(a + b*tan[c + d*x])^3)/(d*(cos[(c + d*x)/2] + sin[(c + d*x)/2])*(a*cos[c + d*x] + b*sin[c + d*x])^3) + (cos[c + d*x]^3*sec[(c + d*x)/2]*(-7*a^2*b*sin[(c + d*x)/2] - 2*b^3*sin[(c + d*x)/2]))*(a + b*tan[c + d*x])^3)/(4*d*(a*cos[c + d*x] + b*sin[c + d*x])^3) - (a^2*b*cos[c + d*x]^3*sec[(c + d*x)/2]^2*tan[(c + d*x)/2]*(a + b*tan[c + d*x])^3)/(8*d*(a*cos[c + d*x] + b*sin[c + d*x])^3)

Maple [A] time = 0.068, size = 254, normalized size = 1.1

$$\frac{b^3}{2d \sin(dx+c) (\cos(dx+c))^2} - \frac{3b^3}{2d \sin(dx+c)} + \frac{3b^3 \ln(\sec(dx+c) + \tan(dx+c))}{2d} - \frac{3ab^2}{2d (\sin(dx+c))^2 \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x)

[Out] 1/2/d*b^3/sin(d*x+c)/cos(d*x+c)^2-3/2/d*b^3/sin(d*x+c)+3/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))-3/2/d*a*b^2/sin(d*x+c)^2/cos(d*x+c)+9/2/d*a*b^2/cos(d*x+c)+9/2/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))-1/d*b*a^2/sin(d*x+c)^3-3/d*b*a^2/sin(d*x+c)+3/d*b*a^2*ln(sec(d*x+c)+tan(d*x+c))-1/4*a^3*cot(d*x+c)*csc(d*x+c)^3/d-3/8*a^3*cot(d*x+c)*csc(d*x+c)/d+3/8/d*a^3*ln(csc(d*x+c)-cot(d*x+c))

Maxima [A] time = 1.1056, size = 338, normalized size = 1.48

$$a^3 \left(\frac{2(3 \cos(dx+c)^3 - 5 \cos(dx+c))}{\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1} - 3 \log(\cos(dx+c) + 1) + 3 \log(\cos(dx+c) - 1) \right) + 12ab^2 \left(\frac{2(3 \cos(dx+c)^2 - 2)}{\cos(dx+c)^3 - \cos(dx+c)} - 3 \log(\cos(dx+c)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] $\frac{1}{16}*(a^3*(2*(3*\cos(d*x + c))^3 - 5*\cos(d*x + c)))/(\cos(d*x + c)^4 - 2*\cos(d*x + c)^2 + 1) - 3*\log(\cos(d*x + c) + 1) + 3*\log(\cos(d*x + c) - 1) + 12*a*b^2*(2*(3*\cos(d*x + c)^2 - 2))/(\cos(d*x + c)^3 - \cos(d*x + c)) - 3*\log(\cos(d*x + c) + 1) + 3*\log(\cos(d*x + c) - 1) - 4*b^3*(2*(3*\sin(d*x + c)^2 - 2))/(\sin(d*x + c)^3 - \sin(d*x + c)) - 3*\log(\sin(d*x + c) + 1) + 3*\log(\sin(d*x + c) - 1) - 8*a^2*b*(2*(3*\sin(d*x + c)^2 + 1))/\sin(d*x + c)^3 - 3*\log(\sin(d*x + c) + 1) + 3*\log(\sin(d*x + c) - 1))/d$

Fricas [B] time = 2.95534, size = 1060, normalized size = 4.63

$6(a^3 + 12ab^2)\cos(dx + c)^5 + 48ab^2\cos(dx + c) - 10(a^3 + 12ab^2)\cos(dx + c)^3 - 3((a^3 + 12ab^2)\cos(dx + c)^6 - 2$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] $\frac{1}{16}*(6*(a^3 + 12*a*b^2)*\cos(d*x + c)^5 + 48*a*b^2*\cos(d*x + c) - 10*(a^3 + 12*a*b^2)*\cos(d*x + c)^3 - 3*((a^3 + 12*a*b^2)*\cos(d*x + c)^6 - 2*(a^3 + 12*a*b^2)*\cos(d*x + c)^4 + (a^3 + 12*a*b^2)*\cos(d*x + c)^2)*\log(1/2*\cos(d*x + c) + 1/2) + 3*((a^3 + 12*a*b^2)*\cos(d*x + c)^6 - 2*(a^3 + 12*a*b^2)*\cos(d*x + c)^4 + (a^3 + 12*a*b^2)*\cos(d*x + c)^2)*\log(-1/2*\cos(d*x + c) + 1/2) + 12*((2*a^2*b + b^3)*\cos(d*x + c)^6 - 2*(2*a^2*b + b^3)*\cos(d*x + c)^4 + (2*a^2*b + b^3)*\cos(d*x + c)^2)*\log(\sin(d*x + c) + 1) - 12*((2*a^2*b + b^3)*\cos(d*x + c)^6 - 2*(2*a^2*b + b^3)*\cos(d*x + c)^4 + (2*a^2*b + b^3)*\cos(d*x + c)^2)*\log(-\sin(d*x + c) + 1) + 8*(3*(2*a^2*b + b^3)*\cos(d*x + c)^4 + b^3 - 4*(2*a^2*b + b^3)*\cos(d*x + c)^2)*\sin(d*x + c))/(d*\cos(d*x + c)^6 - 2*d*\cos(d*x + c)^4 + d*\cos(d*x + c)^2)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**3,x)

[Out] Timed out

Giac [A] time = 2.14968, size = 504, normalized size = 2.2

$a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 8 a^2 b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 8 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 24 a b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 120 a^2 b \tan\left(\frac{1}{2} dx$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] $\frac{1}{64}(a^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^4 - 8a^2 b \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 + 8a^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 24a^2 b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 - 120a^2 b \tan(\frac{1}{2}dx + \frac{1}{2}c) - 32b^3 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 96(2a^2 b + b^3) \log(\abs{\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1}) - 96(2a^2 b + b^3) \log(\abs{\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1}) + 24(a^3 + 12a^2 b^2) \log(\abs{\tan(\frac{1}{2}dx + \frac{1}{2}c)}) + 64(b^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 - 6a^2 b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + b^3 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 6a^2 b^2) / (\tan(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^2 - (50a^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^4 + 600a^2 b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^4 + 120a^2 b \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 + 32b^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 + 8a^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 24a^2 b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 8a^2 b \tan(\frac{1}{2}dx + \frac{1}{2}c) + a^3) / \tan(\frac{1}{2}dx + \frac{1}{2}c)^4) / d$

3.40 $\int \csc^6(c + dx)(a + b \tan(c + dx))^3 dx$

Optimal. Leaf size=167

$$\frac{a(2a^2 + 3b^2) \cot^3(c + dx)}{3d} - \frac{b(6a^2 + b^2) \cot^2(c + dx)}{2d} - \frac{a(a^2 + 6b^2) \cot(c + dx)}{d} + \frac{b(3a^2 + 2b^2) \log(\tan(c + dx))}{d}$$

```
[Out] -((a*(a^2 + 6*b^2)*Cot[c + d*x])/d) - (b*(6*a^2 + b^2)*Cot[c + d*x]^2)/(2*d)
) - (a*(2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (3*a^2*b*Cot[c + d*x]^4)/(4*d)
d) - (a^3*Cot[c + d*x]^5)/(5*d) + (b*(3*a^2 + 2*b^2)*Log[Tan[c + d*x]])/d +
(3*a*b^2*Tan[c + d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.134132, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 948}

$$\frac{a(2a^2 + 3b^2) \cot^3(c + dx)}{3d} - \frac{b(6a^2 + b^2) \cot^2(c + dx)}{2d} - \frac{a(a^2 + 6b^2) \cot(c + dx)}{d} + \frac{b(3a^2 + 2b^2) \log(\tan(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 + 6*b^2)*Cot[c + d*x])/d) - (b*(6*a^2 + b^2)*Cot[c + d*x]^2)/(2*d)
) - (a*(2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (3*a^2*b*Cot[c + d*x]^4)/(4*d)
d) - (a^3*Cot[c + d*x]^5)/(5*d) + (b*(3*a^2 + 2*b^2)*Log[Tan[c + d*x]])/d +
(3*a*b^2*Tan[c + d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_)
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned} \int \csc^6(c + dx)(a + b \tan(c + dx))^3 dx &= \frac{b \operatorname{Subst}\left(\int \frac{(a+x)^3(b^2+x^2)^2}{x^6} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{b \operatorname{Subst}\left(\int \left(3a + \frac{a^3 b^4}{x^6} + \frac{3a^2 b^4}{x^5} + \frac{2a^3 b^2 + 3ab^4}{x^4} + \frac{6a^2 b^2 + b^4}{x^3} + \frac{a^3 + 6ab^2}{x^2} + \frac{3a^2 + 2b^2}{x} + x\right) dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{a(a^2 + 6b^2) \cot(c + dx)}{d} - \frac{b(6a^2 + b^2) \cot^2(c + dx)}{2d} - \frac{a(2a^2 + 3b^2) \cot^3(c + dx)}{3d} + \frac{b(3a^2 + 2b^2) \log(\tan(c + dx))}{d} \end{aligned}$$

Mathematica [B] time = 1.75822, size = 515, normalized size = 3.08

$$\frac{\csc^5(c + dx) \sec^2(c + dx) (40a(5a^2 + 3b^2) \cos(c + dx) + 8(a^3 + 15ab^2) \cos(3(c + dx)) + 360a^2b \sin(c + dx) + 270a^2b \sin(3(c + dx)))}{960d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]

[Out] $-(\text{Csc}[c + d*x]^5 \text{Sec}[c + d*x]^2 (40*a*(5*a^2 + 3*b^2) \text{Cos}[c + d*x] + 8*(a^3 + 15*a*b^2) \text{Cos}[3*(c + d*x)] - 24*a^3 \text{Cos}[5*(c + d*x)] - 360*a*b^2 \text{Cos}[5*(c + d*x)] + 8*a^3 \text{Cos}[7*(c + d*x)] + 120*a*b^2 \text{Cos}[7*(c + d*x)] + 360*a^2*b \text{Sin}[c + d*x] - 240*b^3 \text{Sin}[c + d*x] + 225*a^2*b \text{Log}[\text{Cos}[c + d*x]] \text{Sin}[c + d*x] + 150*b^3 \text{Log}[\text{Cos}[c + d*x]] \text{Sin}[c + d*x] - 225*a^2*b \text{Log}[\text{Sin}[c + d*x]] \text{Sin}[c + d*x] - 150*b^3 \text{Log}[\text{Sin}[c + d*x]] \text{Sin}[c + d*x] + 270*a^2*b \text{Sin}[3*(c + d*x)] + 180*b^3 \text{Sin}[3*(c + d*x)] + 45*a^2*b \text{Log}[\text{Cos}[c + d*x]] \text{Sin}[3*(c + d*x)] + 30*b^3 \text{Log}[\text{Cos}[c + d*x]] \text{Sin}[3*(c + d*x)] - 45*a^2*b \text{Log}[\text{Sin}[c + d*x]] \text{Sin}[3*(c + d*x)] - 30*b^3 \text{Log}[\text{Sin}[c + d*x]] \text{Sin}[3*(c + d*x)] - 90*a^2*b \text{Sin}[5*(c + d*x)] - 60*b^3 \text{Sin}[5*(c + d*x)] - 135*a^2*b \text{Log}[\text{Cos}[c + d*x]] \text{Sin}[5*(c + d*x)] - 90*b^3 \text{Log}[\text{Cos}[c + d*x]] \text{Sin}[5*(c + d*x)] + 135*a^2*b \text{Log}[\text{Sin}[c + d*x]] \text{Sin}[5*(c + d*x)] + 90*b^3 \text{Log}[\text{Sin}[c + d*x]] \text{Sin}[5*(c + d*x)] + 45*a^2*b \text{Log}[\text{Cos}[c + d*x]] \text{Sin}[7*(c + d*x)] + 30*b^3 \text{Log}[\text{Cos}[c + d*x]] \text{Sin}[7*(c + d*x)] - 45*a^2*b \text{Log}[\text{Sin}[c + d*x]] \text{Sin}[7*(c + d*x)] - 30*b^3 \text{Log}[\text{Sin}[c + d*x]] \text{Sin}[7*(c + d*x)]) / (960*d)$

Maple [A] time = 0.069, size = 230, normalized size = 1.4

$$\frac{b^3}{2d(\sin(dx+c))^2(\cos(dx+c))^2} - \frac{b^3}{d(\sin(dx+c))^2} + 2\frac{b^3 \ln(\tan(dx+c))}{d} - \frac{ab^2}{d(\sin(dx+c))^3 \cos(dx+c)} + 4\frac{1}{d \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x)

[Out] $1/2/d*b^3/\sin(d*x+c)^2/\cos(d*x+c)^2 - 1/d*b^3/\sin(d*x+c)^2 + 2/d*b^3*\ln(\tan(d*x+c)) - 1/d*a*b^2/\sin(d*x+c)^3/\cos(d*x+c) + 4/d*a*b^2/\sin(d*x+c)/\cos(d*x+c) - 8/d*a*b^2*\cot(d*x+c) - 3/4/d*b*a^2/\sin(d*x+c)^4 - 3/2/d*b*a^2/\sin(d*x+c)^2 + 3*a^2*b*\ln(\tan(d*x+c))/d - 8/15*a^3*\cot(d*x+c)/d - 1/5/d*a^3*\cot(d*x+c)*\csc(d*x+c)^4 - 4/15/d*a^3*\cot(d*x+c)*\csc(d*x+c)^2$

Maxima [A] time = 1.11848, size = 192, normalized size = 1.15

$$\frac{30b^3 \tan(dx+c)^2 + 180ab^2 \tan(dx+c) + 60(3a^2b + 2b^3) \log(\tan(dx+c)) - \frac{60(a^3 + 6ab^2) \tan(dx+c)^4 + 45a^2b \tan(dx+c) + 30(6a^3 + 6ab^2) \tan(dx+c)^3 + 12a^3 + 20(2a^3 + 3ab^2) \tan(dx+c)}{60d}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] $1/60*(30*b^3*\tan(d*x + c)^2 + 180*a*b^2*\tan(d*x + c) + 60*(3*a^2*b + 2*b^3) \log(\tan(d*x + c)) - (60*(a^3 + 6*a*b^2)*\tan(d*x + c)^4 + 45*a^2*b*\tan(d*x + c) + 30*(6*a^3 + 6*a*b^2)*\tan(d*x + c)^3 + 12*a^3 + 20*(2*a^3 + 3*a*b^2)*\tan(d*x + c)) / 60$

$n(dx + c)^2 / \tan(dx + c)^5 / d$

Fricas [B] time = 2.28379, size = 834, normalized size = 4.99

$$32(a^3 + 15ab^2)\cos(dx + c)^7 - 80(a^3 + 15ab^2)\cos(dx + c)^5 - 180ab^2\cos(dx + c) + 60(a^3 + 15ab^2)\cos(dx + c)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)^6*(a+b*tan(dx+c))^3,x, algorithm="fricas")

[Out] $-1/60*(32*(a^3 + 15*a*b^2)*\cos(dx + c)^7 - 80*(a^3 + 15*a*b^2)*\cos(dx + c)^5 - 180*a*b^2*\cos(dx + c) + 60*(a^3 + 15*a*b^2)*\cos(dx + c)^3 + 30*((3*a^2*b + 2*b^3)*\cos(dx + c)^6 - 2*(3*a^2*b + 2*b^3)*\cos(dx + c)^4 + (3*a^2*b + 2*b^3)*\cos(dx + c)^2)*\log(\cos(dx + c)^2)*\sin(dx + c) - 30*((3*a^2*b + 2*b^3)*\cos(dx + c)^6 - 2*(3*a^2*b + 2*b^3)*\cos(dx + c)^4 + (3*a^2*b + 2*b^3)*\cos(dx + c)^2)*\log(-1/4*\cos(dx + c)^2 + 1/4)*\sin(dx + c) - 15*(2*(3*a^2*b + 2*b^3)*\cos(dx + c)^4 + 2*b^3 - 3*(3*a^2*b + 2*b^3)*\cos(dx + c)^2)*\sin(dx + c))/((d*\cos(dx + c)^6 - 2*d*\cos(dx + c)^4 + d*\cos(dx + c)^2)*\sin(dx + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)**6*(a+b*tan(dx+c))**3,x)

[Out] Timed out

Giac [A] time = 2.03189, size = 255, normalized size = 1.53

$$30b^3 \tan(dx + c)^2 + 180ab^2 \tan(dx + c) + 60(3a^2b + 2b^3) \log(|\tan(dx + c)|) - \frac{411a^2b \tan(dx+c)^5 + 274b^3 \tan(dx+c)^5 + 60a^3}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)^6*(a+b*tan(dx+c))^3,x, algorithm="giac")

[Out] $1/60*(30*b^3*\tan(dx + c)^2 + 180*a*b^2*\tan(dx + c) + 60*(3*a^2*b + 2*b^3)*\log(\text{abs}(\tan(dx + c))) - (411*a^2*b*\tan(dx + c)^5 + 274*b^3*\tan(dx + c)^5 + 60*a^3*\tan(dx + c)^4 + 360*a*b^2*\tan(dx + c)^4 + 180*a^2*b*\tan(dx + c)^3 + 30*b^3*\tan(dx + c)^3 + 40*a^3*\tan(dx + c)^2 + 60*a*b^2*\tan(dx + c)^2 + 45*a^2*b*\tan(dx + c) + 12*a^3)/\tan(dx + c)^5)/d$

3.41 $\int \sin^3(c + dx)(a + b \tan(c + dx))^4 dx$

Optimal. Leaf size=275

$$-\frac{2a^2b^2 \cos^3(c + dx)}{d} + \frac{12a^2b^2 \cos(c + dx)}{d} + \frac{6a^2b^2 \sec(c + dx)}{d} - \frac{4a^3b \sin^3(c + dx)}{3d} - \frac{4a^3b \sin(c + dx)}{d} + \frac{4a^3b \tanh^{-1}(\sin(c + dx))}{d}$$

[Out] (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (a^4*Cos[c + d*x])/d + (12*a^2*b^2*Cos[c + d*x])/d - (3*b^4*Cos[c + d*x])/d + (a^4*Cos[c + d*x]^3)/(3*d) - (2*a^2*b^2*Cos[c + d*x]^3)/d + (b^4*Cos[c + d*x]^3)/(3*d) + (6*a^2*b^2*Sec[c + d*x])/d - (3*b^4*Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) - (4*a^3*b*Sin[c + d*x])/d + (10*a*b^3*Sin[c + d*x])/d - (4*a^3*b*Sin[c + d*x]^3)/(3*d) + (10*a*b^3*Sin[c + d*x]^3)/(3*d) + (2*a*b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/d

Rubi [A] time = 0.247616, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3517, 2633, 2592, 302, 206, 2590, 270, 288}

$$-\frac{2a^2b^2 \cos^3(c + dx)}{d} + \frac{12a^2b^2 \cos(c + dx)}{d} + \frac{6a^2b^2 \sec(c + dx)}{d} - \frac{4a^3b \sin^3(c + dx)}{3d} - \frac{4a^3b \sin(c + dx)}{d} + \frac{4a^3b \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]

[Out] (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (a^4*Cos[c + d*x])/d + (12*a^2*b^2*Cos[c + d*x])/d - (3*b^4*Cos[c + d*x])/d + (a^4*Cos[c + d*x]^3)/(3*d) - (2*a^2*b^2*Cos[c + d*x]^3)/d + (b^4*Cos[c + d*x]^3)/(3*d) + (6*a^2*b^2*Sec[c + d*x])/d - (3*b^4*Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) - (4*a^3*b*Sin[c + d*x])/d + (10*a*b^3*Sin[c + d*x])/d - (4*a^3*b*Sin[c + d*x]^3)/(3*d) + (10*a*b^3*Sin[c + d*x]^3)/(3*d) + (2*a*b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/d

Rule 3517

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2, x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^(n + 1)/2, x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt

Q[m, 2*n - 1]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2590

Int[sin[(e_) + (f_)*(x_)]^(m_)*tan[(e_) + (f_)*(x_)]^(n_), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 288

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1)/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned}
 \int \sin^3(c + dx)(a + b \tan(c + dx))^4 dx &= \int (a^4 \sin^3(c + dx) + 4a^3b \sin^3(c + dx) \tan(c + dx) + 6a^2b^2 \sin^3(c + dx) \tan^2(c + dx) + 4ab^3 \sin^3(c + dx) \tan^3(c + dx) + b^4 \sin^3(c + dx) \tan^4(c + dx)) dx \\
 &= a^4 \int \sin^3(c + dx) dx + (4a^3b) \int \sin^3(c + dx) \tan(c + dx) dx + (6a^2b^2) \int \sin^3(c + dx) \tan^2(c + dx) dx + (4a^3b) \int \sin^3(c + dx) \tan^3(c + dx) dx + b^4 \int \sin^3(c + dx) \tan^4(c + dx) dx \\
 &= -\frac{a^4 \text{Subst}\left(\int (1 - x^2) dx, x, \cos(c + dx)\right)}{d} + \frac{(4a^3b) \text{Subst}\left(\int \frac{x^4}{1 - x^2} dx, x, \sin(c + dx)\right)}{d} \\
 &= -\frac{a^4 \cos(c + dx)}{d} + \frac{a^4 \cos^3(c + dx)}{3d} + \frac{2ab^3 \sin^3(c + dx) \tan^2(c + dx)}{d} + \frac{(4a^3b) \sin^3(c + dx) \tan^3(c + dx)}{d} + \frac{b^4 \sin^3(c + dx) \tan^4(c + dx)}{d} \\
 &= -\frac{a^4 \cos(c + dx)}{d} + \frac{12a^2b^2 \cos(c + dx)}{d} - \frac{3b^4 \cos(c + dx)}{d} + \frac{a^4 \cos^3(c + dx)}{3d} \\
 &= \frac{4a^3b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^4 \cos(c + dx)}{d} + \frac{12a^2b^2 \cos(c + dx)}{d} - \frac{3b^4 \cos(c + dx)}{d} \\
 &= \frac{4a^3b \tanh^{-1}(\sin(c + dx))}{d} - \frac{10ab^3 \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^4 \cos(c + dx)}{d} + \frac{12a^2b^2 \cos(c + dx)}{d} - \frac{3b^4 \cos(c + dx)}{d}
 \end{aligned}$$

Mathematica [B] time = 6.27546, size = 1017, normalized size = 3.7

$$-\frac{(3a^4 - 42b^2a^2 + 11b^4)(a + b \tan(c + dx))^4 \cos^5(c + dx)}{4d(a \cos(c + dx) + b \sin(c + dx))^4} + \frac{ab(a^2 - b^2) \sin(3(c + dx))(a + b \tan(c + dx))^4 \cos^4(c + dx)}{3d(a \cos(c + dx) + b \sin(c + dx))^4}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]

[Out]
$$\begin{aligned} & -(b^2*(-36*a^2 + 17*b^2)*\cos[c + d*x]^4*(a + b*\tan[c + d*x])^4)/(6*d*(a*\cos[c + d*x] + b*\sin[c + d*x])^4) - ((3*a^4 - 42*a^2*b^2 + 11*b^4)*\cos[c + d*x]^5*(a + b*\tan[c + d*x])^4)/(4*d*(a*\cos[c + d*x] + b*\sin[c + d*x])^4) + ((a^4 - 6*a^2*b^2 + b^4)*\cos[c + d*x]^4*\cos[3*(c + d*x)]*(a + b*\tan[c + d*x])^4)/(12*d*(a*\cos[c + d*x] + b*\sin[c + d*x])^4) - (2*(2*a^3*b - 5*a*b^3)*\cos[c + d*x]^4*\log[\cos[(c + d*x)/2] - \sin[(c + d*x)/2]]*(a + b*\tan[c + d*x])^4)/(d*(a*\cos[c + d*x] + b*\sin[c + d*x])^4) + (2*(2*a^3*b - 5*a*b^3)*\cos[c + d*x]^4*\log[\cos[(c + d*x)/2] + \sin[(c + d*x)/2]]*(a + b*\tan[c + d*x])^4)/(d*(a*\cos[c + d*x] + b*\sin[c + d*x])^4) + ((12*a*b^3 + b^4)*\cos[c + d*x]^4*(a + b*\tan[c + d*x])^4)/(12*d*(\cos[(c + d*x)/2] - \sin[(c + d*x)/2])^2*(a*\cos[c + d*x] + b*\sin[c + d*x])^4) + (b^4*\cos[c + d*x]^4*\sin[(c + d*x)/2]*(a + b*\tan[c + d*x])^4)/(6*d*(\cos[(c + d*x)/2] - \sin[(c + d*x)/2])^3*(a*\cos[c + d*x] + b*\sin[c + d*x])^4) - (b^4*\cos[c + d*x]^4*\sin[(c + d*x)/2]*(a + b*\tan[c + d*x])^4)/(6*d*(\cos[(c + d*x)/2] + \sin[(c + d*x)/2])^3*(a*\cos[c + d*x] + b*\sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*\cos[c + d*x]^4*(a + b*\tan[c + d*x])^4)/(12*d*(\cos[(c + d*x)/2] + \sin[(c + d*x)/2])^2*(a*\cos[c + d*x] + b*\sin[c + d*x])^4) + (\cos[c + d*x]^4*(36*a^2*b^2*\sin[(c + d*x)/2] - 17*b^4*\sin[(c + d*x)/2])*(a + b*\tan[c + d*x])^4)/(6*d*(\cos[(c + d*x)/2] - \sin[(c + d*x)/2])*(a*\cos[c + d*x] + b*\sin[c + d*x])^4) + (\cos[c + d*x]^4*(-36*a^2*b^2*\sin[(c + d*x)/2] + 17*b^4*\sin[(c + d*x)/2])*(a + b*\tan[c + d*x])^4)/(6*d*(\cos[(c + d*x)/2] + \sin[(c + d*x)/2])*(a*\cos[c + d*x] + b*\sin[c + d*x])^4) - (a*b*(5*a^2 - 9*b^2)*\cos[c + d*x]^4*\sin[c + d*x]*(a + b*\tan[c + d*x])^4)/(d*(a*\cos[c + d*x] + b*\sin[c + d*x])^4) + (a*b*(a^2 - b^2)*\cos[c + d*x]^4*\sin[3*(c + d*x)]*(a + b*\tan[c + d*x])^4)/(3*d*(a*\cos[c + d*x] + b*\sin[c + d*x])^4) \end{aligned}$$

Maple [A] time = 0.059, size = 412, normalized size = 1.5

$$\frac{b^4 (\sin(dx + c))^8}{3d (\cos(dx + c))^3} - \frac{5b^4 (\sin(dx + c))^8}{3d \cos(dx + c)} - \frac{16b^4 \cos(dx + c)}{3d} - \frac{5b^4 \cos(dx + c) (\sin(dx + c))^6}{3d} - 2 \frac{b^4 \cos(dx + c) (\sin(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x)

[Out]
$$\begin{aligned} & 1/3/d*b^4*\sin(d*x+c)^8/\cos(d*x+c)^3-5/3/d*b^4*\sin(d*x+c)^8/\cos(d*x+c)-16/3*b^4*\cos(d*x+c)/d-5/3/d*b^4*\cos(d*x+c)*\sin(d*x+c)^6-2/d*b^4*\cos(d*x+c)*\sin(d*x+c)^4-8/3/d*b^4*\cos(d*x+c)*\sin(d*x+c)^2+2/d*b^3*a*\sin(d*x+c)^7/\cos(d*x+c)^2+2/d*b^3*a*\sin(d*x+c)^5+10/3*a*b^3*\sin(d*x+c)^3/d+10*a*b^3*\sin(d*x+c)/d-10/d*b^3*a*\ln(\sec(d*x+c)+\tan(d*x+c))+6/d*a^2*b^2*\sin(d*x+c)^6/\cos(d*x+c)+16*a^2*b^2*\cos(d*x+c)/d+6/d*a^2*b^2*\cos(d*x+c)*\sin(d*x+c)^4+8/d*a^2*b^2*\cos(d*x+c)*\sin(d*x+c)^2-4/3*a^3*b*\sin(d*x+c)^3/d-4*a^3*b*\sin(d*x+c)/d+4/d*b*a^3*\ln(\sec(d*x+c)+\tan(d*x+c))-1/3/d*\cos(d*x+c)*\sin(d*x+c)^2*a^4-2/3*a^4*\cos(d*x+c)/d \end{aligned}$$

Maxima [A] time = 1.14764, size = 294, normalized size = 1.07

$$(\cos(dx + c)^3 - 3 \cos(dx + c))a^4 - 2(2 \sin(dx + c)^3 - 3 \log(\sin(dx + c) + 1) + 3 \log(\sin(dx + c) - 1) + 6 \sin(dx + c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*a^4 - 2*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^3*b - 6*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^2*b^2 + (4*sin(d*x + c)^3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 24*sin(d*x + c))*a*b^3 + (cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x + c))*b^4)/d
```

Fricas [A] time = 2.13296, size = 528, normalized size = 1.92

$$\frac{(a^4 - 6a^2b^2 + b^4)\cos(dx + c)^6 - 3(a^4 - 12a^2b^2 + 3b^4)\cos(dx + c)^4 + 3(2a^3b - 5ab^3)\cos(dx + c)^3 \log(\sin(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*((a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^6 - 3*(a^4 - 12*a^2*b^2 + 3*b^4)*cos(d*x + c)^4 + 3*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + b^4 + 9*(2*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2*(2*(a^3*b - a*b^3)*cos(d*x + c)^5 + 3*a*b^3*cos(d*x + c) - 2*(4*a^3*b - 7*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.42 $\int \sin^2(c + dx)(a + b \tan(c + dx))^4 dx$

Optimal. Leaf size=139

$$\frac{b^2(18a^2 - 5b^2)\tan(c + dx)}{2d} - \frac{4ab(a^2 - 2b^2)\log(\cos(c + dx))}{d} + \frac{1}{2}x(-18a^2b^2 + a^4 + 5b^4) + \frac{4ab^3 \tan^2(c + dx)}{d} - \frac{\sin(c + dx)}{2d}$$

```
[Out] ((a^4 - 18*a^2*b^2 + 5*b^4)*x)/2 - (4*a*b*(a^2 - 2*b^2)*Log[Cos[c + d*x]])/
d + (b^2*(18*a^2 - 5*b^2)*Tan[c + d*x])/(2*d) + (4*a*b^3*Tan[c + d*x]^2)/d
+ (5*b^4*Tan[c + d*x]^3)/(6*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c +
d*x])^4)/(2*d)
```

Rubi [A] time = 0.174975, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1645, 801, 635, 203, 260}

$$\frac{b^2(18a^2 - 5b^2)\tan(c + dx)}{2d} - \frac{4ab(a^2 - 2b^2)\log(\cos(c + dx))}{d} + \frac{1}{2}x(-18a^2b^2 + a^4 + 5b^4) + \frac{4ab^3 \tan^2(c + dx)}{d} - \frac{\sin(c + dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((a^4 - 18*a^2*b^2 + 5*b^4)*x)/2 - (4*a*b*(a^2 - 2*b^2)*Log[Cos[c + d*x]])/
d + (b^2*(18*a^2 - 5*b^2)*Tan[c + d*x])/(2*d) + (4*a*b^3*Tan[c + d*x]^2)/d
+ (5*b^4*Tan[c + d*x]^3)/(6*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c +
d*x])^4)/(2*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
+ 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &&
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```


}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \sin^2(c+dx)(a+b \tan(c+dx))^4 dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^2(a+x)^4}{(b^2+x^2)^2} dx, x, b \tan(c+dx)\right)}{d} \\ &= -\frac{\cos(c+dx) \sin(c+dx)(a+b \tan(c+dx))^4}{2d} - \frac{\operatorname{Subst}\left(\int \frac{(a+x)^3(-ab^2-5b^2x)}{b^2+x^2} dx, x, b \tan(c+dx)\right)}{2bd} \\ &= -\frac{\cos(c+dx) \sin(c+dx)(a+b \tan(c+dx))^4}{2d} - \frac{\operatorname{Subst}\left(\int (-18a^2b^2+5b^4-18ab^2x+5b^3x^2) dx, x, b \tan(c+dx)\right)}{2bd} \\ &= \frac{b^2(18a^2-5b^2) \tan(c+dx)}{2d} + \frac{4ab^3 \tan^2(c+dx)}{d} + \frac{5b^4 \tan^3(c+dx)}{6d} - \frac{\cos(c+dx) \sin(c+dx)(a+b \tan(c+dx))^4}{2d} \\ &= \frac{b^2(18a^2-5b^2) \tan(c+dx)}{2d} + \frac{4ab^3 \tan^2(c+dx)}{d} + \frac{5b^4 \tan^3(c+dx)}{6d} - \frac{\cos(c+dx) \sin(c+dx)(a+b \tan(c+dx))^4}{2d} \\ &= \frac{1}{2}(a^4-18a^2b^2+5b^4)x - \frac{4ab(a^2-2b^2) \log(\cos(c+dx))}{d} + \frac{b^2(18a^2-5b^2)}{2d} \end{aligned}$$

Mathematica [A] time = 6.26561, size = 263, normalized size = 1.89

$$b \left(-\frac{(-6a^2b^2+a^4+b^4) \tan^{-1}(\tan(c+dx))}{2b} + 2b(3a^2-b^2) \tan(c+dx) + \frac{1}{2} \left(\frac{-12a^2b^2+a^4+3b^4}{\sqrt{-b^2}} + 4a^3 - 8ab^2 \right) \log\left(\sqrt{-b^2} - b \tan(c+dx)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^4, x]

[Out] (b*(-((a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/(2*b) + 2*a*(a - b)*(a + b)*Cos[c + d*x]^2 + ((4*a^3 - 8*a*b^2 + (a^4 - 12*a^2*b^2 + 3*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]]/2 + ((4*a^3 - 8*a*b^2 - (a^4 - 12*a^2*b^2 + 3*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]]/2 - ((a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*b) + 2*b*(3*a^2 - b^2)*Tan[c + d*x] + 2*a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c + d*x]^3)/3))/d

Maple [B] time = 0.057, size = 368, normalized size = 2.7

$$\frac{b^4 (\sin(dx+c))^7}{3d (\cos(dx+c))^3} - \frac{4b^4 (\sin(dx+c))^7}{3d \cos(dx+c)} - \frac{4b^4 \cos(dx+c) (\sin(dx+c))^5}{3d} - \frac{5b^4 \cos(dx+c) (\sin(dx+c))^3}{3d} - \frac{5b^4 \sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x)`

[Out] $\frac{1}{3}d^4 \sin(d*x+c)^7 / \cos(d*x+c)^3 - \frac{4}{3}d^4 \sin(d*x+c)^7 / \cos(d*x+c) - \frac{4}{3}d^4 \sin(d*x+c)^5 / \cos(d*x+c) + \frac{5}{2}d^4 \sin(d*x+c)^5 / \cos(d*x+c) + \frac{5}{2}d^4 \sin(d*x+c)^3 / \cos(d*x+c) + \frac{2}{d^3} a \sin(d*x+c)^6 / \cos(d*x+c) + \frac{2}{d^3} a \sin(d*x+c)^4 + \frac{4}{d^3} a \sin(d*x+c)^2 + \frac{8}{d^3} a \ln(\cos(d*x+c)) + \frac{6}{d^2} a^2 b^2 \sin(d*x+c)^5 / \cos(d*x+c) + \frac{6}{d^2} a^2 b^2 \cos(d*x+c) \sin(d*x+c)^3 + \frac{9}{d^2} a^2 b^2 \sin(d*x+c) \cos(d*x+c) - \frac{9}{d^2} a^2 b^2 x - \frac{9}{d^2} a^2 b^2 c - \frac{2}{d} b a^3 \sin(d*x+c)^2 - \frac{4}{d} b a^3 \ln(\cos(d*x+c)) - \frac{1}{2} d a^4 \sin(d*x+c) \cos(d*x+c) + \frac{1}{2} a^4 x + \frac{1}{2} d a^4 c$

Maxima [A] time = 1.70871, size = 208, normalized size = 1.5

$$\frac{2b^4 \tan(dx+c)^3 + 12ab^3 \tan(dx+c)^2 + 3(a^4 - 18a^2b^2 + 5b^4)(dx+c) + 12(a^3b - 2ab^3) \log(\tan(dx+c)^2 + 1) + 12(a^4 - 6a^2b^2 + b^4) \tan(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")`

[Out] $\frac{1}{6} (2b^4 \tan(dx+c)^3 + 12a^3b \tan(dx+c)^2 + 3(a^4 - 18a^2b^2 + 5b^4)(dx+c) + 12(a^3b - 2ab^3) \log(\tan(dx+c)^2 + 1) + 12(3a^4 - 2b^4) \tan(dx+c) + 3(4a^3b - 4a^2b^3 - (a^4 - 6a^2b^2 + b^4) \tan(dx+c))) / (\tan(dx+c)^2 + 1) / d$

Fricas [A] time = 2.24796, size = 431, normalized size = 3.1

$$\frac{12(a^3b - ab^3) \cos(dx+c)^5 + 12ab^3 \cos(dx+c) - 24(a^3b - 2ab^3) \cos(dx+c)^3 \log(-\cos(dx+c)) - 3(2a^3b - 2ab^3 - (a^4 - 18a^2b^2 + 5b^4) dx) \cos(dx+c)^3 - (3(a^4 - 6a^2b^2 + b^4) \cos(dx+c)^4 - 2b^4 - 2(18a^2b^2 - 7b^4) \cos(dx+c)^2) \sin(dx+c)}{6d \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")`

[Out] $\frac{1}{6} (12(a^3b - ab^3) \cos(dx+c)^5 + 12a^3b \cos(dx+c) - 24(a^3b - 2ab^3) \cos(dx+c)^3 \log(-\cos(dx+c)) - 3(2a^3b - 2ab^3 - (a^4 - 18a^2b^2 + 5b^4) dx) \cos(dx+c)^3 - (3(a^4 - 6a^2b^2 + b^4) \cos(dx+c)^4 - 2b^4 - 2(18a^2b^2 - 7b^4) \cos(dx+c)^2) \sin(dx+c)) / (d \cos(dx+c)^3)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**4,x)`

[Out] Timed out

Giac [B] time = 7.19152, size = 5307, normalized size = 38.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/6*(3*a^4*d*x*tan(d*x)^5*tan(c)^5 - 54*a^2*b^2*d*x*tan(d*x)^5*tan(c)^5 + 15*b^4*d*x*tan(d*x)^5*tan(c)^5 - 12*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 + 24*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 + 3*a^4*d*x*tan(d*x)^5*tan(c)^3 - 54*a^2*b^2*d*x*tan(d*x)^5*tan(c)^3 + 15*b^4*d*x*tan(d*x)^5*tan(c)^3 - 9*a^4*d*x*tan(d*x)^4*tan(c)^4 + 162*a^2*b^2*d*x*tan(d*x)^4*tan(c)^4 - 45*b^4*d*x*tan(d*x)^4*tan(c)^4 + 3*a^4*d*x*tan(d*x)^3*tan(c)^5 - 54*a^2*b^2*d*x*tan(d*x)^3*tan(c)^5 + 15*b^4*d*x*tan(d*x)^3*tan(c)^5 + 6*a^3*b*tan(d*x)^5*tan(c)^5 + 6*a*b^3*tan(d*x)^5*tan(c)^5 - 12*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^3 + 24*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^3 + 36*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 - 72*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 3*a^4*tan(d*x)^5*tan(c)^4 - 54*a^2*b^2*tan(d*x)^5*tan(c)^4 + 15*b^4*tan(d*x)^5*tan(c)^4 - 12*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^5 + 24*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^5 + 3*a^4*tan(d*x)^4*tan(c)^5 - 54*a^2*b^2*tan(d*x)^4*tan(c)^5 + 15*b^4*tan(d*x)^4*tan(c)^5 - 9*a^4*d*x*tan(d*x)^4*tan(c)^2 + 162*a^2*b^2*d*x*tan(d*x)^4*tan(c)^2 + 12*a^4*d*x*tan(d*x)^3*tan(c)^3 - 216*a^2*b^2*d*x*tan(d*x)^3*tan(c)^3 + 60*b^4*d*x*tan(d*x)^3*tan(c)^3 - 6*a^3*b*tan(d*x)^5*tan(c)^3 + 30*a*b^3*tan(d*x)^5*tan(c)^3 - 9*a^4*d*x*tan(d*x)^2*tan(c)^4 + 162*a^2*b^2*d*x*tan(d*x)^2*tan(c)^4 - 45*b^4*d*x*tan(d*x)^2*tan(c)^4 - 42*a^3*b*tan(d*x)^4*tan(c)^4 + 30*a*b^3*tan(d*x)^4*tan(c)^4 - 6*a^3*b*tan(d*x)^3*tan(c)^5 + 30*a*b^3*tan(d*x)^3*tan(c)^5 + 36*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2 - 72*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2 - 36*a^2*b^2*tan(d*x)^5*tan(c)^2 + 10*b^4*tan(d*x)^5*tan(c)^2 - 48*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 + 96*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 - 12*a^4*tan(d*x)^4*tan(c)^3 + 108*a^2*b^2*tan(d*x)^4*tan(c)^3 - 30*b^4*tan(d*x)^4*tan(c)^3 + 36*a^3*b*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 - 72*a*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 - 12*a^4*tan(d*x)^3*tan(c)^4 + 108*a^2*b^2$$

$$\begin{aligned}
& 2*\tan(d*x)^3*\tan(c)^4 - 30*b^4*\tan(d*x)^3*\tan(c)^4 - 36*a^2*b^2*\tan(d*x)^2* \\
& \tan(c)^5 + 10*b^4*\tan(d*x)^2*\tan(c)^5 + 9*a^4*d*x*\tan(d*x)^3*\tan(c) - 162*a \\
& ^2*b^2*d*x*\tan(d*x)^3*\tan(c) + 45*b^4*d*x*\tan(d*x)^3*\tan(c) + 12*a*b^3*\tan \\
& (d*x)^5*\tan(c) - 12*a^4*d*x*\tan(d*x)^2*\tan(c)^2 + 216*a^2*b^2*d*x*\tan(d*x)^2 \\
& *\tan(c)^2 - 60*b^4*d*x*\tan(d*x)^2*\tan(c)^2 + 18*a^3*b*\tan(d*x)^4*\tan(c)^2 - \\
& 42*a*b^3*\tan(d*x)^4*\tan(c)^2 + 9*a^4*d*x*\tan(d*x)*\tan(c)^3 - 162*a^2*b^2*d \\
& *x*\tan(d*x)*\tan(c)^3 + 45*b^4*d*x*\tan(d*x)*\tan(c)^3 + 96*a^3*b*\tan(d*x)^3*t \\
& \tan(c)^3 - 48*a*b^3*\tan(d*x)^3*\tan(c)^3 + 18*a^3*b*\tan(d*x)^2*\tan(c)^4 - 42* \\
& a*b^3*\tan(d*x)^2*\tan(c)^4 + 12*a*b^3*\tan(d*x)*\tan(c)^5 - 2*b^4*\tan(d*x)^5 - \\
& 36*a^3*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \\
& \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^3*\tan \\
& (c) + 72*a*b^3*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan \\
& (c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^3* \\
& \tan(c) + 72*a^2*b^2*\tan(d*x)^4*\tan(c) - 30*b^4*\tan(d*x)^4*\tan(c) + 48*a^3*b \\
& *\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x) \\
& ^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(c)^2 - 96 \\
& *a*b^3*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan \\
& (d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2*\tan(c)^ \\
& 2 + 18*a^4*\tan(d*x)^3*\tan(c)^2 - 108*a^2*b^2*\tan(d*x)^3*\tan(c)^2 + 10*b^4*t \\
& \tan(d*x)^3*\tan(c)^2 - 36*a^3*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2 \\
& *\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + \\
& 1))*\tan(d*x)*\tan(c)^3 + 72*a*b^3*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 \\
& - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan \\
& (c) + 1))*\tan(d*x)*\tan(c)^3 + 18*a^4*\tan(d*x)^2*\tan(c)^3 - 108*a^2*b^2*\tan \\
& (d*x)^2*\tan(c)^3 + 10*b^4*\tan(d*x)^2*\tan(c)^3 + 72*a^2*b^2*\tan(d*x)*\tan(c)^4 \\
& - 30*b^4*\tan(d*x)*\tan(c)^4 - 2*b^4*\tan(c)^5 - 3*a^4*d*x*\tan(d*x)^2 + 54*a^2 \\
& *b^2*d*x*\tan(d*x)^2 - 15*b^4*d*x*\tan(d*x)^2 - 12*a*b^3*\tan(d*x)^4 + 9*a^4*d \\
& *x*\tan(d*x)*\tan(c) - 162*a^2*b^2*d*x*\tan(d*x)*\tan(c) + 45*b^4*d*x*\tan(d*x)* \\
& \tan(c) - 18*a^3*b*\tan(d*x)^3*\tan(c) + 42*a*b^3*\tan(d*x)^3*\tan(c) - 3*a^4*d* \\
& x*\tan(c)^2 + 54*a^2*b^2*d*x*\tan(c)^2 - 15*b^4*d*x*\tan(c)^2 - 96*a^3*b*\tan(d \\
& *x)^2*\tan(c)^2 + 48*a*b^3*\tan(d*x)^2*\tan(c)^2 - 18*a^3*b*\tan(d*x)*\tan(c)^3 \\
& + 42*a*b^3*\tan(d*x)*\tan(c)^3 - 12*a*b^3*\tan(c)^4 + 12*a^3*b*\log(4*(\tan(c)^2 \\
& + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan \\
& (d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2 - 24*a*b^3*\log(4*(\tan(c)^2 + \\
& 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d \\
& *x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan(d*x)^2 - 36*a^2*b^2*\tan(d*x)^3 + 10*b^4 \\
& *\tan(d*x)^3 - 36*a^3*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d* \\
& x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))*\tan \\
& (d*x)*\tan(c) + 72*a*b^3*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan \\
& (d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1))* \\
& \tan(d*x)*\tan(c) - 12*a^4*\tan(d*x)^2*\tan(c) + 108*a^2*b^2*\tan(d*x)^2*\tan(c) \\
& - 30*b^4*\tan(d*x)^2*\tan(c) + 12*a^3*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan \\
& (c)^2 - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)* \\
& \tan(c) + 1))*\tan(c)^2 - 24*a*b^3*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 \\
& - 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan \\
& (c) + 1))*\tan(c)^2 - 12*a^4*\tan(d*x)*\tan(c)^2 + 108*a^2*b^2*\tan(d*x)*\tan(c)^2 \\
& - 30*b^4*\tan(d*x)*\tan(c)^2 - 36*a^2*b^2*\tan(c)^3 + 10*b^4*\tan(c)^3 - 3*a^4 \\
& *d*x + 54*a^2*b^2*d*x - 15*b^4*d*x + 6*a^3*b*\tan(d*x)^2 - 30*a*b^3*\tan(d*x) \\
& ^2 + 42*a^3*b*\tan(d*x)*\tan(c) - 30*a*b^3*\tan(d*x)*\tan(c) + 6*a^3*b*\tan(c)^2 \\
& - 30*a*b^3*\tan(c)^2 + 12*a^3*b*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - \\
& 2*\tan(d*x)^3*\tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan \\
& (c) + 1)) - 24*a*b^3*\log(4*(\tan(c)^2 + 1)/(\tan(d*x)^4*\tan(c)^2 - 2*\tan(d*x)^3* \\
& \tan(c) + \tan(d*x)^2*\tan(c)^2 + \tan(d*x)^2 - 2*\tan(d*x)*\tan(c) + 1)) + 3*a^4 \\
& *\tan(d*x) - 54*a^2*b^2*\tan(d*x) + 15*b^4*\tan(d*x) + 3*a^4*\tan(c) - 54*a^2*b \\
& ^2*\tan(c) + 15*b^4*\tan(c) - 6*a^3*b - 6*a*b^3)/(d*\tan(d*x)^5*\tan(c)^5 + d*t \\
& \tan(d*x)^5*\tan(c)^3 - 3*d*\tan(d*x)^4*\tan(c)^4 + d*\tan(d*x)^3*\tan(c)^5 - 3*d* \\
& \tan(d*x)^4*\tan(c)^2 + 4*d*\tan(d*x)^3*\tan(c)^3 - 3*d*\tan(d*x)^2*\tan(c)^4 + 3 \\
& *d*\tan(d*x)^3*\tan(c) - 4*d*\tan(d*x)^2*\tan(c)^2 + 3*d*\tan(d*x)*\tan(c)^3 - d* \\
& \tan(d*x)^2 + 3*d*\tan(d*x)*\tan(c) - d*\tan(c)^2 - d)
\end{aligned}$$

3.43 $\int \sin(c + dx)(a + b \tan(c + dx))^4 dx$

Optimal. Leaf size=180

$$\frac{6a^2b^2 \cos(c + dx)}{d} + \frac{6a^2b^2 \sec(c + dx)}{d} - \frac{4a^3b \sin(c + dx)}{d} + \frac{4a^3b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^4 \cos(c + dx)}{d} + \frac{6ab^3 \sin(c + dx)}{d}$$

[Out] $(4a^3b \operatorname{ArcTanh}[\sin(c + dx)])/d - (6a^3b^3 \operatorname{ArcTanh}[\sin(c + dx)])/d - (a^4 \cos(c + dx))/d + (6a^2b^2 \sec(c + dx))/d - (b^4 \cos(c + dx))/d + (6a^2b^2 \sec(c + dx))/d - (2b^4 \sec(c + dx))/d + (b^4 \sec(c + dx)^3)/(3d) - (4a^3b \sin(c + dx))/d + (6a^3b^3 \sin(c + dx))/d + (2a^3b^3 \sin(c + dx) \tan(c + dx)^2)/d$

Rubi [A] time = 0.15661, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3517, 2638, 2592, 321, 206, 2590, 14, 288, 270}

$$\frac{6a^2b^2 \cos(c + dx)}{d} + \frac{6a^2b^2 \sec(c + dx)}{d} - \frac{4a^3b \sin(c + dx)}{d} + \frac{4a^3b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^4 \cos(c + dx)}{d} + \frac{6ab^3 \sin(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\sin(c + dx)(a + b \tan(c + dx))^4, x]$

[Out] $(4a^3b \operatorname{ArcTanh}[\sin(c + dx)])/d - (6a^3b^3 \operatorname{ArcTanh}[\sin(c + dx)])/d - (a^4 \cos(c + dx))/d + (6a^2b^2 \sec(c + dx))/d - (b^4 \cos(c + dx))/d + (6a^2b^2 \sec(c + dx))/d - (2b^4 \sec(c + dx))/d + (b^4 \sec(c + dx)^3)/(3d) - (4a^3b \sin(c + dx))/d + (6a^3b^3 \sin(c + dx))/d + (2a^3b^3 \sin(c + dx) \tan(c + dx)^2)/d$

Rule 3517

$\operatorname{Int}[\sin[(e_.) + (f_.)(x_.)]^{(m_.)}((a_.) + (b_.)\tan[(e_.) + (f_.)(x_.)]^{(n_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{Expand}[\sin[e + f*x]^{m*(a + b*\tan[e + f*x])^n}, x], x] /;$ $\operatorname{FreeQ}\{a, b, e, f, x\} \ \&\& \ \operatorname{IntegerQ}[(m - 1)/2] \ \&\& \ \operatorname{IGtQ}[n, 0]$

Rule 2638

$\operatorname{Int}[\sin[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\cos[c + d*x]/d, x] /;$ $\operatorname{FreeQ}\{c, d, x\}$

Rule 2592

$\operatorname{Int}[(a_.)\sin[(e_.) + (f_.)(x_.)]^{(m_.)}\tan[(e_.) + (f_.)(x_.)]^{(n_.)}, x_Symbol] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\sin[e + f*x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(ff*x)^{(m+n)}(a^2 - ff^2*x^2)^{(n+1)/2}, x], x, (a*\sin[e + f*x])/ff], x] /;$ $\operatorname{FreeQ}\{a, e, f, m, x\} \ \&\& \ \operatorname{IntegerQ}[(n + 1)/2]$

Rule 321

$\operatorname{Int}[(c_.)(x_.)^{(m_.)}((a_.) + (b_.)(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(c^{(n-1)}(c*x)^{(m-n+1)}(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \operatorname{Dist}[(a*c^{(n-1)}(c*x)^{(m-n+1)})/(b*(m+n*p+1)), \operatorname{Int}[(c*x)^{(m-n)}(a + b*x^n)^p, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, p, x\} \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{GtQ}[m, n - 1] \ \&\& \ \operatorname{NeQ}[m + n*p + 1, 0] \ \&\& \ \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2590

Int[sin[(e_) + (f_)*(x_)]^(m_)*tan[(e_) + (f_)*(x_)]^(n_), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 288

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \sin(c + dx)(a + b \tan(c + dx))^4 dx &= \int (a^4 \sin(c + dx) + 4a^3b \sin(c + dx) \tan(c + dx) + 6a^2b^2 \sin(c + dx) \tan^2(c + dx) + 4a^2b^3 \sin(c + dx) \tan^3(c + dx) + b^4 \sin(c + dx) \tan^4(c + dx)) dx \\
 &= a^4 \int \sin(c + dx) dx + (4a^3b) \int \sin(c + dx) \tan(c + dx) dx + (6a^2b^2) \int \sin(c + dx) \tan^2(c + dx) dx + (4a^2b^3) \int \sin(c + dx) \tan^3(c + dx) dx + b^4 \int \sin(c + dx) \tan^4(c + dx) dx \\
 &= -\frac{a^4 \cos(c + dx)}{d} + \frac{(4a^3b) \text{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, \sin(c + dx)\right)}{d} - \frac{(6a^2b^2) \text{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, \sin(c + dx)\right)}{d} + \frac{(4a^2b^3) \text{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, \sin(c + dx)\right)}{d} - \frac{b^4 \text{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, \sin(c + dx)\right)}{d} \\
 &= -\frac{a^4 \cos(c + dx)}{d} - \frac{4a^3b \sin(c + dx)}{d} + \frac{2ab^3 \sin(c + dx) \tan^2(c + dx)}{d} + \frac{(4a^3b) \text{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, \sin(c + dx)\right)}{d} - \frac{(6a^2b^2) \text{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, \sin(c + dx)\right)}{d} + \frac{(4a^2b^3) \text{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, \sin(c + dx)\right)}{d} - \frac{b^4 \text{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, \sin(c + dx)\right)}{d} \\
 &= \frac{4a^3b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^4 \cos(c + dx)}{d} + \frac{6a^2b^2 \cos(c + dx)}{d} - \frac{b^4 \cos(c + dx)}{d} + \frac{4a^2b^3 \tanh^{-1}(\sin(c + dx))}{d} - \frac{6ab^3 \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^4 \cos(c + dx)}{d} + \frac{6a^2b^2 \cos(c + dx)}{d} - \frac{b^4 \cos(c + dx)}{d} \\
 &= \frac{4a^3b \tanh^{-1}(\sin(c + dx))}{d} - \frac{6ab^3 \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^4 \cos(c + dx)}{d} + \frac{6a^2b^2 \cos(c + dx)}{d} - \frac{b^4 \cos(c + dx)}{d}
 \end{aligned}$$

Mathematica [B] time = 5.45856, size = 383, normalized size = 2.13

$$-48ab(a^2 - b^2) \sin(c + dx) - 12(-6a^2b^2 + a^4 + b^4) \cos(c + dx) + \frac{2b^2(36a^2 - 11b^2) \sin\left(\frac{1}{2}(c + dx)\right)}{\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)} + \frac{2b^2(11b^2 - 36a^2) \sin\left(\frac{1}{2}(c + dx)\right)}{\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^4, x]

```
[Out] (72*a^2*b^2 - 22*b^4 - 12*(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x] - 24*a*b*(2*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b*(2*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^3*(12*a + b))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (2*b^2*(36*a^2 - 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (b^3*(-12*a + b))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (2*b^2*(-36*a^2 + 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 48*a*b*(a^2 - b^2)*Sin[c + d*x])/(12*d)
```

Maple [A] time = 0.049, size = 309, normalized size = 1.7

$$\frac{b^4 (\sin(dx+c))^6}{3d (\cos(dx+c))^3} - \frac{b^4 (\sin(dx+c))^6}{d \cos(dx+c)} - \frac{8b^4 \cos(dx+c)}{3d} - \frac{b^4 \cos(dx+c) (\sin(dx+c))^4}{d} - \frac{4b^4 \cos(dx+c) (\sin(dx+c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^4,x)
```

```
[Out] 1/3/d*b^4*sin(d*x+c)^6/cos(d*x+c)^3-1/d*b^4*sin(d*x+c)^6/cos(d*x+c)-8/3*b^4*cos(d*x+c)/d-1/d*b^4*cos(d*x+c)*sin(d*x+c)^4-4/3/d*b^4*cos(d*x+c)*sin(d*x+c)^2+2/d*b^3*a*sin(d*x+c)^5/cos(d*x+c)^2+2*a*b^3*sin(d*x+c)^3/d+6*a*b^3*sin(d*x+c)/d-6/d*b^3*a*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2*b^2*sin(d*x+c)^4/cos(d*x+c)+6/d*a^2*b^2*cos(d*x+c)*sin(d*x+c)^2+12*a^2*b^2*cos(d*x+c)/d-4*a^3*b*sin(d*x+c)/d+4/d*b*a^3*ln(sec(d*x+c)+tan(d*x+c))-a^4*cos(d*x+c)/d
```

Maxima [A] time = 1.09334, size = 224, normalized size = 1.24

$$3ab^3 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} + 3 \log(\sin(dx+c)+1) - 3 \log(\sin(dx+c)-1) - 4 \sin(dx+c) \right) - 18a^2b^2 \left(\frac{1}{\cos(dx+c)} + \cos(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 18*a^2*b^2*(1/cos(d*x + c) + cos(d*x + c)) + b^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)) - 6*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) + 3*a^4*cos(d*x + c))/d
```

Fricas [A] time = 2.06641, size = 414, normalized size = 2.3

$$\frac{3(a^4 - 6a^2b^2 + b^4) \cos(dx+c)^4 - 3(2a^3b - 3ab^3) \cos(dx+c)^3 \log(\sin(dx+c)+1) + 3(2a^3b - 3ab^3) \cos(dx+c)^3}{3d \cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^4 - 3*(2*a^3*b - 3*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*(2*a^3*b - 3*a*b^3)*cos(d*x + c)^3*log
```


$(-\sin(dx + c) + 1) - b^4 - 6*(3*a^2*b^2 - b^4)*\cos(dx + c)^2 - 6*(a*b^3*\cos(dx + c) - 2*(a^3*b - a*b^3)*\cos(dx + c)^3)*\sin(dx + c)/(d*\cos(dx + c)^3)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx))^4 \sin(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)*(a+b*tan(dx+c))**4,x)

[Out] Integral((a + b*tan(c + d*x))**4*sin(c + d*x), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)*(a+b*tan(dx+c))^4,x, algorithm="giac")

[Out] Timed out

3.44 $\int \csc(c + dx)(a + b \tan(c + dx))^4 dx$

Optimal. Leaf size=118

$$\frac{6a^2b^2 \sec(c + dx)}{d} + \frac{4a^3b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^4 \tanh^{-1}(\cos(c + dx))}{d} - \frac{2ab^3 \tanh^{-1}(\sin(c + dx))}{d} + \frac{2ab^3 \tan(c + dx)}{d}$$

[Out] $-(a^4 \operatorname{ArcTanh}[\cos[c + dx]])/d + (4a^3b \operatorname{ArcTanh}[\sin[c + dx]])/d - (2a^2b^2 \operatorname{ArcTanh}[\sin[c + dx]])/d + (6a^2b^2 \sec[c + dx])/d - (b^4 \sec[c + dx])/d + (b^4 \sec[c + dx]^3)/(3d) + (2ab^3 \sec[c + dx] \tan[c + dx])/d$

Rubi [A] time = 0.114219, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3517, 3770, 2606, 8, 2611}

$$\frac{6a^2b^2 \sec(c + dx)}{d} + \frac{4a^3b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^4 \tanh^{-1}(\cos(c + dx))}{d} - \frac{2ab^3 \tanh^{-1}(\sin(c + dx))}{d} + \frac{2ab^3 \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c + dx] * (a + b * \text{Tan}[c + dx])^4, x]$

[Out] $-(a^4 \operatorname{ArcTanh}[\cos[c + dx]])/d + (4a^3b \operatorname{ArcTanh}[\sin[c + dx]])/d - (2a^2b^2 \operatorname{ArcTanh}[\sin[c + dx]])/d + (6a^2b^2 \sec[c + dx])/d - (b^4 \sec[c + dx])/d + (b^4 \sec[c + dx]^3)/(3d) + (2ab^3 \sec[c + dx] \tan[c + dx])/d$

Rule 3517

$\text{Int}[\sin[(e_.) + (f_.)(x_)]^{(m_.)} * ((a_.) + (b_.) * \tan[(e_.) + (f_.)(x_)]^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{Expand}[\sin[e + f*x]^{m*(a + b*\text{Tan}[e + f*x])^n}, x], x] /; \text{FreeQ}\{a, b, e, f\}, x \ \&\& \ \text{IntegerQ}[(m - 1)/2] \ \&\& \ \text{IGtQ}[n, 0]$

Rule 3770

$\text{Int}[\csc[(c_.) + (d_.)(x_)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\cos[c + d*x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2606

$\text{Int}[(a_.) * \sec[(e_.) + (f_.)(x_)]^{(m_.)} * ((b_.) * \tan[(e_.) + (f_.)(x_)]^{(n_.)}), x_Symbol] \rightarrow \text{Dist}[a/f, \text{Subst}[\text{Int}[(a*x)^{(m - 1)} * (-1 + x^2)^{(n - 1)/2}], x], x, \sec[e + f*x]], x] /; \text{FreeQ}\{a, e, f, m\}, x \ \&\& \ \text{IntegerQ}[(n - 1)/2] \ \&\& \ !(\text{IntegerQ}[m/2] \ \&\& \ \text{LtQ}[0, m, n + 1])$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2611

$\text{Int}[(a_.) * \sec[(e_.) + (f_.)(x_)]^{(m_.)} * ((b_.) * \tan[(e_.) + (f_.)(x_)]^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(b*(a*\sec[e + f*x])^{m*(b*\text{Tan}[e + f*x])^{n - 1}})/(f*(m + n - 1)), x] - \text{Dist}[(b^2*(n - 1))/(m + n - 1), \text{Int}[(a*\sec[e + f*x])^{m*(b*\text{Tan}[e + f*x])^{n - 2}}, x], x] /; \text{FreeQ}\{a, b, e, f, m\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{NeQ}[m + n - 1, 0] \ \&\& \ \text{IntegersQ}[2*m, 2*n]$

Rubi steps

$$\begin{aligned}
\int \csc(c+dx)(a+b\tan(c+dx))^4 dx &= \int (a^4 \csc(c+dx) + 4a^3b \sec(c+dx) + 6a^2b^2 \sec(c+dx) \tan(c+dx) + 4ab^3 \sec^3(c+dx) + b^4 \csc(c+dx) \tan^3(c+dx)) dx \\
&= a^4 \int \csc(c+dx) dx + (4a^3b) \int \sec(c+dx) dx + (6a^2b^2) \int \sec(c+dx) \tan(c+dx) dx + 4ab^3 \int \sec^3(c+dx) dx + b^4 \int \csc(c+dx) \tan^3(c+dx) dx \\
&= -\frac{a^4 \tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^3b \tanh^{-1}(\sin(c+dx))}{d} + \frac{2ab^3 \sec(c+dx) \tan(c+dx)}{d} \\
&= -\frac{a^4 \tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^3b \tanh^{-1}(\sin(c+dx))}{d} - \frac{2ab^3 \tanh^{-1}(\sin(c+dx))}{d}
\end{aligned}$$

Mathematica [B] time = 5.1324, size = 352, normalized size = 2.98

$$2b^2 \sin^2\left(\frac{1}{2}(c+dx)\right) \sec^3(c+dx) \left((36a^2 - 5b^2) \cos(2(c+dx)) + 36a^2 + 2b^2 \cos(c+dx) - b^2 \right) + 72a^2b^2 - 48a^3b \log\left(\frac{\cos(c+dx) - \sin(c+dx)}{\cos(c+dx) + \sin(c+dx)}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^4,x]

[Out] (72*a^2*b^2 - 10*b^4 - 12*a^4*Log[Cos[(c + d*x)/2]] - 48*a^3*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^4*Log[Sin[(c + d*x)/2]] + 48*a^3*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 24*a*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*a*b^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + b^4/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 2*b^2*(36*a^2 - b^2 + 2*b^2*Cos[c + d*x] + (36*a^2 - 5*b^2)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2]^2 - (12*a*b^3)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + b^4/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(12*d)

Maple [A] time = 0.069, size = 214, normalized size = 1.8

$$\frac{b^4 (\sin(dx+c))^4}{3d (\cos(dx+c))^3} - \frac{b^4 (\sin(dx+c))^4}{3d \cos(dx+c)} - \frac{b^4 \cos(dx+c) (\sin(dx+c))^2}{3d} - \frac{2b^4 \cos(dx+c)}{3d} + 2 \frac{b^3 a (\sin(dx+c))^3}{d (\cos(dx+c))^2} + 2 \frac{b^2 a^2 (\sin(dx+c))^2}{d \cos(dx+c)} + 2 \frac{b a^3 \sin(dx+c)}{d} + 2 \frac{a^4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^4,x)

[Out] 1/3/d*b^4*sin(d*x+c)^4/cos(d*x+c)^3-1/3/d*b^4*sin(d*x+c)^4/cos(d*x+c)-1/3/d*b^4*cos(d*x+c)*sin(d*x+c)^2-2/3*b^4*cos(d*x+c)/d+2/d*b^3*a*sin(d*x+c)^3/cos(d*x+c)^2+2*a*b^3*sin(d*x+c)/d-2/d*b^3*a*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2*b^2/cos(d*x+c)+4/d*b*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^4*ln(csc(d*x+c)-cot(d*x+c))

Maxima [A] time = 1.10845, size = 188, normalized size = 1.59

$$3ab^3 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1) \right) - 6a^3b(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1))$$

3d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")

[Out]
$$-1/3*(3*a*b^3*(2*\sin(d*x + c)/(\sin(d*x + c)^2 - 1) + \log(\sin(d*x + c) + 1) - \log(\sin(d*x + c) - 1)) - 6*a^3*b*(\log(\sin(d*x + c) + 1) - \log(\sin(d*x + c) - 1)) + 3*a^4*\log(\cot(d*x + c) + \csc(d*x + c)) - 18*a^2*b^2/\cos(d*x + c) + (3*\cos(d*x + c)^2 - 1)*b^4/\cos(d*x + c)^3)/d$$

Fricas [A] time = 2.56534, size = 444, normalized size = 3.76

$$3 a^4 \cos(dx + c)^3 \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) - 3 a^4 \cos(dx + c)^3 \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) - 12 ab^3 \cos(dx + c) \sin(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")

[Out]
$$-1/6*(3*a^4*\cos(d*x + c)^3*\log(1/2*\cos(d*x + c) + 1/2) - 3*a^4*\cos(d*x + c)^3*\log(-1/2*\cos(d*x + c) + 1/2) - 12*a*b^3*\cos(d*x + c)*\sin(d*x + c) - 6*(2*a^3*b - a*b^3)*\cos(d*x + c)^3*\log(\sin(d*x + c) + 1) + 6*(2*a^3*b - a*b^3)*\cos(d*x + c)^3*\log(-\sin(d*x + c) + 1) - 2*b^4 - 6*(6*a^2*b^2 - b^4)*\cos(d*x + c)^2)/(d*\cos(d*x + c)^3)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**4,x)

[Out] Timed out

Giac [A] time = 2.70968, size = 261, normalized size = 2.21

$$3 a^4 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) + 6 (2 a^3 b - ab^3) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 6 (2 a^3 b - ab^3) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) +$$

3d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")

[Out]
$$1/3*(3*a^4*\log(\text{abs}(\tan(1/2*d*x + 1/2*c)))) + 6*(2*a^3*b - a*b^3)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - 6*(2*a^3*b - a*b^3)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) + 4*(3*a*b^3*\tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*\tan(1/2*d*x + 1/2*c)^4 + 18*a^2*b^2*\tan(1/2*d*x + 1/2*c)^2 - 3*b^4*\tan(1/2*d*x + 1/2*c)^2 - 3*a*b^3*\tan(1/2*d*x + 1/2*c) - 9*a^2*b^2 + b^4)/(\tan(1/2*d*x + 1/2*c)^2 - 1)^3/d$$

3.45 $\int \csc^2(c + dx)(a + b \tan(c + dx))^4 dx$

Optimal. Leaf size=83

$$\frac{6a^2b^2 \tan(c + dx)}{d} + \frac{4a^3b \log(\tan(c + dx))}{d} - \frac{a^4 \cot(c + dx)}{d} + \frac{2ab^3 \tan^2(c + dx)}{d} + \frac{b^4 \tan^3(c + dx)}{3d}$$

[Out] $-(a^4 \cot[c + d*x])/d + (4*a^3*b*\log[\tan[c + d*x]])/d + (6*a^2*b^2*\tan[c + d*x])/d + (2*a*b^3*\tan[c + d*x]^2)/d + (b^4*\tan[c + d*x]^3)/(3*d)$

Rubi [A] time = 0.0581838, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 43}

$$\frac{6a^2b^2 \tan(c + dx)}{d} + \frac{4a^3b \log(\tan(c + dx))}{d} - \frac{a^4 \cot(c + dx)}{d} + \frac{2ab^3 \tan^2(c + dx)}{d} + \frac{b^4 \tan^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c + d*x]^2*(a + b*\text{Tan}[c + d*x])^4, x]$

[Out] $-(a^4 \cot[c + d*x])/d + (4*a^3*b*\log[\tan[c + d*x]])/d + (6*a^2*b^2*\tan[c + d*x])/d + (2*a*b^3*\tan[c + d*x]^2)/d + (b^4*\tan[c + d*x]^3)/(3*d)$

Rule 3516

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[b/f, \text{Subst}[\text{Int}[(x^m*(a + x)^n)/(b^2 + x^2)^{(m/2 + 1)}, x], x, b*\text{Tan}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)]^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \csc^2(c + dx)(a + b \tan(c + dx))^4 dx &= \frac{b \text{Subst}\left(\int \frac{(a+x)^4}{x^2} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{b \text{Subst}\left(\int \left(6a^2 + \frac{a^4}{x^2} + \frac{4a^3}{x} + 4ax + x^2\right) dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{a^4 \cot(c + dx)}{d} + \frac{4a^3b \log(\tan(c + dx))}{d} + \frac{6a^2b^2 \tan(c + dx)}{d} + \frac{2ab^3 \tan^2(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 1.16045, size = 162, normalized size = 1.95

$$\frac{\csc(c + dx) \sec^3(c + dx) \left(4(3a^4 + b^4) \cos(2(c + dx)) + (18a^2b^2 + 3a^4 - b^4) \cos(4(c + dx)) + 3(8ab \sin(2(c + dx))\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]

[Out] $-(\text{Csc}[c + d*x] * \text{Sec}[c + d*x]^3 * (4 * (3*a^4 + b^4) * \text{Cos}[2*(c + d*x)] + (3*a^4 + 18*a^2*b^2 - b^4) * \text{Cos}[4*(c + d*x)] + 3*(3*a^4 - 6*a^2*b^2 - b^4 + 8*a*b*(-b^2 + a^2 * \text{Log}[\text{Cos}[c + d*x]] - a^2 * \text{Log}[\text{Sin}[c + d*x]]) * \text{Sin}[2*(c + d*x)] + 4*a^3*b * (\text{Log}[\text{Cos}[c + d*x]] - \text{Log}[\text{Sin}[c + d*x]]) * \text{Sin}[4*(c + d*x)])))/(24*d)$

Maple [A] time = 0.063, size = 90, normalized size = 1.1

$$\frac{b^4 (\sin(dx + c))^3}{3d (\cos(dx + c))^3} + 2 \frac{b^3 a}{d (\cos(dx + c))^2} + 6 \frac{a^2 b^2 \tan(dx + c)}{d} + 4 \frac{ba^3 \ln(\tan(dx + c))}{d} - \frac{a^4 \cot(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x)

[Out] $1/3/d*b^4*\sin(d*x+c)^3/\cos(d*x+c)^3+2/d*b^3*a/\cos(d*x+c)^2+6*a^2*b^2*\tan(d*x+c)/d+4*a^3*b*\ln(\tan(d*x+c))/d-a^4*\cot(d*x+c)/d$

Maxima [A] time = 1.12037, size = 97, normalized size = 1.17

$$\frac{b^4 \tan(dx + c)^3 + 6ab^3 \tan(dx + c)^2 + 12a^3b \log(\tan(dx + c)) + 18a^2b^2 \tan(dx + c) - \frac{3a^4}{\tan(dx + c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")

[Out] $1/3*(b^4*\tan(d*x + c)^3 + 6*a*b^3*\tan(d*x + c)^2 + 12*a^3*b*\log(\tan(d*x + c)) + 18*a^2*b^2*\tan(d*x + c) - 3*a^4/\tan(d*x + c))/d$

Fricas [A] time = 2.04927, size = 389, normalized size = 4.69

$$\frac{6a^3b \cos(dx + c)^3 \log(\cos(dx + c)^2) \sin(dx + c) - 6a^3b \cos(dx + c)^3 \log\left(-\frac{1}{4} \cos(dx + c)^2 + \frac{1}{4}\right) \sin(dx + c) - 6ab^3 \cos(dx + c)^3 \log(\cos(dx + c)^2) \sin(dx + c) - 6ab^3 \cos(dx + c)^3 \log\left(-\frac{1}{4} \cos(dx + c)^2 + \frac{1}{4}\right) \sin(dx + c)}{3d \cos(dx + c)^3 \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")

[Out] $-1/3*(6*a^3*b*\cos(d*x + c)^3*\log(\cos(d*x + c)^2)*\sin(d*x + c) - 6*a^3*b*\cos(d*x + c)^3*\log(-1/4*\cos(d*x + c)^2 + 1/4)*\sin(d*x + c) - 6*a*b^3*\cos(d*x + c)^3*\log(\cos(d*x + c)^2)*\sin(d*x + c) + (3*a^4 + 18*a^2*b^2 - b^4)*\cos(d*x + c)^4 - b^4 - 2*(9*a^2*b^2 - b^4)*\cos(d*x + c)^2)/(d*\cos(d*x + c)^3*\sin(d*x + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**4,x)

[Out] Timed out

Giac [A] time = 2.64793, size = 116, normalized size = 1.4

$$\frac{b^4 \tan(dx+c)^3 + 6ab^3 \tan(dx+c)^2 + 12a^3b \log(|\tan(dx+c)|) + 18a^2b^2 \tan(dx+c) - \frac{3(4a^3b \tan(dx+c) + a^4)}{\tan(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")

[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*a^3*b*log(abs(tan(d*x + c))) + 18*a^2*b^2*tan(d*x + c) - 3*(4*a^3*b*tan(d*x + c) + a^4)/tan(d*x + c))/d

3.46 $\int \csc^3(c + dx)(a + b \tan(c + dx))^4 dx$

Optimal. Leaf size=161

$$\frac{6a^2b^2 \sec(c + dx)}{d} - \frac{6a^2b^2 \tanh^{-1}(\cos(c + dx))}{d} - \frac{4a^3b \csc(c + dx)}{d} + \frac{4a^3b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^4 \tanh^{-1}(\cos(c + dx))}{2d}$$

```
[Out] -(a^4*ArcTanh[Cos[c + d*x]])/(2*d) - (6*a^2*b^2*ArcTanh[Cos[c + d*x]])/d +
(4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (2*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*
a^3*b*Csc[c + d*x])/d - (a^4*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (6*a^2*b^2*
Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Tan[c
+ d*x])/d
```

Rubi [A] time = 0.157433, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3517, 3768, 3770, 2621, 321, 207, 2622, 2606, 30}

$$\frac{6a^2b^2 \sec(c + dx)}{d} - \frac{6a^2b^2 \tanh^{-1}(\cos(c + dx))}{d} - \frac{4a^3b \csc(c + dx)}{d} + \frac{4a^3b \tanh^{-1}(\sin(c + dx))}{d} - \frac{a^4 \tanh^{-1}(\cos(c + dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(a^4*ArcTanh[Cos[c + d*x]])/(2*d) - (6*a^2*b^2*ArcTanh[Cos[c + d*x]])/d +
(4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (2*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*
a^3*b*Csc[c + d*x])/d - (a^4*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (6*a^2*b^2*
Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Tan[c
+ d*x])/d
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n +
1)/2], x], x, a*Csc[e + f*x], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321


```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_.))*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^(n - 1)/2], x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \csc^3(c + dx)(a + b \tan(c + dx))^4 dx &= \int (a^4 \csc^3(c + dx) + 4a^3b \csc^2(c + dx) \sec(c + dx) + 6a^2b^2 \csc(c + dx) \sec^2(c + dx) + 4ab^3 \sec^3(c + dx) + b^4 \tan^4(c + dx)) dx \\ &= a^4 \int \csc^3(c + dx) dx + (4a^3b) \int \csc^2(c + dx) \sec(c + dx) dx + (6a^2b^2) \int \csc(c + dx) \sec^2(c + dx) dx + 4ab^3 \int \sec^3(c + dx) dx + b^4 \int \tan^4(c + dx) dx \\ &= -\frac{a^4 \cot(c + dx) \csc(c + dx)}{2d} + \frac{2ab^3 \sec(c + dx) \tan(c + dx)}{d} + \frac{1}{2}a^4 \int \csc(c + dx) dx + 4ab^3 \int \sec(c + dx) dx + b^4 \int \tan^4(c + dx) dx \\ &= -\frac{a^4 \tanh^{-1}(\cos(c + dx))}{2d} + \frac{2ab^3 \tanh^{-1}(\sin(c + dx))}{d} - \frac{4a^3b \csc(c + dx)}{d} - \frac{4ab^3 \sec(c + dx)}{d} + b^4 \int \tan^4(c + dx) dx \\ &= -\frac{a^4 \tanh^{-1}(\cos(c + dx))}{2d} - \frac{6a^2b^2 \tanh^{-1}(\cos(c + dx))}{d} + \frac{4a^3b \tanh^{-1}(\sin(c + dx))}{d} + b^4 \int \tan^4(c + dx) dx \end{aligned}$$

Mathematica [B] time = 6.19325, size = 1128, normalized size = 7.01

$$\frac{2a^3b \cos^4(c + dx) \tan\left(\frac{1}{2}(c + dx)\right) (a + b \tan(c + dx))^4}{d(a \cos(c + dx) + b \sin(c + dx))^4} + \frac{b^2 (36a^2 + b^2) \cos^4(c + dx) (a + b \tan(c + dx))^4}{6d(a \cos(c + dx) + b \sin(c + dx))^4} - \frac{a^4 \cos^4(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (b^2*(36*a^2 + b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c +
d*x] + b*Sin[c + d*x])^4) - (2*a^3*b*Cos[c + d*x]^4*Cot[(c + d*x)/2]*(a + b
*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a^4*Cos[c + d*
x]^4*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Si
n[c + d*x])^4) + ((-a^4 - 12*a^2*b^2)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2]]*
(a + b*Tan[c + d*x])^4)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a
^3*b + a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a +
b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((a^4 + 12*a^2
*b^2)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(2*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + a*b^3)*Cos[c + d*x]^4*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(a*Cos[c +
d*x] + b*Sin[c + d*x])^4) + (a^4*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*(a + b*
Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 + b
^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*Si
n[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c
+ d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]^
4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-36*a^2*b^2*Sin[(c + d
*x)/2] - b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]
^4*(36*a^2*b^2*Sin[(c + d*x)/2] + b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x]
)^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c +
d*x])^4) - (2*a^3*b*Cos[c + d*x]^4*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^4
)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A] time = 0.075, size = 192, normalized size = 1.2

$$\frac{b^4}{3d(\cos(dx+c))^3} + 2\frac{b^3a \sec(dx+c)\tan(dx+c)}{d} + 2\frac{b^3a \ln(\sec(dx+c) + \tan(dx+c))}{d} + 6\frac{a^2b^2}{d \cos(dx+c)} + 6\frac{a^2b^2 \ln(\sec(dx+c) + \tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x)
```

```
[Out] 1/3/d*b^4/cos(d*x+c)^3+2*a*b^3*sec(d*x+c)*tan(d*x+c)/d+2/d*b^3*a*ln(sec(d*x
+c)+tan(d*x+c))+6/d*a^2*b^2/cos(d*x+c)+6/d*a^2*b^2*ln(csc(d*x+c)-cot(d*x+c)
)-4/d*b*a^3/sin(d*x+c)+4/d*b*a^3*ln(sec(d*x+c)+tan(d*x+c))-1/2*a^4*cot(d*x+
c)*csc(d*x+c)/d+1/2/d*a^4*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.17389, size = 254, normalized size = 1.58

$$3a^4\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) - 12ab^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 36a^2b^2\left(\frac{2}{\cos(dx+c)} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) - 12a^3b\left(\frac{2}{\sin(dx+c)} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^4*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) +
log(cos(d*x + c) - 1)) - 12*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 36*a^2*b^2*(2/cos(d*x + c) -
```

$$\log(\cos(dx + c) + 1) + \log(\cos(dx + c) - 1) - 24a^3b(2/\sin(dx + c) - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1)) + 4b^4/\cos(dx + c)^3/d$$

Fricas [B] time = 3.03541, size = 818, normalized size = 5.08

$$6(a^4 + 12a^2b^2)\cos(dx + c)^4 - 4b^4 - 4(18a^2b^2 - b^4)\cos(dx + c)^2 - 3((a^4 + 12a^2b^2)\cos(dx + c)^5 - (a^4 + 12a^2b^2)\cos(dx + c)^3)/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)^3*(a+b*tan(dx+c))^4,x, algorithm="fricas")

[Out] 1/12*(6*(a^4 + 12*a^2*b^2)*cos(dx + c)^4 - 4*b^4 - 4*(18*a^2*b^2 - b^4)*cos(dx + c)^2 - 3*((a^4 + 12*a^2*b^2)*cos(dx + c)^5 - (a^4 + 12*a^2*b^2)*cos(dx + c)^3)*log(1/2*cos(dx + c) + 1/2) + 3*((a^4 + 12*a^2*b^2)*cos(dx + c)^5 - (a^4 + 12*a^2*b^2)*cos(dx + c)^3)*log(-1/2*cos(dx + c) + 1/2) + 12*((2*a^3*b + a*b^3)*cos(dx + c)^5 - (2*a^3*b + a*b^3)*cos(dx + c)^3)*log(sin(dx + c) + 1) - 12*((2*a^3*b + a*b^3)*cos(dx + c)^5 - (2*a^3*b + a*b^3)*cos(dx + c)^3)*log(-sin(dx + c) + 1) - 24*(a*b^3*cos(dx + c) - (2*a^3*b + a*b^3)*cos(dx + c)^3)*sin(dx + c)/(d*cos(dx + c)^5 - d*cos(dx + c)^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)**3*(a+b*tan(dx+c))**4,x)

[Out] Timed out

Giac [A] time = 2.76962, size = 405, normalized size = 2.52

$$3a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 48a^3b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 48(2a^3b + ab^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 48(2a^3b + ab^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)^3*(a+b*tan(dx+c))^4,x, algorithm="giac")

[Out] 1/24*(3*a^4*tan(1/2*d*x + 1/2*c)^2 - 48*a^3*b*tan(1/2*d*x + 1/2*c) + 48*(2*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*(2*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 12*(a^4 + 12*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))) - 3*(6*a^4*tan(1/2*d*x + 1/2*c)^2 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a^3*b*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1/2*c)^2 + 16*(6*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 3*b^4*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b^3*tan(1/2*d*x + 1/2*c) - 18*a^2*b^2 - b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3/d

3.47 $\int \csc^4(c + dx)(a + b \tan(c + dx))^4 dx$

Optimal. Leaf size=137

$$\frac{b^2(6a^2 + b^2)\tan(c + dx)}{d} - \frac{a^2(a^2 + 6b^2)\cot(c + dx)}{d} + \frac{4ab(a^2 + b^2)\log(\tan(c + dx))}{d} - \frac{2a^3b\cot^2(c + dx)}{d} - \frac{a^4\cot^3(c + dx)}{3d}$$

[Out] $-\frac{(a^2(a^2 + 6b^2)\cot[c + dx])}{d} - \frac{(2a^3b\cot[c + dx]^2)}{d} - \frac{(a^4\cot[c + dx]^3)}{(3d)} + \frac{(4ab(a^2 + b^2)\log[\tan[c + dx]])}{d} + \frac{(b^2(6a^2 + b^2)\tan[c + dx])}{d} + \frac{(2ab^3\tan[c + dx]^2)}{d} + \frac{(b^4\tan[c + dx]^3)}{(3d)}$

Rubi [A] time = 0.104227, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 894}

$$\frac{b^2(6a^2 + b^2)\tan(c + dx)}{d} - \frac{a^2(a^2 + 6b^2)\cot(c + dx)}{d} + \frac{4ab(a^2 + b^2)\log(\tan(c + dx))}{d} - \frac{2a^3b\cot^2(c + dx)}{d} - \frac{a^4\cot^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c + dx]^4(a + b\tan[c + dx])^4, x]$

[Out] $-\frac{(a^2(a^2 + 6b^2)\cot[c + dx])}{d} - \frac{(2a^3b\cot[c + dx]^2)}{d} - \frac{(a^4\cot[c + dx]^3)}{(3d)} + \frac{(4ab(a^2 + b^2)\log[\tan[c + dx]])}{d} + \frac{(b^2(6a^2 + b^2)\tan[c + dx])}{d} + \frac{(2ab^3\tan[c + dx]^2)}{d} + \frac{(b^4\tan[c + dx]^3)}{(3d)}$

Rule 3516

$\text{Int}[\sin[(e_.) + (f_.)(x_.)]^{(m_.)}((a_.) + (b_.)\tan[(e_.) + (f_.)(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[b/f, \text{Subst}[\text{Int}[(x^m(a + x)^n)/(b^2 + x^2)^{(m/2 + 1)}], x], x, b\tan[e + f*x]], x] /;$ FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 894

$\text{Int}[(d_. + (e_.)(x_.))^{(m_.)}((f_.) + (g_.)(x_.))^{(n_.)}((a_.) + (c_.)(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m(f + g*x)^n(a + c*x^2)^p, x], x] /;$ FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \csc^4(c + dx)(a + b \tan(c + dx))^4 dx &= \frac{b \text{Subst}\left(\int \frac{(a+x)^4(b^2+x^2)}{x^4} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{b \text{Subst}\left(\int \left(6a^2\left(1 + \frac{b^2}{6a^2}\right) + \frac{a^4b^2}{x^4} + \frac{4a^3b^2}{x^3} + \frac{a^4+6a^2b^2}{x^2} + \frac{4a(a^2+b^2)}{x} + 4ax + x^2\right) dx, x\right)}{d} \\ &= -\frac{a^2(a^2 + 6b^2)\cot(c + dx)}{d} - \frac{2a^3b\cot^2(c + dx)}{d} - \frac{a^4\cot^3(c + dx)}{3d} + \frac{4ab(a^2 + b^2)\log(\tan(c + dx))}{d} + \frac{b^2(6a^2 + b^2)\tan(c + dx)}{d} + \frac{2ab^3\tan^2(c + dx)}{d} + \frac{b^4\tan^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 3.67706, size = 188, normalized size = 1.37

$$\frac{\sin(c + dx) \tan^3(c + dx) (a \cot(c + dx) + b)^4 \left(-2b^2 (9a^2 + b^2) \sin(c + dx) \cos^2(c + dx) + \cos(c + dx) (6a^3b \cot^2(c + dx) + \dots) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^4,x]

[Out] $-\left((b + a \cot[c + d*x])^4 \sin[c + d*x] (\cos[c + d*x] (-6a^3b^3 + 6a^3b^2 \cot[c + d*x]^2 + a^4 \cot[c + d*x]^3) + 2a \cos[c + d*x]^3 (a(a^2 + 9b^2) \cot[c + d*x] + 6b(a^2 + b^2) (\log[\cos[c + d*x]] - \log[\sin[c + d*x]])) - b^4 \sin[c + d*x] - 2b^2(9a^2 + b^2) \cos[c + d*x]^2 \sin[c + d*x] \tan[c + d*x]^3) / (3d(a \cos[c + d*x] + b \sin[c + d*x])^4) \right)$

Maple [A] time = 0.077, size = 184, normalized size = 1.3

$$\frac{2b^4 \tan(dx + c)}{3d} + \frac{b^4 \tan(dx + c) (\sec(dx + c))^2}{3d} + 2 \frac{b^3 a}{d (\cos(dx + c))^2} + 4 \frac{b^3 a \ln(\tan(dx + c))}{d} + 6 \frac{a^2 b^2}{d \sin(dx + c) \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x)

[Out] $2/3/d*b^4*\tan(d*x+c)+1/3/d*b^4*\tan(d*x+c)*\sec(d*x+c)^2+2/d*b^3*a/\cos(d*x+c)^2+4/d*b^3*a*\ln(\tan(d*x+c))+6/d*a^2*b^2/\sin(d*x+c)/\cos(d*x+c)-12/d*a^2*b^2*\cot(d*x+c)-2/d*b*a^3/\sin(d*x+c)^2+4*a^3*b*\ln(\tan(d*x+c))/d-2/3*a^4*\cot(d*x+c)/d-1/3/d*a^4*\cot(d*x+c)*\csc(d*x+c)^2$

Maxima [A] time = 1.07015, size = 162, normalized size = 1.18

$$\frac{b^4 \tan(dx + c)^3 + 6ab^3 \tan(dx + c)^2 + 12(a^3b + ab^3) \log(\tan(dx + c)) + 3(6a^2b^2 + b^4) \tan(dx + c) - \frac{6a^3b \tan(dx + c)}{3d}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="maxima")

[Out] $1/3*(b^4*\tan(d*x + c)^3 + 6*a*b^3*\tan(d*x + c)^2 + 12*(a^3*b + a*b^3)*\log(\tan(d*x + c)) + 3*(6*a^2*b^2 + b^4)*\tan(d*x + c) - (6*a^3*b*\tan(d*x + c) + a^4 + 3*(a^4 + 6*a^2*b^2)*\tan(d*x + c)^2)/\tan(d*x + c)^3)/d$

Fricas [A] time = 2.19935, size = 633, normalized size = 4.62

$$2(a^4 + 18a^2b^2 + b^4) \cos(dx + c)^6 + 18a^2b^2 \cos(dx + c)^2 - 3(a^4 + 18a^2b^2 + b^4) \cos(dx + c)^4 + b^4 + 6((a^3b + ab^3) \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="fricas")

```
[Out] -1/3*(2*(a^4 + 18*a^2*b^2 + b^4)*cos(d*x + c)^6 + 18*a^2*b^2*cos(d*x + c)^2
- 3*(a^4 + 18*a^2*b^2 + b^4)*cos(d*x + c)^4 + b^4 + 6*((a^3*b + a*b^3)*cos
(d*x + c)^5 - (a^3*b + a*b^3)*cos(d*x + c)^3)*log(cos(d*x + c)^2)*sin(d*x +
c) - 6*((a^3*b + a*b^3)*cos(d*x + c)^5 - (a^3*b + a*b^3)*cos(d*x + c)^3)*l
og(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 6*(a*b^3*cos(d*x + c) - (a^3*b
+ a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/((d*cos(d*x + c)^5 - d*cos(d*x + c)
^3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**4,x)
```

[Out] Timed out

Giac [A] time = 2.64509, size = 217, normalized size = 1.58

$$\frac{b^4 \tan(dx + c)^3 + 6ab^3 \tan(dx + c)^2 + 18a^2b^2 \tan(dx + c) + 3b^4 \tan(dx + c) + 12(a^3b + ab^3) \log(|\tan(dx + c)|) - \frac{22a^3b^4}{3d}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 18*a^2*b^2*tan(d*x + c)
+ 3*b^4*tan(d*x + c) + 12*(a^3*b + a*b^3)*log(abs(tan(d*x + c)))) - (22*a^3*
b*tan(d*x + c)^3 + 22*a*b^3*tan(d*x + c)^3 + 3*a^4*tan(d*x + c)^2 + 18*a^2*
b^2*tan(d*x + c)^2 + 6*a^3*b*tan(d*x + c) + a^4)/tan(d*x + c)^3/d
```

3.48 $\int \csc^5(c + dx)(a + b \tan(c + dx))^4 dx$

Optimal. Leaf size=274

$$\frac{9a^2b^2 \sec(c + dx)}{d} - \frac{9a^2b^2 \tanh^{-1}(\cos(c + dx))}{d} - \frac{3a^2b^2 \csc^2(c + dx) \sec(c + dx)}{d} - \frac{4a^3b \csc^3(c + dx)}{3d} - \frac{4a^3b \csc(c + dx)}{d}$$

```
[Out] (-3*a^4*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a^2*b^2*ArcTanh[Cos[c + d*x]])/d
- (b^4*ArcTanh[Cos[c + d*x]])/d + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (6*a*
b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (6*a*b^3*Csc[c +
d*x])/d - (3*a^4*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (4*a^3*b*Csc[c + d*x]^3
)/(3*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) + (9*a^2*b^2*Sec[c + d*x]
)/d + (b^4*Sec[c + d*x])/d - (3*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/d + (2
*a*b^3*Csc[c + d*x]*Sec[c + d*x]^2)/d + (b^4*Sec[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.240883, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3517, 3768, 3770, 2621, 302, 207, 2622, 288, 321}

$$\frac{9a^2b^2 \sec(c + dx)}{d} - \frac{9a^2b^2 \tanh^{-1}(\cos(c + dx))}{d} - \frac{3a^2b^2 \csc^2(c + dx) \sec(c + dx)}{d} - \frac{4a^3b \csc^3(c + dx)}{3d} - \frac{4a^3b \csc(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-3*a^4*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a^2*b^2*ArcTanh[Cos[c + d*x]])/d
- (b^4*ArcTanh[Cos[c + d*x]])/d + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (6*a*
b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (6*a*b^3*Csc[c +
d*x])/d - (3*a^4*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (4*a^3*b*Csc[c + d*x]^3
)/(3*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) + (9*a^2*b^2*Sec[c + d*x]
)/d + (b^4*Sec[c + d*x])/d - (3*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/d + (2
*a*b^3*Csc[c + d*x]*Sec[c + d*x]^2)/d + (b^4*Sec[c + d*x]^3)/(3*d)
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] := Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] := -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n +
1)/2], x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
```

+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 302

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2622

Int[csc[(e_) + (f_)*(x_)]^(n_)*((a_)*sec[(e_) + (f_)*(x_)]^(m_)), x_Symbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 288

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned}
 \int \csc^5(c + dx)(a + b \tan(c + dx))^4 dx &= \int (a^4 \csc^5(c + dx) + 4a^3b \csc^4(c + dx) \sec(c + dx) + 6a^2b^2 \csc^3(c + dx) \sec^2(c + dx) + 4a^2b^2 \csc^2(c + dx) \sec^3(c + dx) + 4ab^3 \csc(c + dx) \sec^4(c + dx) + b^4 \sec^5(c + dx)) dx \\
 &= a^4 \int \csc^5(c + dx) dx + (4a^3b) \int \csc^4(c + dx) \sec(c + dx) dx + (6a^2b^2) \int \csc^3(c + dx) \sec^2(c + dx) dx + (4a^2b^2) \int \csc^2(c + dx) \sec^3(c + dx) dx + (4ab^3) \int \csc(c + dx) \sec^4(c + dx) dx + b^4 \int \sec^5(c + dx) dx \\
 &= -\frac{a^4 \cot(c + dx) \csc^3(c + dx)}{4d} + \frac{1}{4} (3a^4) \int \csc^3(c + dx) dx - \frac{(4a^3b) \text{Subst}\left(\int \frac{1}{-1 + x^2/a^2} dx, x, a \sec(c + dx)\right)}{4d} \\
 &= -\frac{3a^4 \cot(c + dx) \csc(c + dx)}{8d} - \frac{a^4 \cot(c + dx) \csc^3(c + dx)}{4d} - \frac{3a^2b^2 \csc^2(c + dx) \sec(c + dx)}{d} \\
 &= -\frac{3a^4 \tanh^{-1}(\cos(c + dx))}{8d} - \frac{4a^3b \csc(c + dx)}{d} - \frac{6ab^3 \csc(c + dx)}{d} - \frac{3a^4 \cot(c + dx) \csc^3(c + dx)}{4d} \\
 &= -\frac{3a^4 \tanh^{-1}(\cos(c + dx))}{8d} - \frac{9a^2b^2 \tanh^{-1}(\cos(c + dx))}{d} - \frac{b^4 \tanh^{-1}(\cos(c + dx))}{d}
 \end{aligned}$$

Mathematica [B] time = 6.2872, size = 1491, normalized size = 5.44

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]

[Out] $(b^2(36a^2 + 7b^2)\cos[c + dx]^4(a + b\tan[c + dx])^4)/(6d(a\cos[c + dx] + b\sin[c + dx])^4) + ((-7a^3b\cos[(c + dx)/2] - 6ab^3\cos[(c + dx)/2])\cos[c + dx]^4\csc[(c + dx)/2](a + b\tan[c + dx])^4)/(3d(a\cos[c + dx] + b\sin[c + dx])^4) - (3(a^4 + 8a^2b^2)\cos[c + dx]^4\csc[(c + dx)/2]^2(a + b\tan[c + dx])^4)/(32d(a\cos[c + dx] + b\sin[c + dx])^4) - (a^3b\cos[c + dx]^4\cot[(c + dx)/2]\csc[(c + dx)/2]^2(a + b\tan[c + dx])^4)/(6d(a\cos[c + dx] + b\sin[c + dx])^4) - (a^4\cos[c + dx]^4\csc[(c + dx)/2]^4(a + b\tan[c + dx])^4)/(64d(a\cos[c + dx] + b\sin[c + dx])^4) + ((-3a^4 - 72a^2b^2 - 8b^4)\cos[c + dx]^4\log[\cos[(c + dx)/2]](a + b\tan[c + dx])^4)/(8d(a\cos[c + dx] + b\sin[c + dx])^4) - (2(2a^3b + 3ab^3)\cos[c + dx]^4\log[\cos[(c + dx)/2] - \sin[(c + dx)/2]](a + b\tan[c + dx])^4)/(d(a\cos[c + dx] + b\sin[c + dx])^4) + ((3a^4 + 72a^2b^2 + 8b^4)\cos[c + dx]^4\log[\sin[(c + dx)/2]](a + b\tan[c + dx])^4)/(8d(a\cos[c + dx] + b\sin[c + dx])^4) + (2(2a^3b + 3ab^3)\cos[c + dx]^4\log[\cos[(c + dx)/2] + \sin[(c + dx)/2]](a + b\tan[c + dx])^4)/(d(a\cos[c + dx] + b\sin[c + dx])^4) + (3(a^4 + 8a^2b^2)\cos[c + dx]^4\sec[(c + dx)/2]^2(a + b\tan[c + dx])^4)/(32d(a\cos[c + dx] + b\sin[c + dx])^4) + (a^4\cos[c + dx]^4\sec[(c + dx)/2]^4(a + b\tan[c + dx])^4)/(64d(a\cos[c + dx] + b\sin[c + dx])^4) + ((12ab^3 + b^4)\cos[c + dx]^4(a + b\tan[c + dx])^4)/(12d(\cos[(c + dx)/2] - \sin[(c + dx)/2])^2(a\cos[c + dx] + b\sin[c + dx])^4) + (b^4\cos[c + dx]^4\sin[(c + dx)/2](a + b\tan[c + dx])^4)/(6d(\cos[(c + dx)/2] - \sin[(c + dx)/2])^3(a\cos[c + dx] + b\sin[c + dx])^4) - (b^4\cos[c + dx]^4\sin[(c + dx)/2](a + b\tan[c + dx])^4)/(6d(\cos[(c + dx)/2] + \sin[(c + dx)/2])^3(a\cos[c + dx] + b\sin[c + dx])^4) + ((-12ab^3 + b^4)\cos[c + dx]^4(a + b\tan[c + dx])^4)/(12d(\cos[(c + dx)/2] + \sin[(c + dx)/2])^2(a\cos[c + dx] + b\sin[c + dx])^4) + (\cos[c + dx]^4\sec[(c + dx)/2](-7a^3b\sin[(c + dx)/2] - 6ab^3\sin[(c + dx)/2])(a + b\tan[c + dx])^4)/(3d(a\cos[c + dx] + b\sin[c + dx])^4) + (\cos[c + dx]^4(-36a^2b^2\sin[(c + dx)/2] - 7b^4\sin[(c + dx)/2])(a + b\tan[c + dx])^4)/(6d(\cos[(c + dx)/2] + \sin[(c + dx)/2])(a\cos[c + dx] + b\sin[c + dx])^4) + (\cos[c + dx]^4(36a^2b^2\sin[(c + dx)/2] + 7b^4\sin[(c + dx)/2])(a + b\tan[c + dx])^4)/(6d(\cos[(c + dx)/2] - \sin[(c + dx)/2])(a\cos[c + dx] + b\sin[c + dx])^4) - (a^3b\cos[c + dx]^4\sec[(c + dx)/2]^2\tan[(c + dx)/2](a + b\tan[c + dx])^4)/(6d(a\cos[c + dx] + b\sin[c + dx])^4)$

Maple [A] time = 0.082, size = 317, normalized size = 1.2

$$\frac{b^4}{3d(\cos(dx+c))^3} + \frac{b^4}{d\cos(dx+c)} + \frac{b^4 \ln(\csc(dx+c) - \cot(dx+c))}{d} + 2 \frac{b^3 a}{d \sin(dx+c)(\cos(dx+c))^2} - 6 \frac{b^3 a}{d \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x)

[Out] $1/3/d*b^4/\cos(d*x+c)^3+1/d*b^4/\cos(d*x+c)+1/d*b^4*\ln(\csc(d*x+c)-\cot(d*x+c))+2/d*b^3*a/\sin(d*x+c)/\cos(d*x+c)^2-6/d*b^3*a/\sin(d*x+c)+6/d*b^3*a*\ln(\sec(d*x+c)+\tan(d*x+c))-3/d*a^2*b^2/\sin(d*x+c)^2/\cos(d*x+c)+9/d*a^2*b^2/\cos(d*x+c)+9/d*a^2*b^2*\ln(\csc(d*x+c)-\cot(d*x+c))-4/3/d*b*a^3/\sin(d*x+c)^3-4/d*b*a^3/\sin(d*x+c)+4/d*b*a^3*\ln(\sec(d*x+c)+\tan(d*x+c))-1/4*a^4*\cot(d*x+c)*\csc(d*x+c)^3/d-3/8*a^4*\cot(d*x+c)*\csc(d*x+c)/d+3/8/d*a^4*\ln(\csc(d*x+c)-\cot(d*x+c))$

Maxima [A] time = 1.17311, size = 410, normalized size = 1.5

$$3a^4 \left(\frac{2(3\cos(dx+c)^3 - 5\cos(dx+c))}{\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1} - 3\log(\cos(dx+c)+1) + 3\log(\cos(dx+c)-1) \right) + 72a^2b^2 \left(\frac{2(3\cos(dx+c)^2 - 2)}{\cos(dx+c)^3 - \cos(dx+c)} - 3\log(\cos(dx+c)+1) + 3\log(\cos(dx+c)-1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="maxima")

[Out] 1/48*(3*a^4*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 72*a^2*b^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 48*a*b^3*(2*(3*sin(d*x + c)^2 - 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 8*b^4*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 32*a^3*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d

Fricas [B] time = 3.98277, size = 1289, normalized size = 4.7

$$6(3a^4 + 72a^2b^2 + 8b^4)\cos(dx+c)^6 - 10(3a^4 + 72a^2b^2 + 8b^4)\cos(dx+c)^4 + 16b^4 + 16(18a^2b^2 + b^4)\cos(dx+c)^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="fricas")

[Out] 1/48*(6*(3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^6 - 10*(3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 16*b^4 + 16*(18*a^2*b^2 + b^4)*cos(d*x + c)^2 - 3*((3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 - 2*(3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^5 + (3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) + 3*((3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 - 2*(3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^5 + (3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) + 48*((2*a^3*b + 3*a*b^3)*cos(d*x + c)^7 - 2*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + (2*a^3*b + 3*a*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 48*((2*a^3*b + 3*a*b^3)*cos(d*x + c)^7 - 2*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + (2*a^3*b + 3*a*b^3)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) + 32*(3*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + 3*a*b^3*cos(d*x + c) - 4*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^7 - 2*d*cos(d*x + c)^5 + d*cos(d*x + c)^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**4,x)

[Out] Timed out

Giac [A] time = 2.68359, size = 647, normalized size = 2.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="giac")

[Out] $\frac{1}{192}(3a^4 \tan(\frac{1}{2}dx + \frac{1}{2}c)^4 - 32a^3b \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 + 24a^4 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 144a^2b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 - 480a^3b \tan(\frac{1}{2}dx + \frac{1}{2}c) - 384ab^3 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 384(2a^3b + 3ab^3) \log(\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1) - 384(2a^3b + 3ab^3) \log(\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1) + 24(3a^4 + 72a^2b^2 + 8b^4) \log(\tan(\frac{1}{2}dx + \frac{1}{2}c))) + 256(3ab^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^5 - 9a^2b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^4 - 3b^4 \tan(\frac{1}{2}dx + \frac{1}{2}c)^4 + 18a^2b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 3b^4 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 - 3ab^3 \tan(\frac{1}{2}dx + \frac{1}{2}c) - 9a^2b^2 - 2b^4) / (\tan(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^3 - (150a^4 \tan(\frac{1}{2}dx + \frac{1}{2}c)^4 + 3600a^2b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^4 + 400b^4 \tan(\frac{1}{2}dx + \frac{1}{2}c)^4 + 480a^3b \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 + 384ab^3 \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 + 24a^4 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 144a^2b^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 32a^3b \tan(\frac{1}{2}dx + \frac{1}{2}c) + 3a^4) / \tan(\frac{1}{2}dx + \frac{1}{2}c)^4) / d$

3.49 $\int \csc^6(c + dx)(a + b \tan(c + dx))^4 dx$

Optimal. Leaf size=194

$$\frac{2b^2(3a^2 + b^2)\tan(c + dx)}{d} - \frac{2a^2(a^2 + 3b^2)\cot^3(c + dx)}{3d} - \frac{2ab(2a^2 + b^2)\cot^2(c + dx)}{d} - \frac{(12a^2b^2 + a^4 + b^4)\cot(c + dx)}{d}$$

```
[Out] -(((a^4 + 12*a^2*b^2 + b^4)*Cot[c + d*x])/d) - (2*a*b*(2*a^2 + b^2)*Cot[c + d*x]^2)/d - (2*a^2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (a^3*b*Cot[c + d*x]^4)/d - (a^4*Cot[c + d*x]^5)/(5*d) + (4*a*b*(a^2 + 2*b^2)*Log[Tan[c + d*x]])/d + (2*b^2*(3*a^2 + b^2)*Tan[c + d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d + (b^4*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.157077, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 948}

$$\frac{2b^2(3a^2 + b^2)\tan(c + dx)}{d} - \frac{2a^2(a^2 + 3b^2)\cot^3(c + dx)}{3d} - \frac{2ab(2a^2 + b^2)\cot^2(c + dx)}{d} - \frac{(12a^2b^2 + a^4 + b^4)\cot(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^4 + 12*a^2*b^2 + b^4)*Cot[c + d*x])/d) - (2*a*b*(2*a^2 + b^2)*Cot[c + d*x]^2)/d - (2*a^2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (a^3*b*Cot[c + d*x]^4)/d - (a^4*Cot[c + d*x]^5)/(5*d) + (4*a*b*(a^2 + 2*b^2)*Log[Tan[c + d*x]])/d + (2*b^2*(3*a^2 + b^2)*Tan[c + d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d + (b^4*Tan[c + d*x]^3)/(3*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned} \int \csc^6(c + dx)(a + b \tan(c + dx))^4 dx &= \frac{b \operatorname{Subst}\left(\int \frac{(a+x)^4(b^2+x^2)^2}{x^6} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{b \operatorname{Subst}\left(\int \left(2(3a^2 + b^2) + \frac{a^4 b^4}{x^6} + \frac{4a^3 b^4}{x^5} + \frac{2a^2 b^2(a^2 + 3b^2)}{x^4} + \frac{4ab^2(2a^2 + b^2)}{x^3} + \frac{a^4 + 12a^2 b^2 + b^4}{x^2}\right) dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{(a^4 + 12a^2 b^2 + b^4)\cot(c + dx)}{d} - \frac{2ab(2a^2 + b^2)\cot^2(c + dx)}{d} - \frac{2a^2(a^2 + 3b^2)}{3d} \end{aligned}$$

Mathematica [A] time = 3.91889, size = 233, normalized size = 1.2

$$(a + b \tan(c + dx))^4 \left(-5b^2 (18a^2 + 5b^2) \sin(c + dx) \cos^3(c + dx) + 2a \cos^2(c + dx) (15b (a^2 + b^2) \cot^2(c + dx) + a (2 \right.$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]

[Out] -((15*a^3*b*Cot[c + d*x]^4 + 3*a^4*Cot[c + d*x]^5 + 2*a*Cos[c + d*x]^2*(-15*b^3 + 15*b*(a^2 + b^2)*Cot[c + d*x]^2 + a*(2*a^2 + 15*b^2)*Cot[c + d*x]^3) + Cos[c + d*x]^4*((8*a^4 + 150*a^2*b^2 + 15*b^4)*Cot[c + d*x] + 60*a*b*(a^2 + 2*b^2)*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) - 5*b^2*(18*a^2 + 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x] - (5*b^4*Sin[2*(c + d*x)]/2)*(a + b*Tan[c + d*x])^4)/(15*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)

Maple [A] time = 0.082, size = 301, normalized size = 1.6

$$\frac{b^4}{3d \sin(dx + c) (\cos(dx + c))^3} + \frac{4b^4}{3d \sin(dx + c) \cos(dx + c)} - \frac{8b^4 \cot(dx + c)}{3d} + 2 \frac{b^3 a}{d (\sin(dx + c))^2 (\cos(dx + c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x)

[Out] 1/3/d*b^4/sin(d*x+c)/cos(d*x+c)^3+4/3/d*b^4/sin(d*x+c)/cos(d*x+c)-8/3/d*b^4*cot(d*x+c)+2/d*b^3*a/sin(d*x+c)^2/cos(d*x+c)^2-4/d*b^3*a/sin(d*x+c)^2+8/d*b^3*a*ln(tan(d*x+c))-2/d*a^2*b^2/sin(d*x+c)^3/cos(d*x+c)+8/d*a^2*b^2/sin(d*x+c)/cos(d*x+c)-16/d*a^2*b^2*cot(d*x+c)-1/d*b*a^3/sin(d*x+c)^4-2/d*b*a^3/sin(d*x+c)^2+4*a^3*b*ln(tan(d*x+c))/d-8/15*a^4*cot(d*x+c)/d-1/5/d*a^4*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^4*cot(d*x+c)*csc(d*x+c)^2

Maxima [A] time = 1.06929, size = 231, normalized size = 1.19

$$5b^4 \tan(dx + c)^3 + 30ab^3 \tan(dx + c)^2 + 60(a^3b + 2ab^3) \log(\tan(dx + c)) + 30(3a^2b^2 + b^4) \tan(dx + c) - \frac{15a^3b \tan(dx + c)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="maxima")

[Out] 1/15*(5*b^4*tan(d*x + c)^3 + 30*a*b^3*tan(d*x + c)^2 + 60*(a^3*b + 2*a*b^3)*log(tan(d*x + c)) + 30*(3*a^2*b^2 + b^4)*tan(d*x + c) - (15*a^3*b*tan(d*x + c) + 15*(a^4 + 12*a^2*b^2 + b^4)*tan(d*x + c)^4 + 3*a^4 + 30*(2*a^3*b + a*b^3)*tan(d*x + c)^3 + 10*(a^4 + 3*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c)^5/d

Fricas [B] time = 2.63441, size = 922, normalized size = 4.75

$$8(a^4 + 30a^2b^2 + 5b^4) \cos(dx + c)^8 - 20(a^4 + 30a^2b^2 + 5b^4) \cos(dx + c)^6 + 15(a^4 + 30a^2b^2 + 5b^4) \cos(dx + c)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^4 + 30*a^2*b^2 + 5*b^4)*cos(d*x + c)^8 - 20*(a^4 + 30*a^2*b^2 +
5*b^4)*cos(d*x + c)^6 + 15*(a^4 + 30*a^2*b^2 + 5*b^4)*cos(d*x + c)^4 - 5*b
^4 - 10*(9*a^2*b^2 + b^4)*cos(d*x + c)^2 + 30*((a^3*b + 2*a*b^3)*cos(d*x +
c)^7 - 2*(a^3*b + 2*a*b^3)*cos(d*x + c)^5 + (a^3*b + 2*a*b^3)*cos(d*x + c)^
3)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*((a^3*b + 2*a*b^3)*cos(d*x + c)^7
- 2*(a^3*b + 2*a*b^3)*cos(d*x + c)^5 + (a^3*b + 2*a*b^3)*cos(d*x + c)^3)*lo
g(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 15*(2*(a^3*b + 2*a*b^3)*cos(d*x
+ c)^5 + 2*a*b^3*cos(d*x + c) - 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*x
+ c))/((d*cos(d*x + c)^7 - 2*d*cos(d*x + c)^5 + d*cos(d*x + c)^3)*sin(d*x
+ c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.65297, size = 317, normalized size = 1.63

$$5b^4 \tan(dx + c)^3 + 30ab^3 \tan(dx + c)^2 + 90a^2b^2 \tan(dx + c) + 30b^4 \tan(dx + c) + 60(a^3b + 2ab^3) \log(|\tan(dx + c)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/15*(5*b^4*tan(d*x + c)^3 + 30*a*b^3*tan(d*x + c)^2 + 90*a^2*b^2*tan(d*x +
c) + 30*b^4*tan(d*x + c) + 60*(a^3*b + 2*a*b^3)*log(abs(tan(d*x + c)))) - (
137*a^3*b*tan(d*x + c)^5 + 274*a*b^3*tan(d*x + c)^5 + 15*a^4*tan(d*x + c)^4
+ 180*a^2*b^2*tan(d*x + c)^4 + 15*b^4*tan(d*x + c)^4 + 60*a^3*b*tan(d*x +
c)^3 + 30*a*b^3*tan(d*x + c)^3 + 10*a^4*tan(d*x + c)^2 + 30*a^2*b^2*tan(d*x
+ c)^2 + 15*a^3*b*tan(d*x + c) + 3*a^4)/tan(d*x + c)^5/d
```

3.50 $\int \csc^7(c + dx)(a + b \tan(c + dx))^4 dx$

Optimal. Leaf size=402

$$\frac{45a^2b^2 \sec(c + dx)}{4d} - \frac{45a^2b^2 \tanh^{-1}(\cos(c + dx))}{4d} - \frac{3a^2b^2 \csc^4(c + dx) \sec(c + dx)}{2d} - \frac{15a^2b^2 \csc^2(c + dx) \sec(c + dx)}{4d}$$

```
[Out] (-5*a^4*ArcTanh[Cos[c + d*x]])/(16*d) - (45*a^2*b^2*ArcTanh[Cos[c + d*x]])/(4*d) - (5*b^4*ArcTanh[Cos[c + d*x]])/(2*d) + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (10*a*b^3*Csc[c + d*x])/d - (5*a^4*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (4*a^3*b*Csc[c + d*x]^3)/(3*d) - (10*a*b^3*Csc[c + d*x]^3)/(3*d) - (5*a^4*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (4*a^3*b*Csc[c + d*x]^5)/(5*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^5)/(6*d) + (45*a^2*b^2*Sec[c + d*x])/(4*d) + (5*b^4*Sec[c + d*x])/(2*d) - (15*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/(4*d) - (3*a^2*b^2*Csc[c + d*x]^4*Sec[c + d*x])/(2*d) + (2*a*b^3*Csc[c + d*x]^3*Sec[c + d*x]^2)/d + (5*b^4*Sec[c + d*x]^3)/(6*d) - (b^4*Csc[c + d*x]^2*Sec[c + d*x]^3)/(2*d)
```

Rubi [A] time = 0.305542, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3517, 3768, 3770, 2621, 302, 207, 2622, 288, 321}

$$\frac{45a^2b^2 \sec(c + dx)}{4d} - \frac{45a^2b^2 \tanh^{-1}(\cos(c + dx))}{4d} - \frac{3a^2b^2 \csc^4(c + dx) \sec(c + dx)}{2d} - \frac{15a^2b^2 \csc^2(c + dx) \sec(c + dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-5*a^4*ArcTanh[Cos[c + d*x]])/(16*d) - (45*a^2*b^2*ArcTanh[Cos[c + d*x]])/(4*d) - (5*b^4*ArcTanh[Cos[c + d*x]])/(2*d) + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (10*a*b^3*Csc[c + d*x])/d - (5*a^4*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (4*a^3*b*Csc[c + d*x]^3)/(3*d) - (10*a*b^3*Csc[c + d*x]^3)/(3*d) - (5*a^4*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (4*a^3*b*Csc[c + d*x]^5)/(5*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^5)/(6*d) + (45*a^2*b^2*Sec[c + d*x])/(4*d) + (5*b^4*Sec[c + d*x])/(2*d) - (15*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/(4*d) - (3*a^2*b^2*Csc[c + d*x]^4*Sec[c + d*x])/(2*d) + (2*a*b^3*Csc[c + d*x]^3*Sec[c + d*x]^2)/d + (5*b^4*Sec[c + d*x]^3)/(6*d) - (b^4*Csc[c + d*x]^2*Sec[c + d*x]^3)/(2*d)
```

Rule 3517

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Csc[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)*(a_.)]^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_)), x_Symbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^(n - 1)*(c*x)^(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \csc^7(c+dx)(a+b \tan(c+dx))^4 dx &= \int (a^4 \csc^7(c+dx) + 4a^3b \csc^6(c+dx) \sec(c+dx) + 6a^2b^2 \csc^5(c+dx) \sec^2(c+dx) \\
&+ 4a^2b^3 \csc^4(c+dx) \sec^3(c+dx) + 4a^2b^4 \csc^3(c+dx) \sec^4(c+dx) + 4a^2b^4 \csc^2(c+dx) \sec^5(c+dx) \\
&+ 4a^2b^4 \csc(c+dx) \sec^6(c+dx) + 4a^2b^4 \sec^7(c+dx)) dx \\
&= a^4 \int \csc^7(c+dx) dx + (4a^3b) \int \csc^6(c+dx) \sec(c+dx) dx + (6a^2b^2) \int \csc^5(c+dx) \sec^2(c+dx) dx \\
&+ (4a^2b^3) \int \csc^4(c+dx) \sec^3(c+dx) dx + (4a^2b^4) \int \csc^3(c+dx) \sec^4(c+dx) dx + (4a^2b^4) \int \csc^2(c+dx) \sec^5(c+dx) dx \\
&+ (4a^2b^4) \int \csc(c+dx) \sec^6(c+dx) dx + (4a^2b^4) \int \sec^7(c+dx) dx \\
&= -\frac{a^4 \cot(c+dx) \csc^5(c+dx)}{6d} + \frac{1}{6} (5a^4) \int \csc^5(c+dx) dx - \frac{(4a^3b) \text{Subst}\left(\int \csc^4(u) du, c+dx, x\right)}{6d} \\
&+ \frac{(6a^2b^2) \text{Subst}\left(\int \csc^3(u) \sec^2(u) du, c+dx, x\right)}{6d} + \frac{(4a^2b^3) \text{Subst}\left(\int \csc^2(u) \sec^3(u) du, c+dx, x\right)}{6d} \\
&+ \frac{(4a^2b^4) \text{Subst}\left(\int \csc(u) \sec^4(u) du, c+dx, x\right)}{6d} + \frac{(4a^2b^4) \text{Subst}\left(\int \sec^5(u) du, c+dx, x\right)}{6d} \\
&= -\frac{5a^4 \cot(c+dx) \csc^3(c+dx)}{24d} - \frac{a^4 \cot(c+dx) \csc^5(c+dx)}{6d} - \frac{3a^2b^2 \csc^4(c+dx) \sec^2(c+dx)}{6d} \\
&- \frac{4a^2b^3 \csc^3(c+dx) \sec^3(c+dx)}{6d} - \frac{4a^2b^4 \csc^2(c+dx) \sec^4(c+dx)}{6d} - \frac{4a^2b^4 \csc(c+dx) \sec^5(c+dx)}{6d} - \frac{4a^2b^4 \sec^7(c+dx)}{6d} \\
&= -\frac{4a^3b \csc(c+dx)}{d} - \frac{5a^4 \cot(c+dx) \csc(c+dx)}{16d} - \frac{4a^3b \csc^3(c+dx)}{3d} - \frac{5a^4 \cot(c+dx) \csc^3(c+dx)}{16d} \\
&= -\frac{5a^4 \tanh^{-1}(\cos(c+dx))}{16d} + \frac{4a^3b \tanh^{-1}(\sin(c+dx))}{d} - \frac{4a^3b \csc(c+dx)}{d} - \frac{4a^3b \csc^3(c+dx)}{3d} \\
&= -\frac{5a^4 \tanh^{-1}(\cos(c+dx))}{16d} - \frac{45a^2b^2 \tanh^{-1}(\cos(c+dx))}{4d} - \frac{5b^4 \tanh^{-1}(\cos(c+dx))}{2d}
\end{aligned}$$

Mathematica [A] time = 6.25887, size = 660, normalized size = 1.64

$$\frac{5(36a^2b^2 + a^4 + 8b^4) \cos^4(c+dx) \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) (a+b \tan(c+dx))^4}{16d(a \cos(c+dx) + b \sin(c+dx))^4} - \frac{2(2a^3b + 5ab^3) \cos^4(c+dx) (a+b \tan(c+dx))^4}{d(a \cos(c+dx) + b \sin(c+dx))^4}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^7*(a + b*Tan[c + d*x])^4, x]

[Out] $(-5*(a^4 + 36*a^2*b^2 + 8*b^4)*\text{Cos}[c + d*x]^4*\text{Log}[\text{Cos}[(c + d*x)/2]]*(a + b*\text{Tan}[c + d*x])^4)/(16*d*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^4) - (2*(2*a^3*b + 5*a*b^3)*\text{Cos}[c + d*x]^4*\text{Log}[\text{Cos}[(c + d*x)/2] - \text{Sin}[(c + d*x)/2]]*(a + b*\text{Tan}[c + d*x])^4)/(d*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^4) + (5*(a^4 + 36*a^2*b^2 + 8*b^4)*\text{Cos}[c + d*x]^4*\text{Log}[\text{Sin}[(c + d*x)/2]]*(a + b*\text{Tan}[c + d*x])^4)/(16*d*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^4) + (2*(2*a^3*b + 5*a*b^3)*\text{Cos}[c + d*x]^4*\text{Log}[\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2]]*(a + b*\text{Tan}[c + d*x])^4)/(d*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^4) + (\text{Cot}[c + d*x]*\text{Csc}[c + d*x]^5*(-2545*a^4 + 540*a^2*b^2 + 5240*b^4 - 2760*a^4*\text{Cos}[2*(c + d*x)] - 7200*a^2*b^2*\text{Cos}[2*(c + d*x)] - 6720*b^4*\text{Cos}[2*(c + d*x)] + 60*a^4*\text{Cos}[4*(c + d*x)] + 2160*a^2*b^2*\text{Cos}[4*(c + d*x)] + 480*b^4*\text{Cos}[4*(c + d*x)] + 200*a^4*\text{Cos}[6*(c + d*x)] + 7200*a^2*b^2*\text{Cos}[6*(c + d*x)] + 1600*b^4*\text{Cos}[6*(c + d*x)] - 75*a^4*\text{Cos}[8*(c + d*x)] - 2700*a^2*b^2*\text{Cos}[8*(c + d*x)] - 600*b^4*\text{Cos}[8*(c + d*x)] - 15744*a^3*b*\text{Sin}[2*(c + d*x)] - 8640*a*b^3*\text{Sin}[2*(c + d*x)] - 1152*a^3*b*\text{Sin}[4*(c + d*x)] - 2880*a*b^3*\text{Sin}[4*(c + d*x)] + 3200*a^3*b*\text{Sin}[6*(c + d*x)] + 8000*a*b^3*\text{Sin}[6*(c + d*x)] - 960*a^3*b*\text{Sin}[8*(c + d*x)] - 2400*a*b^3*\text{Sin}[8*(c + d*x)])*(a + b*\text{Tan}[c + d*x])^4)/(30720*d*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^4)$

Maple [A] time = 0.127, size = 442, normalized size = 1.1

$$\frac{b^4}{3d(\sin(dx+c))^2(\cos(dx+c))^3} - \frac{5b^4}{6d(\sin(dx+c))^2\cos(dx+c)} + \frac{5b^4}{2d\cos(dx+c)} + \frac{5b^4 \ln(\csc(dx+c) - \cot(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x)

[Out] $\frac{1}{3} \frac{b^4}{\sin(d*x+c)^2 \cos(d*x+c)^3} - \frac{5}{6} \frac{b^4}{\sin(d*x+c)^2 \cos(d*x+c)} + \frac{5}{2} \frac{b^4}{\cos(d*x+c)} + \frac{5}{2} \frac{b^4 \ln(\csc(d*x+c) - \cot(d*x+c))}{\cos(d*x+c)} - \frac{4}{3} \frac{b^3 a}{\sin(d*x+c)^3 \cos(d*x+c)^2} + \frac{10}{3} \frac{b^3 a}{\sin(d*x+c) \cos(d*x+c)^2} - \frac{10}{d} \frac{b^3 a}{\sin(d*x+c)} + \frac{10}{d} \frac{b^3 a \ln(\sec(d*x+c) + \tan(d*x+c))}{\sin(d*x+c)^4 \cos(d*x+c)} - \frac{15}{4} \frac{d a^2 b^2}{\sin(d*x+c)^2 \cos(d*x+c)} + \frac{45}{4} \frac{d a^2 b^2}{\cos(d*x+c)} + \frac{45}{4} \frac{d a^2 b^2 \ln(\csc(d*x+c) - \cot(d*x+c))}{\sin(d*x+c)^5} - \frac{4}{3} \frac{d b a^3}{\sin(d*x+c)^5} - \frac{4}{3} \frac{d b a^3 \ln(\sec(d*x+c) + \tan(d*x+c))}{\sin(d*x+c)^5} - \frac{1}{6} a^4 \cot(d*x+c) \csc(d*x+c)^5 - \frac{5}{24} a^4 \cot(d*x+c) \csc(d*x+c)^3 - \frac{5}{16} a^4 \cot(d*x+c) \csc(d*x+c) + \frac{5}{16} \frac{d a^4 \ln(\csc(d*x+c) - \cot(d*x+c))}{d}$

Maxima [A] time = 1.15415, size = 522, normalized size = 1.3

$$5a^4 \left(\frac{2(15 \cos(dx+c)^5 - 40 \cos(dx+c)^3 + 33 \cos(dx+c))}{\cos(dx+c)^6 - 3 \cos(dx+c)^4 + 3 \cos(dx+c)^2 - 1} - 15 \log(\cos(dx+c) + 1) + 15 \log(\cos(dx+c) - 1) \right) + 40b^4 \left(\frac{2(15 \cos(dx+c)^4 - 10 \cos(dx+c)^2 - 2)}{\cos(dx+c)^5 - \cos(dx+c)^3} - 15 \log(\cos(dx+c) + 1) + 15 \log(\cos(dx+c) - 1) \right) + 180a^2 b^2 \left(\frac{2(15 \cos(dx+c)^4 - 25 \cos(dx+c)^2 + 8)}{\cos(dx+c)^5 - 2 \cos(dx+c)^3 + \cos(dx+c)} - 15 \log(\cos(dx+c) + 1) + 15 \log(\cos(dx+c) - 1) \right) - \frac{160 a^3 b^3 (2(15 \sin(dx+c)^4 - 10 \sin(dx+c)^2 - 2))}{(\sin(dx+c)^5 - \sin(dx+c)^3) - 15 \log(\sin(dx+c) + 1) + 15 \log(\sin(dx+c) - 1)} - \frac{64 a^3 b^3 (2(15 \sin(dx+c)^4 + 5 \sin(dx+c)^2 + 3))}{\sin(dx+c)^5 - 15 \log(\sin(dx+c) + 1) + 15 \log(\sin(dx+c) - 1)} \Big/ d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="maxima")

[Out] $\frac{1}{480} \frac{(5a^4(2(15 \cos(dx+c)^5 - 40 \cos(dx+c)^3 + 33 \cos(dx+c)) / (\cos(dx+c)^6 - 3 \cos(dx+c)^4 + 3 \cos(dx+c)^2 - 1) - 15 \log(\cos(dx+c) + 1) + 15 \log(\cos(dx+c) - 1)) + 40b^4(2(15 \cos(dx+c)^4 - 10 \cos(dx+c)^2 - 2) / (\cos(dx+c)^5 - \cos(dx+c)^3) - 15 \log(\cos(dx+c) + 1) + 15 \log(\cos(dx+c) - 1)) + 180a^2 b^2(2(15 \cos(dx+c)^4 - 25 \cos(dx+c)^2 + 8) / (\cos(dx+c)^5 - 2 \cos(dx+c)^3 + \cos(dx+c)) - 15 \log(\cos(dx+c) + 1) + 15 \log(\cos(dx+c) - 1)) - 160a^3 b^3(2(15 \sin(dx+c)^4 - 10 \sin(dx+c)^2 - 2) / (\sin(dx+c)^5 - \sin(dx+c)^3) - 15 \log(\sin(dx+c) + 1) + 15 \log(\sin(dx+c) - 1)) - 64a^3 b^3(2(15 \sin(dx+c)^4 + 5 \sin(dx+c)^2 + 3) / \sin(dx+c)^5 - 15 \log(\sin(dx+c) + 1) + 15 \log(\sin(dx+c) - 1)))}{d}$

Fricas [A] time = 4.56755, size = 1661, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="fricas")

[Out] $\frac{1}{480} (150(a^4 + 36a^2 b^2 + 8b^4) \cos(dx+c)^8 - 400(a^4 + 36a^2 b^2 + 8b^4) \cos(dx+c)^6 + 330(a^4 + 36a^2 b^2 + 8b^4) \cos(dx+c)^4 - 160b^4 - 480(6a^2 b^2 + b^4) \cos(dx+c)^2 - 75((a^4 + 36a^2 b^2 + 8b^4) \cos(dx+c)^9 - 3(a^4 + 36a^2 b^2 + 8b^4) \cos(dx+c)^7 + 3(a^4 + 36a^2 b^2 + 8b^4) \cos(dx+c)^5 - (a^4 + 36a^2 b^2 + 8b^4) \cos(dx+c)^3) \log(1/2 \cos(dx+c) + 1/2) + 75((a^4 + 36a^2 b^2 + 8b^4) \cos(dx+c)^9 - 3(a^4 + 36a^2 b^2 + 8b^4) \cos(dx+c)^7 + 3(a^4 + 36a^2 b^2 + 8b^4) \cos(dx+c)^5 - (a^4 + 36a^2 b^2 + 8b^4) \cos(dx+c)^3) \log(-1/2 \cos(dx+c) + 1/2) + 480((2a^3 b + 5a^2 b^2) \cos(dx+c)^9 - 3(2a^3 b + 5a^2 b^2) \cos(dx+c)^7 + 3(2a^3 b + 5a^2 b^2) \cos(dx+c)^5 - (2a^3 b + 5a^2 b^2) \cos(dx+c)^3) \log(\sin(dx+c) + 1) - 480((2a^3 b + 5a^2 b^2) \cos(dx+c)^9 - 3(2a^3 b + 5a^2 b^2) \cos(dx+c)^7 + 3(2a^3 b + 5a^2 b^2) \cos(dx+c)^5 - (2a^3 b + 5a^2 b^2) \cos(dx+c)^3) \log(\sin(dx+c) - 1)) \Big/ d$

$$5*a*b^3*\cos(d*x + c)^5 - (2*a^3*b + 5*a*b^3)*\cos(d*x + c)^3*\log(-\sin(d*x + c) + 1) + 64*(15*(2*a^3*b + 5*a*b^3)*\cos(d*x + c)^7 - 35*(2*a^3*b + 5*a*b^3)*\cos(d*x + c)^5 - 15*a*b^3*\cos(d*x + c) + 23*(2*a^3*b + 5*a*b^3)*\cos(d*x + c)^3)*\sin(d*x + c))/(d*\cos(d*x + c)^9 - 3*d*\cos(d*x + c)^7 + 3*d*\cos(d*x + c)^5 - d*\cos(d*x + c)^3)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**7*(a+b*tan(d*x+c))**4,x)

[Out] Timed out

Giac [A] time = 2.80009, size = 873, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="giac")

[Out]
$$\frac{1}{1920}*(5*a^4*\tan(1/2*d*x + 1/2*c)^6 - 48*a^3*b*\tan(1/2*d*x + 1/2*c)^5 + 45*a^4*\tan(1/2*d*x + 1/2*c)^4 + 180*a^2*b^2*\tan(1/2*d*x + 1/2*c)^4 - 560*a^3*b*\tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*\tan(1/2*d*x + 1/2*c)^3 + 225*a^4*\tan(1/2*d*x + 1/2*c)^2 + 2880*a^2*b^2*\tan(1/2*d*x + 1/2*c)^2 + 240*b^4*\tan(1/2*d*x + 1/2*c)^2 - 5280*a^3*b*\tan(1/2*d*x + 1/2*c) - 8640*a*b^3*\tan(1/2*d*x + 1/2*c) + 3840*(2*a^3*b + 5*a*b^3)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - 3840*(2*a^3*b + 5*a*b^3)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) + 600*(a^4 + 36*a^2*b^2 + 8*b^4)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))) + 1280*(6*a*b^3*\tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b^2*\tan(1/2*d*x + 1/2*c)^4 - 9*b^4*\tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^2*\tan(1/2*d*x + 1/2*c)^2 + 12*b^4*\tan(1/2*d*x + 1/2*c)^2 - 6*a*b^3*\tan(1/2*d*x + 1/2*c) - 18*a^2*b^2 - 7*b^4)/(\tan(1/2*d*x + 1/2*c)^2 - 1)^3 - (1470*a^4*\tan(1/2*d*x + 1/2*c)^6 + 52920*a^2*b^2*\tan(1/2*d*x + 1/2*c)^6 + 11760*b^4*\tan(1/2*d*x + 1/2*c)^6 + 5280*a^3*b*\tan(1/2*d*x + 1/2*c)^5 + 8640*a*b^3*\tan(1/2*d*x + 1/2*c)^5 + 225*a^4*\tan(1/2*d*x + 1/2*c)^4 + 2880*a^2*b^2*\tan(1/2*d*x + 1/2*c)^4 + 240*b^4*\tan(1/2*d*x + 1/2*c)^4 + 560*a^3*b*\tan(1/2*d*x + 1/2*c)^3 + 320*a*b^3*\tan(1/2*d*x + 1/2*c)^3 + 45*a^4*\tan(1/2*d*x + 1/2*c)^2 + 180*a^2*b^2*\tan(1/2*d*x + 1/2*c)^2 + 48*a^3*b*\tan(1/2*d*x + 1/2*c) + 5*a^4)/\tan(1/2*d*x + 1/2*c)^6)/d$$

3.51 $\int \frac{\sin^5(c+dx)}{a+b \tan(c+dx)} dx$

Optimal. Leaf size=274

$$\frac{a^2 b \sin^3(c+dx)}{3d(a^2+b^2)^2} + \frac{b \sin^5(c+dx)}{5d(a^2+b^2)} + \frac{a^4 b \sin(c+dx)}{d(a^2+b^2)^3} - \frac{a \cos^5(c+dx)}{5d(a^2+b^2)} + \frac{2a \cos^3(c+dx)}{3d(a^2+b^2)} - \frac{ab^2 \cos^3(c+dx)}{3d(a^2+b^2)^2} + \frac{a^3 b^2 \cos(c+dx)}{d(a^2+b^2)}$$

[Out] (a^5*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(7/2)*d) + (a^3*b^2*Cos[c + d*x])/((a^2 + b^2)^3*d) + (a*b^2*Cos[c + d*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) - (a*b^2*Cos[c + d*x]^3)/(3*(a^2 + b^2)^2*d) + (2*a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) - (a*Cos[c + d*x]^5)/(5*(a^2 + b^2)*d) + (a^4*b*Sin[c + d*x])/((a^2 + b^2)^3*d) + (a^2*b*Sin[c + d*x]^3)/(3*(a^2 + b^2)^2*d) + (b*Sin[c + d*x]^5)/(5*(a^2 + b^2)*d)

Rubi [A] time = 0.353902, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3518, 3109, 2564, 30, 2633, 3099, 3074, 206, 2638}

$$\frac{a^2 b \sin^3(c+dx)}{3d(a^2+b^2)^2} + \frac{b \sin^5(c+dx)}{5d(a^2+b^2)} + \frac{a^4 b \sin(c+dx)}{d(a^2+b^2)^3} - \frac{a \cos^5(c+dx)}{5d(a^2+b^2)} + \frac{2a \cos^3(c+dx)}{3d(a^2+b^2)} - \frac{ab^2 \cos^3(c+dx)}{3d(a^2+b^2)^2} + \frac{a^3 b^2 \cos(c+dx)}{d(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^5/(a + b*Tan[c + d*x]),x]

[Out] (a^5*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(7/2)*d) + (a^3*b^2*Cos[c + d*x])/((a^2 + b^2)^3*d) + (a*b^2*Cos[c + d*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) - (a*b^2*Cos[c + d*x]^3)/(3*(a^2 + b^2)^2*d) + (2*a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) - (a*Cos[c + d*x]^5)/(5*(a^2 + b^2)*d) + (a^4*b*Sin[c + d*x])/((a^2 + b^2)^3*d) + (a^2*b*Sin[c + d*x]^3)/(3*(a^2 + b^2)^2*d) + (b*Sin[c + d*x]^5)/(5*(a^2 + b^2)*d)

Rule 3518

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3109

Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)} dx &= \int \frac{\cos(c+dx)\sin^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)} dx \\
&= \frac{a \int \sin^5(c+dx) dx}{a^2+b^2} + \frac{b \int \cos(c+dx)\sin^4(c+dx) dx}{a^2+b^2} - \frac{(ab) \int \frac{\sin^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)} dx}{a^2+b^2} \\
&= \frac{a^2b \sin^3(c+dx)}{3(a^2+b^2)^2 d} - \frac{(a^3b) \int \frac{\sin^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)} dx}{(a^2+b^2)^2} - \frac{(ab^2) \int \sin^3(c+dx) dx}{(a^2+b^2)^2} - \frac{a \operatorname{Subst}\left(\int (1-\right.}{(a^2+b^2)^2} \\
&= -\frac{a \cos(c+dx)}{(a^2+b^2)d} + \frac{2a \cos^3(c+dx)}{3(a^2+b^2)d} - \frac{a \cos^5(c+dx)}{5(a^2+b^2)d} + \frac{a^4b \sin(c+dx)}{(a^2+b^2)^3 d} + \frac{a^2b \sin^3(c+dx)}{3(a^2+b^2)^2 d} + \frac{bs}{5} \\
&= \frac{a^3b^2 \cos(c+dx)}{(a^2+b^2)^3 d} + \frac{ab^2 \cos(c+dx)}{(a^2+b^2)^2 d} - \frac{a \cos(c+dx)}{(a^2+b^2)d} - \frac{ab^2 \cos^3(c+dx)}{3(a^2+b^2)^2 d} + \frac{2a \cos^3(c+dx)}{3(a^2+b^2)d} - \frac{a}{5} \\
&= \frac{a^5b \tanh^{-1}\left(\frac{b \cos(c+dx)-a \sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2} d} + \frac{a^3b^2 \cos(c+dx)}{(a^2+b^2)^3 d} + \frac{ab^2 \cos(c+dx)}{(a^2+b^2)^2 d} - \frac{a \cos(c+dx)}{(a^2+b^2)d} - \frac{ab^2}{3(a^2+b^2)}
\end{aligned}$$

Mathematica [A] time = 3.12434, size = 289, normalized size = 1.05

$$\sqrt{a^2+b^2} (120a^2b^3 \sin(c+dx) - 50a^2b^3 \sin(3(c+dx)) + 6a^2b^3 \sin(5(c+dx)) - 6a^3b^2 \cos(5(c+dx)) - 30a(-4a^2b^2 + 5a^4$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^5/(a + b*Tan[c + d*x]),x]

[Out] (-480*a^5*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2]*(-30*a*(5*a^4 - 4*a^2*b^2 - b^4)*Cos[c + d*x] + 5*a*(5*a^4 + 6*a^2*b^2 + b^4)*Cos[3*(c + d*x)] - 3*a^5*Cos[5*(c + d*x)] - 6*a^3*b^2*Cos[5*(c + d*x)] - 3*a*b^4*Cos[5*(c + d*x)] + 330*a^4*b*Sin[c + d*x] + 120*a^2*b^3*Sin[c + d*x] + 30*b^5*Sin[c + d*x] - 35*a^4*b*Sin[3*(c + d*x)] - 50*a^2*b^3*Sin[3*(c + d*x)] - 15*b^5*Sin[3*(c + d*x)] + 3*a^4*b*Sin[5*(c + d*x)] + 6*a^2*b^3*Sin[5*(c + d*x)] + 3*b^5*Sin[5*(c + d*x)]))/(240*(a^2 + b^2)^(7/2)*d)

Maple [A] time = 0.079, size = 361, normalized size = 1.3

$$\frac{1}{d} \left(-2 \frac{1}{(a^4 + 2a^2b^2 + b^4)(a^2 + b^2)(1 + (\tan(1/2 dx + c/2))^2)^5} \left(-a^4b(\tan(1/2 dx + c/2))^9 - a^3b^2(\tan(1/2 dx + c/2))^8 + \dots \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^5/(a+b*tan(d*x+c)),x)

[Out] 1/d*(-2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(-a^4*b*tan(1/2*d*x+1/2*c)^9-a^3*b^2*tan(1/2*d*x+1/2*c)^8+(-16/3*a^4*b-4/3*a^2*b^3)*tan(1/2*d*x+1/2*c)^7+(-6*a^3*b^2-2*a*b^4)*tan(1/2*d*x+1/2*c)^6+(-178/15*a^4*b-136/15*a^2*b^3-16/5*b^5)*tan(1/2*d*x+1/2*c)^5+(16/3*a^5+2/3*a*b^4)*tan(1/2*d*x+1/2*c)^4+(-16/3*a^4*b-4/3*a^2*b^3)*tan(1/2*d*x+1/2*c)^3+(-2*a^3*b^2+8/3*a^5-2/3*a*b^4)*tan(1/2*d

$$\frac{(x+\frac{1}{2}c)^2 - a^4 b \tan(\frac{1}{2}dx + \frac{1}{2}c) + 8/15 a^5 - 3/5 a^3 b^2 - 2/15 a b^4}{(1 + \tan(\frac{1}{2}dx + \frac{1}{2}c))^2} - 64 a^5 b / (32 a^6 + 96 a^4 b^2 + 96 a^2 b^4 + 32 b^6) / (a^2 + b^2)^{1/2} \operatorname{arctanh}(\frac{1}{2}(2 a \tan(\frac{1}{2}dx + \frac{1}{2}c) - 2 b) / (a^2 + b^2)^{1/2})$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)^5/(a+b*tan(dx+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 2.3272, size = 840, normalized size = 3.07

$$15 \sqrt{a^2 + b^2} a^5 b \log\left(\frac{2 ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 - 2 a^2 - b^2 - 2 \sqrt{a^2 + b^2} (b \cos(dx+c) - a \sin(dx+c))}{2 ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 + b^2}\right) - 6 (a^7 + 3 a^5 b^2 + 3 a^3 b^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)^5/(a+b*tan(dx+c)),x, algorithm="fricas")

[Out] $\frac{1}{30} (15 \sqrt{a^2 + b^2} a^5 b \log((2 a b \cos(dx + c) \sin(dx + c) + (a^2 - b^2) \cos(dx + c)^2 - 2 a^2 - b^2 - 2 \sqrt{a^2 + b^2} (b \cos(dx + c) - a \sin(dx + c))) / (2 a b \cos(dx + c) \sin(dx + c) + (a^2 - b^2) \cos(dx + c)^2 + b^2)) - 6 (a^7 + 3 a^5 b^2 + 3 a^3 b^4 + a b^6) \cos(dx + c)^5 + 10 (2 a^7 + 5 a^5 b^2 + 4 a^3 b^4 + a b^6) \cos(dx + c)^3 - 30 (a^7 + a^5 b^2) \cos(dx + c) + 2 (23 a^6 b + 34 a^4 b^3 + 14 a^2 b^5 + 3 b^7 + 3 (a^6 b + 3 a^4 b^3 + 3 a^2 b^5 + b^7) \cos(dx + c)^4 - (11 a^6 b + 28 a^4 b^3 + 23 a^2 b^5 + 6 b^7) \cos(dx + c)^2) \sin(dx + c)) / ((a^8 + 4 a^6 b^2 + 6 a^4 b^4 + 4 a^2 b^6 + b^8) d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)**5/(a+b*tan(dx+c)),x)

[Out] Timed out

Giac [A] time = 1.34169, size = 626, normalized size = 2.28

$$\frac{15 a^5 b \log\left(\frac{\left|2 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2 b - 2 \sqrt{a^2 + b^2}\right|}{\left|2 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2 b + 2 \sqrt{a^2 + b^2}\right|}\right)}{(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) \sqrt{a^2 + b^2}} + \frac{2 \left(15 a^4 b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 + 15 a^3 b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^8 + 80 a^4 b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 20 a^2 b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 90 a^3 b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 15 a^2 b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 60 a^3 b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 15 a^2 b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 60 a^3 b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 15 a^2 b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 60 a^3 b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 15 a^2 b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 60 a^3 b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 15 a^2 b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 60 a^3 b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 15 a^2 b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 60 a^3 b^2\right)}{(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) \sqrt{a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/15*(15*a^5*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/
abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(15*a^4*b*tan(1/2*d*x + 1/2*c)^9 +
15*a^3*b^2*tan(1/2*d*x + 1/2*c)^8 + 80*a^4*b*tan(1/2*d*x + 1/2*c)^7 + 20*a^
2*b^3*tan(1/2*d*x + 1/2*c)^7 + 90*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 + 30*a*b^4
*tan(1/2*d*x + 1/2*c)^6 + 178*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 136*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^5 + 48*b^5*tan(1/2*d*x + 1/2*c)^5 - 80*a^5*tan(1/2*d*x +
1/2*c)^4 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^4 + 80*a^4*b*tan(1/2*d*x + 1/2*c)
^3 + 20*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^5*tan(1/2*d*x + 1/2*c)^2 + 30
*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 10*a*b^4*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*
b*tan(1/2*d*x + 1/2*c) - 8*a^5 + 9*a^3*b^2 + 2*a*b^4)/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)^5))/d
```


3.52 $\int \frac{\sin^4(c+dx)}{a+b \tan(c+dx)} dx$

Optimal. Leaf size=158

$$\frac{\cos^4(c+dx)(a \tan(c+dx)+b)}{4d(a^2+b^2)} - \frac{\cos^2(c+dx)(a(5a^2+b^2)\tan(c+dx)+4b(2a^2+b^2))}{8d(a^2+b^2)^2} + \frac{a^4b \log(a \cos(c+dx)+b)}{d(a^2+b^2)^3}$$

[Out] (a*(3*a^4 - 6*a^2*b^2 - b^4)*x)/(8*(a^2 + b^2)^3) + (a^4*b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (Cos[c + d*x]^4*(b + a*Tan[c + d*x]))/(4*(a^2 + b^2)*d) - (Cos[c + d*x]^2*(4*b*(2*a^2 + b^2) + a*(5*a^2 + b^2)*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)

Rubi [A] time = 0.338949, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1647, 801, 635, 203, 260}

$$\frac{\cos^4(c+dx)(a \tan(c+dx)+b)}{4d(a^2+b^2)} - \frac{\cos^2(c+dx)(a(5a^2+b^2)\tan(c+dx)+4b(2a^2+b^2))}{8d(a^2+b^2)^2} + \frac{a^4b \log(a \cos(c+dx)+b)}{d(a^2+b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x]), x]

[Out] (a*(3*a^4 - 6*a^2*b^2 - b^4)*x)/(8*(a^2 + b^2)^3) + (a^4*b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (Cos[c + d*x]^4*(b + a*Tan[c + d*x]))/(4*(a^2 + b^2)*d) - (Cos[c + d*x]^2*(4*b*(2*a^2 + b^2) + a*(5*a^2 + b^2)*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1)], x], x, b*Tan[e + f*x], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 1647

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q]/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}

}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{\sin^4(c + dx)}{a + b \tan(c + dx)} dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^4}{(a+x)(b^2+x^2)^3} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{\cos^4(c + dx)(b + a \tan(c + dx))}{4(a^2 + b^2)d} - \frac{\operatorname{Subst}\left(\int \frac{\frac{a^2b^4}{a^2+b^2} - \frac{3ab^4x}{a^2+b^2} - 4b^2x^2}{(a+x)(b^2+x^2)^2} dx, x, b \tan(c + dx)\right)}{4bd} \\ &= \frac{\cos^4(c + dx)(b + a \tan(c + dx))}{4(a^2 + b^2)d} - \frac{\cos^2(c + dx)(4b(2a^2 + b^2) + a(5a^2 + b^2)\tan(c + dx))}{8(a^2 + b^2)^2d} + \dots \\ &= \frac{\cos^4(c + dx)(b + a \tan(c + dx))}{4(a^2 + b^2)d} - \frac{\cos^2(c + dx)(4b(2a^2 + b^2) + a(5a^2 + b^2)\tan(c + dx))}{8(a^2 + b^2)^2d} + \dots \\ &= \frac{a^4b \log(a + b \tan(c + dx))}{(a^2 + b^2)^3d} + \frac{\cos^4(c + dx)(b + a \tan(c + dx))}{4(a^2 + b^2)d} - \frac{\cos^2(c + dx)(4b(2a^2 + b^2) + a(5a^2 + b^2)\tan(c + dx))}{8(a^2 + b^2)^2d} + \dots \\ &= \frac{a^4b \log(a + b \tan(c + dx))}{(a^2 + b^2)^3d} + \frac{\cos^4(c + dx)(b + a \tan(c + dx))}{4(a^2 + b^2)d} - \frac{\cos^2(c + dx)(4b(2a^2 + b^2) + a(5a^2 + b^2)\tan(c + dx))}{8(a^2 + b^2)^2d} + \dots \\ &= \frac{a(3a^4 - 6a^2b^2 - b^4)x}{8(a^2 + b^2)^3} + \frac{a^4b \log(\cos(c + dx))}{(a^2 + b^2)^3d} + \frac{a^4b \log(a + b \tan(c + dx))}{(a^2 + b^2)^3d} + \frac{\cos^4(c + dx)(b + a \tan(c + dx))}{4(a^2 + b^2)d} \end{aligned}$$

Mathematica [A] time = 2.59212, size = 249, normalized size = 1.58

$$\frac{a(6a^2b^3 + 5a^4b + b^5) \sin(2(c + dx)) - 4b^2(a^2 + b^2)^2 \cos^4(c + dx) + 8b^2(3a^2b^2 + 2a^4 + b^4) \cos^2(c + dx) + 2ab(6a^2b^2 + \dots}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x]), x]

[Out] -(2*a*b*(5*a^4 + 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]] + 8*b^2*(2*a^4 + 3*a^2*b^2 + b^4)*Cos[c + d*x]^2 - 4*b^2*(a^2 + b^2)^2*Cos[c + d*x]^4 + 8*a^4*(b^2 + a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*b^2*Log[a + b*Tan

$$[c + d*x]] + (b^2 - a*\text{Sqrt}[-b^2])*\text{Log}[\text{Sqrt}[-b^2] + b*\text{Tan}[c + d*x]]) - 4*a*b*(a^2 + b^2)^2*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x] + a*(5*a^4*b + 6*a^2*b^3 + b^5)*\text{Sin}[2*(c + d*x)]/(16*b*(a^2 + b^2)^3*d)$$

Maple [B] time = 0.068, size = 565, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c)), x)

[Out]
$$-5/8/d/(a^2+b^2)^3/(1+\tan(d*x+c))^2*\tan(d*x+c)^3*a^5-3/4/d/(a^2+b^2)^3/(1+\tan(d*x+c))^2*\tan(d*x+c)^2*\tan(d*x+c)^3*a^3*b^2-1/8/d/(a^2+b^2)^3/(1+\tan(d*x+c))^2*\tan(d*x+c)^3*a*b^4-1/d/(a^2+b^2)^3/(1+\tan(d*x+c))^2*\tan(d*x+c)^2*a^4*b-3/2/d/(a^2+b^2)^3/(1+\tan(d*x+c))^2*\tan(d*x+c)^2*a^2*b^3-1/2/d/(a^2+b^2)^3/(1+\tan(d*x+c))^2*\tan(d*x+c)^2*b^5-3/8/d/(a^2+b^2)^3/(1+\tan(d*x+c))^2*\tan(d*x+c)*a^5-1/4/d/(a^2+b^2)^3/(1+\tan(d*x+c))^2*\tan(d*x+c)*a^3*b^2+1/8/d/(a^2+b^2)^3/(1+\tan(d*x+c))^2*\tan(d*x+c)*a*b^4-3/4/d/(a^2+b^2)^3/(1+\tan(d*x+c))^2*a^4*b-1/d/(a^2+b^2)^3/(1+\tan(d*x+c))^2*a^2*b^3-1/4/d/(a^2+b^2)^3/(1+\tan(d*x+c))^2*b^5-1/2/d/(a^2+b^2)^3*\ln(1+\tan(d*x+c))^2*a^4*b+3/8/d/(a^2+b^2)^3*\arctan(\tan(d*x+c))*a^5-3/4/d/(a^2+b^2)^3*\arctan(\tan(d*x+c))*a^3*b^2-1/8/d/(a^2+b^2)^3*\arctan(\tan(d*x+c))*a*b^4+1/d*b/(a^2+b^2)^3*a^4*\ln(a+b*\tan(d*x+c))$$

Maxima [A] time = 1.65528, size = 378, normalized size = 2.39

$$\frac{8a^4b \log(b \tan(dx+c)+a)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{4a^4b \log(\tan(dx+c)^2+1)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{(3a^5-6a^3b^2-ab^4)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{(5a^3+ab^2) \tan(dx+c)^3+6a^2b+2b^3+4(2a^2b+b^3) \tan(dx+c)^2+(3a^3+(a^4+2a^2b^2+b^4) \tan(dx+c)^4+a^4+2a^2b^2+b^4+2(a^4+2a^2b^2+b^4) \tan(dx+c)^2))}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)), x, algorithm="maxima")

[Out]
$$1/8*(8*a^4*b*\log(b*\tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 4*a^4*b*\log(\tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*a^5 - 6*a^3*b^2 - a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((5*a^3 + a*b^2)*\tan(d*x + c)^3 + 6*a^2*b + 2*b^3 + 4*(2*a^2*b + b^3)*\tan(d*x + c)^2 + (3*a^3 - a*b^2)*\tan(d*x + c)))/((a^4 + 2*a^2*b^2 + b^4)*\tan(d*x + c)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*\tan(d*x + c)^2))/d$$

Fricas [A] time = 2.28896, size = 481, normalized size = 3.04

$$\frac{4a^4b \log(2ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 + b^2) + 2(a^4b + 2a^2b^3 + b^5) \cos(dx+c)^4 + (3a^5 - 6a^3b^2 - ab^4)(dx+c)}{8(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)), x, algorithm="fricas")

```
[Out] 1/8*(4*a^4*b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 - 6*a^3*b^2 - a*b^4)*d*x - 4*(2*a^4*b + 3*a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - (5*a^5 + 6*a^3*b^2 + a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.19573, size = 451, normalized size = 2.85

$$\frac{8a^4b^2 \log(|b \tan(dx+c)+a|)}{a^6b+3a^4b^3+3a^2b^5+b^7} - \frac{4a^4b \log(\tan(dx+c)^2+1)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{(3a^5-6a^3b^2-ab^4)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{6a^4b \tan(dx+c)^4 - 5a^5 \tan(dx+c)^3 - 6a^3b^2 \tan(dx+c)^3 - ab^4 \tan(dx+c)}{8d}$$

8d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*a^4*b^2*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 4*a^4*b*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*a^5 - 6*a^3*b^2 - a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (6*a^4*b*tan(d*x + c)^4 - 5*a^5*tan(d*x + c)^3 - 6*a^3*b^2*tan(d*x + c)^3 - a*b^4*tan(d*x + c)^3 + 4*a^4*b*tan(d*x + c)^2 - 12*a^2*b^3*tan(d*x + c)^2 - 4*b^5*tan(d*x + c)^2 - 3*a^5*tan(d*x + c) - 2*a^3*b^2*tan(d*x + c) + a*b^4*tan(d*x + c) - 8*a^2*b^3 - 2*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(d*x + c)^2 + 1)^2))/d
```

3.53 $\int \frac{\sin^3(c+dx)}{a+b \tan(c+dx)} dx$

Optimal. Leaf size=168

$$\frac{b \sin^3(c+dx)}{3d(a^2+b^2)} + \frac{a^2 b \sin(c+dx)}{d(a^2+b^2)^2} + \frac{a \cos^3(c+dx)}{3d(a^2+b^2)} - \frac{a \cos(c+dx)}{d(a^2+b^2)} + \frac{ab^2 \cos(c+dx)}{d(a^2+b^2)^2} + \frac{a^3 b \tanh^{-1}\left(\frac{b \cos(c+dx) - a \sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{5/2}}$$

```
[Out] (a^3*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(5/2)*d) + (a*b^2*Cos[c + d*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/((
a^2 + b^2)*d) + (a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) + (a^2*b*Sin[c + d*x]
)/((a^2 + b^2)^2*d) + (b*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```

Rubi [A] time = 0.22299, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3518, 3109, 2564, 30, 2633, 3099, 3074, 206, 2638}

$$\frac{b \sin^3(c+dx)}{3d(a^2+b^2)} + \frac{a^2 b \sin(c+dx)}{d(a^2+b^2)^2} + \frac{a \cos^3(c+dx)}{3d(a^2+b^2)} - \frac{a \cos(c+dx)}{d(a^2+b^2)} + \frac{ab^2 \cos(c+dx)}{d(a^2+b^2)^2} + \frac{a^3 b \tanh^{-1}\left(\frac{b \cos(c+dx) - a \sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a^3*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(5/2)*d) + (a*b^2*Cos[c + d*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/((
a^2 + b^2)*d) + (a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) + (a^2*b*Sin[c + d*x]
)/((a^2 + b^2)^2*d) + (b*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] := Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
+ b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] := Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^(n - 1)/2, x], x, a*
Sin[e + f*x], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 3099

Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2 + b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]^(m - 1), x], x) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1]

Rule 3074

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]^(-1), x_Symbol] := -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sin^3(c + dx)}{a + b \tan(c + dx)} dx &= \int \frac{\cos(c + dx) \sin^3(c + dx)}{a \cos(c + dx) + b \sin(c + dx)} dx \\ &= \frac{a \int \sin^3(c + dx) dx}{a^2 + b^2} + \frac{b \int \cos(c + dx) \sin^2(c + dx) dx}{a^2 + b^2} - \frac{(ab) \int \frac{\sin^2(c + dx)}{a \cos(c + dx) + b \sin(c + dx)} dx}{a^2 + b^2} \\ &= \frac{a^2 b \sin(c + dx)}{(a^2 + b^2)^2 d} - \frac{(a^3 b) \int \frac{1}{a \cos(c + dx) + b \sin(c + dx)} dx}{(a^2 + b^2)^2} - \frac{(ab^2) \int \sin(c + dx) dx}{(a^2 + b^2)^2} - \frac{a \operatorname{Subst}\left(\int (1 - x^2)^{-1/2} dx\right)}{(a^2 + b^2)^2} \\ &= \frac{ab^2 \cos(c + dx)}{(a^2 + b^2)^2 d} - \frac{a \cos(c + dx)}{(a^2 + b^2) d} + \frac{a \cos^3(c + dx)}{3(a^2 + b^2) d} + \frac{a^2 b \sin(c + dx)}{(a^2 + b^2)^2 d} + \frac{b \sin^3(c + dx)}{3(a^2 + b^2) d} + \frac{(a^3 b) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 - x^2}} dx\right)}{(a^2 + b^2)^2} \\ &= \frac{a^3 b \tanh^{-1}\left(\frac{b \cos(c + dx) - a \sin(c + dx)}{\sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)^{5/2} d} + \frac{ab^2 \cos(c + dx)}{(a^2 + b^2)^2 d} - \frac{a \cos(c + dx)}{(a^2 + b^2) d} + \frac{a \cos^3(c + dx)}{3(a^2 + b^2) d} + \frac{a^2 b \sin(c + dx)}{(a^2 + b^2)^2 d} + \frac{b \sin^3(c + dx)}{3(a^2 + b^2) d} \end{aligned}$$

Mathematica [A] time = 1.37492, size = 139, normalized size = 0.83

$$\frac{\sqrt{a^2 + b^2} \left((3ab^2 - 9a^3) \cos(c + dx) + a(a^2 + b^2) \cos(3(c + dx)) - 2b \sin(c + dx) \left((a^2 + b^2) \cos(2(c + dx)) - 7a^2 - b^2 \right) \right)}{12d(a^2 + b^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^3/(a + b*Tan[c + d*x]),x]

[Out] $(-24*a^3*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2]*((-9*a^3 + 3*a*b^2)*Cos[c + d*x] + a*(a^2 + b^2)*Cos[3*(c + d*x)] - 2*b*(-7*a^2 - b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*(a^2 + b^2)^{(5/2)*d})$

Maple [A] time = 0.065, size = 205, normalized size = 1.2

$$\frac{1}{d} \left(-2 \frac{-ba^2 (\tan(1/2 dx + c/2))^5 - ab^2 (\tan(1/2 dx + c/2))^4 + (-10/3 ba^2 - 4/3 b^3) (\tan(1/2 dx + c/2))^3 + 2 a^3 (\tan(1/2 dx + c/2))^2 - 2 b^4 (\tan(1/2 dx + c/2))}{(a^4 + 2 a^2 b^2 + b^4) (1 + (\tan(1/2 dx + c/2))^2)^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^3/(a+b*tan(d*x+c)),x)

[Out] $1/d*(-2/(a^4+2*a^2*b^2+b^4)*(-b*a^2*\tan(1/2*d*x+1/2*c)^5-a*b^2*\tan(1/2*d*x+1/2*c)^4+(-10/3*b*a^2-4/3*b^3)*\tan(1/2*d*x+1/2*c)^3+2*a^3*\tan(1/2*d*x+1/2*c)^2-b*a^2*\tan(1/2*d*x+1/2*c)+2/3*a^3-1/3*a*b^2)/(1+\tan(1/2*d*x+1/2*c)^2)^3-16*b*a^3/(8*a^4+16*a^2*b^2+8*b^4)/(a^2+b^2)^{(1/2)*arctanh(1/2*(2*a*\tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^{(1/2)})}$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 2.27774, size = 597, normalized size = 3.55

$$\frac{3 \sqrt{a^2 + b^2} a^3 b \log \left(\frac{2 ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 - 2 a^2 - b^2 - 2 \sqrt{a^2 + b^2} (b \cos(dx+c) - a \sin(dx+c))}{2 ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 + b^2} \right) + 2 (a^5 + 2 a^3 b^2 + ab^4) \cos(dx+c)}{6 (a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a^3*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*s
in(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2)) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - 6*(a^5 + a^3*b^2)*c
os(d*x + c) + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cos(
d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38606, size = 325, normalized size = 1.93

$$\frac{3a^3b \log\left(\frac{2a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 2b - 2\sqrt{a^2 + b^2}}{2a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 2b + 2\sqrt{a^2 + b^2}}\right)}{(a^4 + 2a^2b^2 + b^4)\sqrt{a^2 + b^2}} + \frac{2\left(3a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 3ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 + 10a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 4b^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 6a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 2a^5 + 2a^3b^2 + a^2b^4 + b^5\right)}{(a^4 + 2a^2b^2 + b^4)\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)^3}$$

$3d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^3*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/ab
s(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
b^4)*sqrt(a^2 + b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 3*a*b^2*tan(1/2
*d*x + 1/2*c)^4 + 10*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 4*b^3*tan(1/2*d*x + 1/2
*c)^2 - 6*a^3*tan(1/2*d*x + 1/2*c) + 2*a^5 + 2*a^3*b^2 + a^2*b^4 + b^5)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```


3.54 $\int \frac{\sin^2(c+dx)}{a+b \tan(c+dx)} dx$

Optimal. Leaf size=94

$$-\frac{\cos^2(c+dx)(a \tan(c+dx)+b)}{2d(a^2+b^2)} + \frac{a^2 b \log(a \cos(c+dx)+b \sin(c+dx))}{d(a^2+b^2)^2} + \frac{ax(a^2-b^2)}{2(a^2+b^2)^2}$$

[Out] (a*(a^2 - b^2)*x)/(2*(a^2 + b^2)^2) + (a^2*b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^2*d) - (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 + b^2)*d)

Rubi [A] time = 0.163218, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1647, 801, 635, 203, 260}

$$-\frac{\cos^2(c+dx)(a \tan(c+dx)+b)}{2d(a^2+b^2)} + \frac{a^2 b \log(a \cos(c+dx)+b \sin(c+dx))}{d(a^2+b^2)^2} + \frac{ax(a^2-b^2)}{2(a^2+b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x]), x]

[Out] (a*(a^2 - b^2)*x)/(2*(a^2 + b^2)^2) + (a^2*b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^2*d) - (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 + b^2)*d)

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 1647

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(c + dx)}{a + b \tan(c + dx)} dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^2}{(a+x)(b^2+x^2)^2} dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{\cos^2(c + dx)(b + a \tan(c + dx))}{2(a^2 + b^2)d} - \frac{\operatorname{Subst}\left(\int \frac{-\frac{a^2b^2}{a^2+b^2} + \frac{ab^2x}{a^2+b^2}}{(a+x)(b^2+x^2)} dx, x, b \tan(c + dx)\right)}{2bd} \\ &= -\frac{\cos^2(c + dx)(b + a \tan(c + dx))}{2(a^2 + b^2)d} - \frac{\operatorname{Subst}\left(\int \left(-\frac{2a^2b^2}{(a^2+b^2)^2(a+x)} - \frac{ab^2(a^2-b^2-2ax)}{(a^2+b^2)^2(b^2+x^2)}\right) dx, x, b \tan(c + dx)\right)}{2bd} \\ &= \frac{a^2b \log(a + b \tan(c + dx))}{(a^2 + b^2)^2 d} - \frac{\cos^2(c + dx)(b + a \tan(c + dx))}{2(a^2 + b^2)d} + \frac{(ab) \operatorname{Subst}\left(\int \frac{a^2-b^2-2ax}{b^2+x^2} dx, x, b \tan(c + dx)\right)}{2(a^2 + b^2)^2 d} \\ &= \frac{a^2b \log(a + b \tan(c + dx))}{(a^2 + b^2)^2 d} - \frac{\cos^2(c + dx)(b + a \tan(c + dx))}{2(a^2 + b^2)d} - \frac{(a^2b) \operatorname{Subst}\left(\int \frac{x}{b^2+x^2} dx, x, b \tan(c + dx)\right)}{(a^2 + b^2)^2 d} \\ &= \frac{a(a^2 - b^2)x}{2(a^2 + b^2)^2} + \frac{a^2b \log(\cos(c + dx))}{(a^2 + b^2)^2 d} + \frac{a^2b \log(a + b \tan(c + dx))}{(a^2 + b^2)^2 d} - \frac{\cos^2(c + dx)(b + a \tan(c + dx))}{2(a^2 + b^2)d} \end{aligned}$$

Mathematica [A] time = 0.722595, size = 170, normalized size = 1.81

$$\frac{2b^2(a^2 + b^2)\cos^2(c + dx) + 2ab(a^2 + b^2)\tan^{-1}(\tan(c + dx)) + a\left(b(a^2 + b^2)\sin(2(c + dx)) + 2a\left(-2b^2\log(a + b \tan(c + dx))\right)\right)}{4bd(a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x]),x]

[Out] -(2*a*b*(a^2 + b^2)*ArcTan[Tan[c + d*x]] + 2*b^2*(a^2 + b^2)*Cos[c + d*x]^2 + a*(2*a*((b^2 + a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*b^2*Log[a + b*Tan[c + d*x]] + (b^2 - a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]]) + b*(a^2 + b^2)*Sin[2*(c + d*x)])/(4*b*(a^2 + b^2)^2*d)

Maple [B] time = 0.059, size = 238, normalized size = 2.5

$$\frac{a^3 \tan(dx + c)}{2d(a^2 + b^2)^2(1 + (\tan(dx + c))^2)} - \frac{a \tan(dx + c) b^2}{2d(a^2 + b^2)^2(1 + (\tan(dx + c))^2)} - \frac{ba^2}{2d(a^2 + b^2)^2(1 + (\tan(dx + c))^2)} - \frac{b^3}{2d(a^2 + b^2)^2(1 + (\tan(dx + c))^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(d*x+c)^2/(a+b*tan(d*x+c)),x)`

[Out]
$$\frac{-1/2/d/(a^2+b^2)^2/(1+\tan(d*x+c))^2*\tan(d*x+c)*a^3-1/2/d/(a^2+b^2)^2/(1+\tan(d*x+c))^2*\tan(d*x+c)*a*b^2-1/2/d/(a^2+b^2)^2/(1+\tan(d*x+c))^2*b*a^2-1/2/d/(a^2+b^2)^2/(1+\tan(d*x+c))^2*b^3-1/2/d/(a^2+b^2)^2*\ln(1+\tan(d*x+c))^2*a^2*b+1/2/d/(a^2+b^2)^2*\arctan(\tan(d*x+c))*a^3-1/2/d/(a^2+b^2)^2*\arctan(\tan(d*x+c))*a*b^2+1/d*b*a^2/(a^2+b^2)^2*\ln(a+b*\tan(d*x+c))}{2d}$$

Maxima [A] time = 1.5721, size = 194, normalized size = 2.06

$$\frac{\frac{2a^2b \log(b \tan(dx+c)+a)}{a^4+2a^2b^2+b^4} - \frac{a^2b \log(\tan(dx+c)^2+1)}{a^4+2a^2b^2+b^4} + \frac{(a^3-ab^2)(dx+c)}{a^4+2a^2b^2+b^4} - \frac{a \tan(dx+c)+b}{(a^2+b^2) \tan(dx+c)^2+a^2+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out]
$$\frac{1/2*(2*a^2*b*\log(b*\tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - a^2*b*\log(\tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 - a*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (a*\tan(d*x + c) + b)/((a^2 + b^2)*\tan(d*x + c)^2 + a^2 + b^2))/d}$$

Fricas [A] time = 2.12276, size = 278, normalized size = 2.96

$$\frac{a^2b \log(2ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 + b^2) + (a^3 - ab^2)dx - (a^2b + b^3) \cos(dx+c)^2 - (a^3 + b^3) \sin(dx+c)^2}{2(a^4 + 2a^2b^2 + b^4)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out]
$$\frac{1/2*(a^2*b*\log(2*a*b*\cos(d*x + c)*\sin(d*x + c) + (a^2 - b^2)*\cos(d*x + c)^2 + b^2) + (a^3 - a*b^2)*d*x - (a^2*b + b^3)*\cos(d*x + c)^2 - (a^3 + a*b^2)*\cos(d*x + c)*\sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)}$$

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)**2/(a+b*tan(d*x+c)),x)`

[Out] Exception raised: AttributeError

Giac [B] time = 1.14829, size = 248, normalized size = 2.64

$$\frac{\frac{2a^2b^2 \log(|b \tan(dx+c)+a|)}{a^4b+2a^2b^3+b^5} - \frac{a^2b \log(\tan(dx+c)^2+1)}{a^4+2a^2b^2+b^4} + \frac{(a^3-ab^2)(dx+c)}{a^4+2a^2b^2+b^4} + \frac{a^2b \tan(dx+c)^2 - a^3 \tan(dx+c) - ab^2 \tan(dx+c) - b^3}{(a^4+2a^2b^2+b^4)(\tan(dx+c)^2+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] 1/2*(2*a^2*b^2*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - a^2*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 - a*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (a^2*b*tan(d*x + c)^2 - a^3*tan(d*x + c) - a*b^2*tan(d*x + c) - b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)))/d

3.55 $\int \frac{\sin(c+dx)}{a+b \tan(c+dx)} dx$

Optimal. Leaf size=90

$$\frac{b \sin(c+dx)}{d(a^2+b^2)} - \frac{a \cos(c+dx)}{d(a^2+b^2)} + \frac{ab \tanh^{-1}\left(\frac{b \cos(c+dx) - a \sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}$$

[Out] (a*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) + (b*Sin[c + d*x])/((a^2 + b^2)*d)

Rubi [A] time = 0.105716, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3518, 3109, 2637, 2638, 3074, 206}

$$\frac{b \sin(c+dx)}{d(a^2+b^2)} - \frac{a \cos(c+dx)}{d(a^2+b^2)} + \frac{ab \tanh^{-1}\left(\frac{b \cos(c+dx) - a \sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]/(a + b*Tan[c + d*x]), x]

[Out] (a*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) + (b*Sin[c + d*x])/((a^2 + b^2)*d)

Rule 3518

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3109

Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3074

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(-1), x_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\sin(c+dx)}{a+b \tan(c+dx)} dx &= \int \frac{\cos(c+dx) \sin(c+dx)}{a \cos(c+dx) + b \sin(c+dx)} dx \\ &= \frac{a \int \sin(c+dx) dx}{a^2+b^2} + \frac{b \int \cos(c+dx) dx}{a^2+b^2} - \frac{(ab) \int \frac{1}{a \cos(c+dx)+b \sin(c+dx)} dx}{a^2+b^2} \\ &= -\frac{a \cos(c+dx)}{(a^2+b^2)d} + \frac{b \sin(c+dx)}{(a^2+b^2)d} + \frac{(ab) \text{Subst}\left(\int \frac{1}{a^2+b^2-x^2} dx, x, b \cos(c+dx) - a \sin(c+dx)\right)}{(a^2+b^2)d} \\ &= \frac{ab \tanh^{-1}\left(\frac{b \cos(c+dx) - a \sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} - \frac{a \cos(c+dx)}{(a^2+b^2)d} + \frac{b \sin(c+dx)}{(a^2+b^2)d} \end{aligned}$$

Mathematica [A] time = 0.336073, size = 79, normalized size = 0.88

$$\frac{\sqrt{a^2+b^2}(b \sin(c+dx) - a \cos(c+dx)) - 2ab \tanh^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) - b}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]/(a + b*Tan[c + d*x]), x]

[Out] (-2*a*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2]*(-(a*Cos[c + d*x]) + b*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)

Maple [A] time = 0.05, size = 100, normalized size = 1.1

$$\frac{1}{d} \left(-2 \frac{-\tan(1/2 dx + c/2) b + a}{(a^2 + b^2) (1 + (\tan(1/2 dx + c/2))^2)} - 4 \frac{ab}{(2a^2 + 2b^2) \sqrt{a^2 + b^2}} \text{Artanh} \left(\frac{1}{2} \frac{2a \tan(1/2 dx + c/2) - 2b}{\sqrt{a^2 + b^2}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)/(a+b*tan(d*x+c)), x)

[Out] 1/d*(-2/(a^2+b^2)*(-tan(1/2*d*x+1/2*c)*b+a)/(1+tan(1/2*d*x+1/2*c)^2)-4*a*b/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.0465, size = 435, normalized size = 4.83

$$\frac{\sqrt{a^2 + b^2} ab \log\left(\frac{2ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 - 2a^2 - b^2 - 2\sqrt{a^2 + b^2}(b \cos(dx+c) - a \sin(dx+c))}{2ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 + b^2}\right) - 2(a^3 + ab^2) \cos(dx+c) + 2(a^4 + 2a^2b^2 + b^4)d}{2(a^4 + 2a^2b^2 + b^4)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{2}(\sqrt{a^2 + b^2})ab \log\left(\frac{(2ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 - 2a^2 - b^2 - 2\sqrt{a^2 + b^2}(b \cos(dx+c) - a \sin(dx+c)))}{(2ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 + b^2)}\right) - 2(a^3 + ab^2) \cos(dx+c) + 2(a^4 + 2a^2b^2 + b^4)d$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c + dx)}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x)

[Out] Integral(sin(c + d*x)/(a + b*tan(c + d*x)), x)

Giac [A] time = 1.33428, size = 159, normalized size = 1.77

$$\frac{ab \log\left(\frac{\left|2a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2b - 2\sqrt{a^2 + b^2}\right|}{\left|2a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2b + 2\sqrt{a^2 + b^2}\right|}\right)}{(a^2 + b^2)^{\frac{3}{2}}} + \frac{2\left(b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - a\right)}{(a^2 + b^2)\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1\right)} \cdot d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] $(a*b*\log(\text{abs}(2*a*\tan(1/2*d*x + 1/2*c) - 2*b - 2*\sqrt{a^2 + b^2}))/\text{abs}(2*a*\tan(1/2*d*x + 1/2*c) - 2*b + 2*\sqrt{a^2 + b^2}))/((a^2 + b^2)^{(3/2)} + 2*(b*\tan(1/2*d*x + 1/2*c) - a)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d$

$$3.56 \quad \int \frac{\csc(c+dx)}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=66

$$\frac{b \tanh^{-1}\left(\frac{b \cos(c+dx) - a \sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}} - \frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + (b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d)

Rubi [A] time = 0.128764, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3518, 3110, 3770, 3074, 206}

$$\frac{b \tanh^{-1}\left(\frac{b \cos(c+dx) - a \sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}} - \frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]/(a + b*Tan[c + d*x]),x]

[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + (b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d)

Rule 3518

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3110

Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3074

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]^(-1), x_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{a+b\tan(c+dx)} dx &= \int \frac{\cot(c+dx)}{a\cos(c+dx)+b\sin(c+dx)} dx \\
&= \int \left(\frac{\csc(c+dx)}{a} - \frac{b}{a(a\cos(c+dx)+b\sin(c+dx))} \right) dx \\
&= \frac{\int \csc(c+dx) dx}{a} - \frac{b \int \frac{1}{a\cos(c+dx)+b\sin(c+dx)} dx}{a} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{b \operatorname{Subst}\left(\int \frac{1}{a^2+b^2-x^2} dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{ad} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{b \tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}
\end{aligned}$$

Mathematica [A] time = 0.105158, size = 75, normalized size = 1.14

$$\frac{2b \tanh^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) - b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]/(a + b*Tan[c + d*x]), x]

[Out] ((-2*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])/(a*d)

Maple [A] time = 0.056, size = 65, normalized size = 1.

$$-2 \frac{b}{ad\sqrt{a^2+b^2}} \operatorname{Artanh}\left(\frac{1}{2} \frac{2a \tan(1/2 dx + c/2) - 2b}{\sqrt{a^2+b^2}}\right) + \frac{1}{ad} \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)/(a+b*tan(d*x+c)), x)

[Out] -2/d*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+1/d/a*ln(tan(1/2*d*x+1/2*c))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.37691, size = 450, normalized size = 6.82

$$\frac{\sqrt{a^2 + b^2} b \log\left(\frac{2ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 - 2a^2 - b^2 - 2\sqrt{a^2 + b^2}(b \cos(dx+c) - a \sin(dx+c))}{2ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 + b^2}\right) - (a^2 + b^2) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right)}{2(a^3 + ab^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(sqrt(a^2 + b^2)*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - (a^2 + b^2)*log(1/2*cos(d*x + c) + 1/2) + (a^2 + b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3 + a*b^2)*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(c + dx)}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x)

[Out] Integral(csc(c + d*x)/(a + b*tan(c + d*x)), x)

Giac [A] time = 1.43594, size = 127, normalized size = 1.92

$$\frac{b \log\left(\frac{\left|2a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2b - 2\sqrt{a^2 + b^2}\right|}{\left|2a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2b + 2\sqrt{a^2 + b^2}\right|}\right)}{\sqrt{a^2 + b^2} a} + \frac{\log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] (b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2)))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(abs(tan(1/2*d*x + 1/2*c)))/a)/d

$$3.57 \quad \int \frac{\csc^2(c+dx)}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=50

$$-\frac{b \log(\tan(c+dx))}{a^2 d} + \frac{b \log(a+b \tan(c+dx))}{a^2 d} - \frac{\cot(c+dx)}{ad}$$

[Out] $-(\text{Cot}[c + d*x]/(a*d)) - (b*\text{Log}[\text{Tan}[c + d*x]])/(a^2*d) + (b*\text{Log}[a + b*\text{Tan}[c + d*x]])/(a^2*d)$

Rubi [A] time = 0.0602185, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 44}

$$-\frac{b \log(\tan(c+dx))}{a^2 d} + \frac{b \log(a+b \tan(c+dx))}{a^2 d} - \frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c + d*x]^2/(a + b*\text{Tan}[c + d*x]), x]$

[Out] $-(\text{Cot}[c + d*x]/(a*d)) - (b*\text{Log}[\text{Tan}[c + d*x]])/(a^2*d) + (b*\text{Log}[a + b*\text{Tan}[c + d*x]])/(a^2*d)$

Rule 3516

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] := \text{Dist}[b/f, \text{Subst}[\text{Int}[(x^m*(a+x)^n)/(b^2 + x^2)^{(m/2 + 1)}], x], x, b*\text{Tan}[e + f*x], x] /; \text{FreeQ}[\{a, b, e, f, n\}, x] \&\& \text{IntegerQ}[m/2]$

Rule 44

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{ILtQ}[m, 0] \&\& \text{IntegerQ}[n] \&\& !(\text{IGtQ}[n, 0] \&\& \text{LtQ}[m + n + 2, 0])$

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(c+dx)}{a+b \tan(c+dx)} dx &= \frac{b \text{Subst}\left(\int \frac{1}{x^2(a+x)} dx, x, b \tan(c+dx)\right)}{d} \\ &= \frac{b \text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{1}{a^2x} + \frac{1}{a^2(a+x)}\right) dx, x, b \tan(c+dx)\right)}{d} \\ &= -\frac{\cot(c+dx)}{ad} - \frac{b \log(\tan(c+dx))}{a^2 d} + \frac{b \log(a+b \tan(c+dx))}{a^2 d} \end{aligned}$$

Mathematica [A] time = 0.130935, size = 47, normalized size = 0.94

$$\frac{b(\log(a \cos(c+dx) + b \sin(c+dx)) - \log(\sin(c+dx))) - a \cot(c+dx)}{a^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x]),x]

[Out] $(-(a*\cot[c + d*x]) + b*(-\log[\sin[c + d*x]] + \log[a*\cos[c + d*x] + b*\sin[c + d*x]]))/a^2*d$

Maple [A] time = 0.062, size = 53, normalized size = 1.1

$$-\frac{1}{ad \tan(dx + c)} - \frac{b \ln(\tan(dx + c))}{a^2 d} + \frac{b \ln(a + b \tan(dx + c))}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c)),x)

[Out] $-1/d/a/\tan(d*x+c)-b*\ln(\tan(d*x+c))/a^2/d+b*\ln(a+b*\tan(d*x+c))/a^2/d$

Maxima [A] time = 1.06775, size = 63, normalized size = 1.26

$$\frac{\frac{b \log(b \tan(dx+c)+a)}{a^2} - \frac{b \log(\tan(dx+c))}{a^2} - \frac{1}{a \tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] $(b*\log(b*\tan(d*x + c) + a)/a^2 - b*\log(\tan(d*x + c))/a^2 - 1/(a*\tan(d*x + c)))/d$

Fricas [A] time = 2.05537, size = 246, normalized size = 4.92

$$\frac{b \log\left(2ab \cos(dx + c) \sin(dx + c) + (a^2 - b^2) \cos(dx + c)^2 + b^2\right) \sin(dx + c) - b \log\left(-\frac{1}{4} \cos(dx + c)^2 + \frac{1}{4}\right) \sin(dx + c)}{2a^2d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] $1/2*(b*\log(2*a*b*\cos(d*x + c)*\sin(d*x + c) + (a^2 - b^2)*\cos(d*x + c)^2 + b^2)*\sin(d*x + c) - b*\log(-1/4*\cos(d*x + c)^2 + 1/4)*\sin(d*x + c) - 2*a*\cos(d*x + c))/(a^2*d*\sin(d*x + c))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(c + dx)}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

Giac [A] time = 1.20827, size = 81, normalized size = 1.62

$$\frac{\frac{b \log(b \tan(dx+c)+a)}{a^2} - \frac{b \log(|\tan(dx+c)|)}{a^2} + \frac{b \tan(dx+c)-a}{a^2 \tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(b*tan(d*x + c) + a))/a^2 - b*log(abs(tan(d*x + c)))/a^2 + (b*tan
(d*x + c) - a)/(a^2*tan(d*x + c)))/d
```

3.58 $\int \frac{\csc^3(c+dx)}{a+b \tan(c+dx)} dx$

Optimal. Leaf size=122

$$-\frac{b^2 \tanh^{-1}(\cos(c+dx))}{a^3 d} + \frac{b\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b \cos(c+dx)-a \sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3 d} + \frac{b \csc(c+dx)}{a^2 d} - \frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)}{2a}$$

[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) - (b^2*ArcTanh[Cos[c + d*x]])/(a^3*d) + (b*sqrt[a^2 + b^2]*ArcTanh[(b*cos[c + d*x] - a*sin[c + d*x])/Sqrt[a^2 + b^2]])/(a^3*d) + (b*Csc[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)

Rubi [A] time = 0.307234, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3518, 3110, 3768, 3770, 2621, 321, 207, 2622, 3104, 3074, 206}

$$-\frac{b^2 \tanh^{-1}(\cos(c+dx))}{a^3 d} + \frac{b\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b \cos(c+dx)-a \sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3 d} + \frac{b \csc(c+dx)}{a^2 d} - \frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^3/(a + b*Tan[c + d*x]),x]

[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) - (b^2*ArcTanh[Cos[c + d*x]])/(a^3*d) + (b*sqrt[a^2 + b^2]*ArcTanh[(b*cos[c + d*x] - a*sin[c + d*x])/Sqrt[a^2 + b^2]])/(a^3*d) + (b*Csc[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)

Rule 3518

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*cos[e + f*x] + b*sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3110

Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> -Simp[(b*cos[c + d*x]*(b*csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 2621

Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n +

1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 321

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2622

Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_)), x_Symbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 3104

Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b^2, Int[Cos[c + d*x]^(m + 2)/(a*cos[c + d*x] + b*sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3074

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]^(-1), x_Symbol] := -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*cos[c + d*x] - a*sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)} dx &= \int \frac{\cot(c+dx)\csc^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)} dx \\
&= \int \left(\frac{\csc^3(c+dx)}{a} - \frac{b\csc^2(c+dx)\sec(c+dx)}{a^2} + \frac{b^2\csc(c+dx)\sec^2(c+dx)}{a^3} - \frac{b^3\sec^3(c+dx)}{a^3(a\cos(c+dx)+b\sin(c+dx))} \right) dx \\
&= \frac{\int \csc^3(c+dx) dx}{a} - \frac{b \int \csc^2(c+dx)\sec(c+dx) dx}{a^2} + \frac{b^2 \int \csc(c+dx)\sec^2(c+dx) dx}{a^3} - \frac{b^3 \int \sec^3(c+dx) dx}{a^3(a\cos(c+dx)+b\sin(c+dx))} \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{b^2\sec(c+dx)}{a^3d} + \frac{\int \csc(c+dx) dx}{2a} + \frac{b \int \sec(c+dx) dx}{a^2} - \frac{b^3 \int \sec^3(c+dx) dx}{a^3(a\cos(c+dx)+b\sin(c+dx))} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} + \frac{b \tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{b \csc(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{b^3 \int \sec^3(c+dx) dx}{a^3(a\cos(c+dx)+b\sin(c+dx))} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2 \tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3d} + \frac{b^3 \int \sec^3(c+dx) dx}{a^3(a\cos(c+dx)+b\sin(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.826587, size = 179, normalized size = 1.47

$$-16b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)+a^2\left(-\csc^2\left(\frac{1}{2}(c+dx)\right)\right)+a^2\sec^2\left(\frac{1}{2}(c+dx)\right)+4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4a^2$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^3/(a + b*Tan[c + d*x]),x]

[Out] (-16*b*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + 4*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 - 4*a^2*Log[Cos[(c + d*x)/2]] - 8*b^2*Log[Cos[(c + d*x)/2]] + 4*a^2*Log[Sin[(c + d*x)/2]] + 8*b^2*Log[Sin[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 + 4*a*b*Tan[(c + d*x)/2])/(8*a^3*d)

Maple [A] time = 0.069, size = 162, normalized size = 1.3

$$\frac{1}{8ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{b}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\frac{b\sqrt{a^2+b^2}}{da^3}\operatorname{Arctanh}\left(\frac{1}{2}\frac{2a\tan\left(\frac{1}{2}dx+\frac{c}{2}\right)-2b}{\sqrt{a^2+b^2}}\right)-\frac{1}{8ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^3/(a+b*tan(d*x+c)),x)

[Out] 1/8/d/a*tan(1/2*d*x+1/2*c)^2+1/2/d/a^2*tan(1/2*d*x+1/2*c)*b-2/d/a^3*b*(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/8/d/a/tan(1/2*d*x+1/2*c)^2+1/2/d/a*ln(tan(1/2*d*x+1/2*c))+1/d/a^3*ln(tan(1/2*d*x+1/2*c))*b^2+1/2/d*b/a^2/tan(1/2*d*x+1/2*c)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.68281, size = 644, normalized size = 5.28

$$2a^2 \cos(dx+c) - 4ab \sin(dx+c) + 2(b \cos(dx+c)^2 - b) \sqrt{a^2 + b^2} \log\left(\frac{2ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 - 2a^2 - b^2 - 2}{2ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 - 2a^2 - b^2 - 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{4} * (2 * a^2 * \cos(d * x + c) - 4 * a * b * \sin(d * x + c) + 2 * (b * \cos(d * x + c)^2 - b) * \sqrt{a^2 + b^2} * \log((2 * a * b * \cos(d * x + c) * \sin(d * x + c) + (a^2 - b^2) * \cos(d * x + c)^2 - 2 * a^2 - b^2 - 2 * \sqrt{a^2 + b^2} * (b * \cos(d * x + c) - a * \sin(d * x + c)))) / (2 * a * b * \cos(d * x + c) * \sin(d * x + c) + (a^2 - b^2) * \cos(d * x + c)^2 + b^2) - ((a^2 + 2 * b^2) * \cos(d * x + c)^2 - a^2 - 2 * b^2) * \log(1/2 * \cos(d * x + c) + 1/2) + ((a^2 + 2 * b^2) * \cos(d * x + c)^2 - a^2 - 2 * b^2) * \log(-1/2 * \cos(d * x + c) + 1/2)) / (a^3 * d * \cos(d * x + c)^2 - a^3 * d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(c + dx)}{a + b \tan(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**3/(a+b*tan(d*x+c)),x)

[Out] Integral(csc(c + d*x)**3/(a + b*tan(c + d*x)), x)

Giac [A] time = 1.35144, size = 282, normalized size = 2.31

$$\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 4b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^2} + \frac{4(a^2 + 2b^2) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^3} + \frac{8(a^2 b + b^3) \log\left(\frac{|2a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2b - 2\sqrt{a^2 + b^2}|}{|2a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2b + 2\sqrt{a^2 + b^2}|}\right)}{\sqrt{a^2 + b^2} a^3} - \frac{6a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 12ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{8} * ((a * \tan(1/2 * d * x + 1/2 * c))^2 + 4 * b * \tan(1/2 * d * x + 1/2 * c)) / a^2 + 4 * (a^2 + 2 * b^2) * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c))) / a^3 + 8 * (a^2 * b + b^3) * \log(\text{abs}(2 * a * \tan(1/2 * d * x + 1/2 * c) - 2 * b - 2 * \sqrt{a^2 + b^2}) / \text{abs}(2 * a * \tan(1/2 * d * x + 1/2 * c) - 2 * b + 2 * \sqrt{a^2 + b^2})) / (\sqrt{a^2 + b^2} * a^3) - (6 * a^2 * \tan(1/2 * d * x + 1/2 * c)^2 + 12 * b^2 * \tan(1/2 * d * x + 1/2 * c)^2 - 4 * a * b * \tan(1/2 * d * x + 1/2 * c) + a^2) / (a^3 * \tan(1/2 * d * x + 1/2 * c)^2) / d$

$$3.59 \quad \int \frac{\csc^4(c+dx)}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=108

$$-\frac{(a^2 + b^2) \cot(c + dx)}{a^3 d} - \frac{b(a^2 + b^2) \log(\tan(c + dx))}{a^4 d} + \frac{b(a^2 + b^2) \log(a + b \tan(c + dx))}{a^4 d} + \frac{b \cot^2(c + dx)}{2a^2 d} - \frac{\cot^3(c + dx)}{3ad}$$

[Out] -(((a^2 + b^2)*Cot[c + d*x])/(a^3*d)) + (b*Cot[c + d*x]^2)/(2*a^2*d) - Cot[c + d*x]^3/(3*a*d) - (b*(a^2 + b^2)*Log[Tan[c + d*x]])/(a^4*d) + (b*(a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(a^4*d)

Rubi [A] time = 0.104453, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 894}

$$-\frac{(a^2 + b^2) \cot(c + dx)}{a^3 d} - \frac{b(a^2 + b^2) \log(\tan(c + dx))}{a^4 d} + \frac{b(a^2 + b^2) \log(a + b \tan(c + dx))}{a^4 d} + \frac{b \cot^2(c + dx)}{2a^2 d} - \frac{\cot^3(c + dx)}{3ad}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x]),x]

[Out] -(((a^2 + b^2)*Cot[c + d*x])/(a^3*d)) + (b*Cot[c + d*x]^2)/(2*a^2*d) - Cot[c + d*x]^3/(3*a*d) - (b*(a^2 + b^2)*Log[Tan[c + d*x]])/(a^4*d) + (b*(a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(a^4*d)

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.)), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 894

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{\csc^4(c + dx)}{a + b \tan(c + dx)} dx &= \frac{b \operatorname{Subst}\left(\int \frac{b^2 + x^2}{x^4(a + x)} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{b \operatorname{Subst}\left(\int \left(\frac{b^2}{ax^4} - \frac{b^2}{a^2x^3} + \frac{a^2 + b^2}{a^3x^2} + \frac{-a^2 - b^2}{a^4x} + \frac{a^2 + b^2}{a^4(a + x)}\right) dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{(a^2 + b^2) \cot(c + dx)}{a^3 d} + \frac{b \cot^2(c + dx)}{2a^2 d} - \frac{\cot^3(c + dx)}{3ad} - \frac{b(a^2 + b^2) \log(\tan(c + dx))}{a^4 d} + \frac{b(a^2 + b^2) \log(a + b \tan(c + dx))}{a^4 d} \end{aligned}$$

Mathematica [A] time = 0.455689, size = 95, normalized size = 0.88

$$\frac{-2 \cot(c + dx) (a^3 \csc^2(c + dx) + 2a^3 + 3ab^2) - 6b(a^2 + b^2) (\log(\sin(c + dx)) - \log(a \cos(c + dx) + b \sin(c + dx))) + 3a^2}{6a^4 d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x]),x]

[Out] $(3a^2b\text{Csc}[c + dx]^2 - 2\text{Cot}[c + dx]*(2a^3 + 3ab^2 + a^3\text{Csc}[c + dx]^2) - 6b*(a^2 + b^2)*(Log[\text{Sin}[c + dx]] - Log[a\text{Cos}[c + dx] + b\text{Sin}[c + dx]]))/(6a^4d)$

Maple [A] time = 0.076, size = 144, normalized size = 1.3

$$\frac{1}{3ad(\tan(dx+c))^3} - \frac{1}{ad\tan(dx+c)} - \frac{b^2}{da^3\tan(dx+c)} + \frac{b}{2a^2d(\tan(dx+c))^2} - \frac{b\ln(\tan(dx+c))}{a^2d} - \frac{b^3\ln(\tan(dx+c))}{da^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c)),x)

[Out] $-1/3/d/a/\tan(dx+c)^3 - 1/d/a/\tan(dx+c) - 1/d/a^3/\tan(dx+c)*b^2 + 1/2/d*b/a^2/\tan(dx+c)^2 - b*\ln(\tan(dx+c))/a^2/d - 1/d/a^4*b^3*\ln(\tan(dx+c)) + b*\ln(a+b*\tan(dx+c))/a^2/d + 1/d/a^4*b^3*\ln(a+b*\tan(dx+c))$

Maxima [A] time = 1.06377, size = 131, normalized size = 1.21

$$\frac{\frac{6(a^2b+b^3)\log(b\tan(dx+c)+a)}{a^4} - \frac{6(a^2b+b^3)\log(\tan(dx+c))}{a^4} + \frac{3ab\tan(dx+c) - 6(a^2+b^2)\tan(dx+c)^2 - 2a^2}{a^3\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] $1/6*(6*(a^2*b + b^3)*\log(b*\tan(dx + c) + a)/a^4 - 6*(a^2*b + b^3)*\log(\tan(dx + c))/a^4 + (3*a*b*\tan(dx + c) - 6*(a^2 + b^2)*\tan(dx + c)^2 - 2*a^2)/(a^3*\tan(dx + c)^3))/d$

Fricas [B] time = 2.15412, size = 500, normalized size = 4.63

$$2(2a^3 + 3ab^2)\cos(dx+c)^3 + 3a^2b\sin(dx+c) + 3(a^2b + b^3 - (a^2b + b^3)\cos(dx+c)^2)\log(2ab\cos(dx+c)\sin(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] $-1/6*(2*(2a^3 + 3ab^2)*\cos(dx + c)^3 + 3a^2b*\sin(dx + c) + 3*(a^2*b + b^3 - (a^2*b + b^3)*\cos(dx + c)^2)*\log(2*a*b*\cos(dx + c)*\sin(dx + c) + (a^2 - b^2)*\cos(dx + c)^2 + b^2)*\sin(dx + c) - 3*(a^2*b + b^3 - (a^2*b + b^3)*\cos(dx + c)^2)*\log(-1/4*\cos(dx + c)^2 + 1/4)*\sin(dx + c) - 6*(a^3 + a*b^2)*\cos(dx + c))/(a^4*d*\cos(dx + c)^2 - a^4*d*\sin(dx + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c)),x)

[Out] Timed out

Giac [A] time = 1.18105, size = 194, normalized size = 1.8

$$\frac{6(a^2b+b^3)\log(|\tan(dx+c)|)}{a^4} - \frac{6(a^2b^2+b^4)\log(|b\tan(dx+c)+a|)}{a^4b} - \frac{11a^2b\tan(dx+c)^3+11b^3\tan(dx+c)^3-6a^3\tan(dx+c)^2-6ab^2\tan(dx+c)^2+3a^2b\tan(dx+c)^2-2a^3}{a^4\tan(dx+c)^3} \cdot \frac{1}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")

[Out]
$$-1/6*(6*(a^2*b + b^3)*\log(\text{abs}(\tan(d*x + c)))/a^4 - 6*(a^2*b^2 + b^4)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^4*b) - (11*a^2*b*\tan(d*x + c)^3 + 11*b^3*\tan(d*x + c)^3 - 6*a^3*\tan(d*x + c)^2 - 6*a*b^2*\tan(d*x + c)^2 + 3*a^2*b*\tan(d*x + c)^2 - 2*a^3)/(a^4*\tan(d*x + c)^3))/d$$

$$3.60 \quad \int \frac{\csc^6(c+dx)}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=169

$$-\frac{(2a^2+b^2)\cot^3(c+dx)}{3a^3d} + \frac{b(2a^2+b^2)\cot^2(c+dx)}{2a^4d} - \frac{(a^2+b^2)^2\cot(c+dx)}{a^5d} - \frac{b(a^2+b^2)^2\log(\tan(c+dx))}{a^6d} + \frac{b(a^2+b^2)^2\log(\tan(c+dx))}{a^6d}$$

[Out] -(((a^2 + b^2)^2*Cot[c + d*x])/(a^5*d)) + (b*(2*a^2 + b^2)*Cot[c + d*x]^2)/(2*a^4*d) - ((2*a^2 + b^2)*Cot[c + d*x]^3)/(3*a^3*d) + (b*Cot[c + d*x]^4)/(4*a^2*d) - Cot[c + d*x]^5/(5*a*d) - (b*(a^2 + b^2)^2*Log[Tan[c + d*x]])/(a^6*d) + (b*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(a^6*d)

Rubi [A] time = 0.150783, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 894}

$$-\frac{(2a^2+b^2)\cot^3(c+dx)}{3a^3d} + \frac{b(2a^2+b^2)\cot^2(c+dx)}{2a^4d} - \frac{(a^2+b^2)^2\cot(c+dx)}{a^5d} - \frac{b(a^2+b^2)^2\log(\tan(c+dx))}{a^6d} + \frac{b(a^2+b^2)^2\log(\tan(c+dx))}{a^6d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x]),x]

[Out] -(((a^2 + b^2)^2*Cot[c + d*x])/(a^5*d)) + (b*(2*a^2 + b^2)*Cot[c + d*x]^2)/(2*a^4*d) - ((2*a^2 + b^2)*Cot[c + d*x]^3)/(3*a^3*d) + (b*Cot[c + d*x]^4)/(4*a^2*d) - Cot[c + d*x]^5/(5*a*d) - (b*(a^2 + b^2)^2*Log[Tan[c + d*x]])/(a^6*d) + (b*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(a^6*d)

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1)], x], x, b*Tan[e + f*x], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 894

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{\csc^6(c+dx)}{a+b \tan(c+dx)} dx &= \frac{b \operatorname{Subst}\left(\int \frac{(b^2+x^2)^2}{x^6(a+x)} dx, x, b \tan(c+dx)\right)}{d} \\ &= \frac{b \operatorname{Subst}\left(\int \left(\frac{b^4}{ax^6} - \frac{b^4}{a^2x^5} + \frac{2a^2b^2+b^4}{a^3x^4} + \frac{b^2(-2a^2-b^2)}{a^4x^3} + \frac{(a^2+b^2)^2}{a^5x^2} - \frac{(a^2+b^2)^2}{a^6x} + \frac{(a^2+b^2)^2}{a^6(a+x)}\right) dx, x, b \tan(c+dx)\right)}{d} \\ &= -\frac{(a^2+b^2)^2\cot(c+dx)}{a^5d} + \frac{b(2a^2+b^2)\cot^2(c+dx)}{2a^4d} - \frac{(2a^2+b^2)\cot^3(c+dx)}{3a^3d} + \frac{b\cot^4(c+dx)}{4a^2d} \end{aligned}$$

Mathematica [A] time = 2.16387, size = 150, normalized size = 0.89

$$\frac{15b \left(2a^2 (a^2 + b^2) \csc^2(c + dx) - 4(a^2 + b^2)^2 (\log(\sin(c + dx)) - \log(a \cos(c + dx) + b \sin(c + dx))) + a^4 \csc^4(c + dx) \right)}{60a^6d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x]),x]

[Out] (-4*Cot[c + d*x]*(8*a^5 + 25*a^3*b^2 + 15*a*b^4 + a^3*(4*a^2 + 5*b^2)*Csc[c + d*x]^2 + 3*a^5*Csc[c + d*x]^4) + 15*b*(2*a^2*(a^2 + b^2)*Csc[c + d*x]^2 + a^4*Csc[c + d*x]^4 - 4*(a^2 + b^2)^2*(Log[Sin[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c + d*x]])))/(60*a^6*d)

Maple [A] time = 0.083, size = 273, normalized size = 1.6

$$\frac{1}{5ad(\tan(dx+c))^5} - \frac{2}{3ad(\tan(dx+c))^3} - \frac{b^2}{3da^3(\tan(dx+c))^3} - \frac{1}{ad \tan(dx+c)} - 2 \frac{b^2}{da^3 \tan(dx+c)} - \frac{b^4}{da^5 \tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c)),x)

[Out] -1/5/d/a/tan(d*x+c)^5-2/3/d/a/tan(d*x+c)^3-1/3/d/a^3/tan(d*x+c)^3*b^2-1/d/a/tan(d*x+c)-2/d/a^3/tan(d*x+c)*b^2-1/d/a^5/tan(d*x+c)*b^4+1/4/d*b/a^2/tan(d*x+c)^4+1/d*b/a^2/tan(d*x+c)^2+1/2/d/a^4*b^3/tan(d*x+c)^2-b*ln(tan(d*x+c))/a^2/d-2/d/a^4*b^3*ln(tan(d*x+c))-1/d/a^6*b^5*ln(tan(d*x+c))+b*ln(a+b*tan(d*x+c))/a^2/d+2/d/a^4*b^3*ln(a+b*tan(d*x+c))+1/d/a^6*b^5*ln(a+b*tan(d*x+c))

Maxima [A] time = 1.09497, size = 227, normalized size = 1.34

$$\frac{60(a^4b+2a^2b^3+b^5)\log(b\tan(dx+c)+a)}{a^6} - \frac{60(a^4b+2a^2b^3+b^5)\log(\tan(dx+c))}{a^6} + \frac{15a^3b\tan(dx+c)-60(a^4+2a^2b^2+b^4)\tan(dx+c)^4-12a^4+30(2a^3b+ab^3)\tan(dx+c)}{a^5 \tan(dx+c)^5}$$

60 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] 1/60*(60*(a^4*b + 2*a^2*b^3 + b^5)*log(b*tan(d*x + c) + a)/a^6 - 60*(a^4*b + 2*a^2*b^3 + b^5)*log(tan(d*x + c))/a^6 + (15*a^3*b*tan(d*x + c) - 60*(a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^4 - 12*a^4 + 30*(2*a^3*b + a*b^3)*tan(d*x + c)^3 - 20*(2*a^4 + a^2*b^2)*tan(d*x + c)^2)/(a^5*tan(d*x + c)^5)/d

Fricas [B] time = 2.41224, size = 902, normalized size = 5.34

$$4(8a^5 + 25a^3b^2 + 15ab^4) \cos(dx + c)^5 - 20(4a^5 + 11a^3b^2 + 6ab^4) \cos(dx + c)^3 - 30(a^4b + 2a^2b^3 + b^5 + (a^4b + 2a^2b^3 + b^5) \tan(dx + c)^2) \sin(dx + c)^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out]
$$-1/60*(4*(8*a^5 + 25*a^3*b^2 + 15*a*b^4)*\cos(d*x + c)^5 - 20*(4*a^5 + 11*a^3*b^2 + 6*a*b^4)*\cos(d*x + c)^3 - 30*(a^4*b + 2*a^2*b^3 + b^5 + (a^4*b + 2*a^2*b^3 + b^5)*\cos(d*x + c)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5)*\cos(d*x + c)^2) * \log(2*a*b*\cos(d*x + c)*\sin(d*x + c) + (a^2 - b^2)*\cos(d*x + c)^2 + b^2)*\sin(d*x + c) + 30*(a^4*b + 2*a^2*b^3 + b^5 + (a^4*b + 2*a^2*b^3 + b^5)*\cos(d*x + c)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5)*\cos(d*x + c)^2) * \log(-1/4*\cos(d*x + c)^2 + 1/4)*\sin(d*x + c) + 60*(a^5 + 2*a^3*b^2 + a*b^4)*\cos(d*x + c) - 15*(3*a^4*b + 2*a^2*b^3 - 2*(a^4*b + a^2*b^3)*\cos(d*x + c)^2)*\sin(d*x + c))/((a^6*d*\cos(d*x + c)^4 - 2*a^6*d*\cos(d*x + c)^2 + a^6*d)*\sin(d*x + c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c)),x)

[Out] Timed out

Giac [A] time = 1.24413, size = 339, normalized size = 2.01

$$\frac{60(a^4b+2a^2b^3+b^5)\log(|\tan(dx+c)|)}{a^6} - \frac{60(a^4b^2+2a^2b^4+b^6)\log(|b\tan(dx+c)+a|)}{a^6b} - \frac{137a^4b\tan(dx+c)^5+274a^2b^3\tan(dx+c)^5+137b^5\tan(dx+c)^5-60a^6}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="giac")

[Out]
$$-1/60*(60*(a^4*b + 2*a^2*b^3 + b^5)*\log(\text{abs}(\tan(d*x + c)))/a^6 - 60*(a^4*b^2 + 2*a^2*b^4 + b^6)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^6*b) - (137*a^4*b*\tan(d*x + c)^5 + 274*a^2*b^3*\tan(d*x + c)^5 + 137*b^5*\tan(d*x + c)^5 - 60*a^5*\tan(d*x + c)^4 - 120*a^3*b^2*\tan(d*x + c)^4 - 60*a*b^4*\tan(d*x + c)^4 + 60*a^4*b*\tan(d*x + c)^3 + 30*a^2*b^3*\tan(d*x + c)^3 - 40*a^5*\tan(d*x + c)^2 - 20*a^3*b^2*\tan(d*x + c)^2 + 15*a^4*b*\tan(d*x + c) - 12*a^5)/(a^6*\tan(d*x + c)^5))/d$$

3.61 $\int \frac{\sin^6(c+dx)}{(a+b \tan(c+dx))^2} dx$

Optimal. Leaf size=297

$$-\frac{a^6 b}{d(a^2 + b^2)^4 (a + b \tan(c + dx))} - \frac{\cos^6(c + dx) \left((a^2 - b^2) \tan(c + dx) + 2ab \right)}{6d(a^2 + b^2)^2} + \frac{\cos^4(c + dx) \left((-18a^2 b^2 + 13a^4 - 7b^4) \tan(c + dx) + 12ab(a^2 - b^2) \right)}{24d(a^2 + b^2)^3}$$

[Out] $((5a^8 - 80a^6b^2 + 50a^4b^4 + 8a^2b^6 + b^8)x)/(16(a^2 + b^2)^5) + (2a^5b(a^2 - 3b^2) \text{Log}[a \text{Cos}[c + dx] + b \text{Sin}[c + dx]])/(a^2 + b^2)^5d - (a^6b)/((a^2 + b^2)^4d(a + b \text{Tan}[c + dx])) - (\text{Cos}[c + dx]^6(2ab + (a^2 - b^2) \text{Tan}[c + dx]))/(6(a^2 + b^2)^2d) + (\text{Cos}[c + dx]^4(12ab(3a^2 + b^2) + (13a^4 - 18a^2b^2 - 7b^4) \text{Tan}[c + dx]))/(24(a^2 + b^2)^3d) - (\text{Cos}[c + dx]^2(48a^5b + (11a^6 - 43a^4b^2 - 7a^2b^4 - b^6) \text{Tan}[c + dx]))/(16(a^2 + b^2)^4d)$

Rubi [A] time = 0.913641, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^6 b}{d(a^2 + b^2)^4 (a + b \tan(c + dx))} - \frac{\cos^6(c + dx) \left((a^2 - b^2) \tan(c + dx) + 2ab \right)}{6d(a^2 + b^2)^2} + \frac{\cos^4(c + dx) \left((-18a^2 b^2 + 13a^4 - 7b^4) \tan(c + dx) + 12ab(a^2 - b^2) \right)}{24d(a^2 + b^2)^3}$$

Antiderivative was successfully verified.

[In] `Int[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]`

[Out] $((5a^8 - 80a^6b^2 + 50a^4b^4 + 8a^2b^6 + b^8)x)/(16(a^2 + b^2)^5) + (2a^5b(a^2 - 3b^2) \text{Log}[a \text{Cos}[c + dx] + b \text{Sin}[c + dx]])/(a^2 + b^2)^5d - (a^6b)/((a^2 + b^2)^4d(a + b \text{Tan}[c + dx])) - (\text{Cos}[c + dx]^6(2ab + (a^2 - b^2) \text{Tan}[c + dx]))/(6(a^2 + b^2)^2d) + (\text{Cos}[c + dx]^4(12ab(3a^2 + b^2) + (13a^4 - 18a^2b^2 - 7b^4) \text{Tan}[c + dx]))/(24(a^2 + b^2)^3d) - (\text{Cos}[c + dx]^2(48a^5b + (11a^6 - 43a^4b^2 - 7a^2b^4 - b^6) \text{Tan}[c + dx]))/(16(a^2 + b^2)^4d)$

Rule 3516

`Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]`

Rule 1647

`Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_.) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]`

Rule 1629

`Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,`

d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{\sin^6(c + dx)}{(a + b \tan(c + dx))^2} dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^6}{(a+x)^2(b^2+x^2)^4} dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{\cos^6(c + dx)(2ab + (a^2 - b^2) \tan(c + dx))}{6(a^2 + b^2)^2 d} - \operatorname{Subst}\left(\int \frac{-\frac{a^2 b^6 (a^2 - b^2)}{(a^2 + b^2)^2} + \frac{2ab^6(5a^2 + b^2)x}{(a^2 + b^2)^2} + \frac{b^4(6a^4 + 17a^2)}{(a^2 + b^2)^2}}{(a+x)^2(b^2+x^2)^3} dx, x, b \tan(c + dx)\right) \\ &= -\frac{\cos^6(c + dx)(2ab + (a^2 - b^2) \tan(c + dx))}{6(a^2 + b^2)^2 d} + \frac{\cos^4(c + dx)(12ab(3a^2 + b^2) + (13a^4 - 13b^4))}{24(a^2 + b^2)^3 d} \\ &= -\frac{\cos^6(c + dx)(2ab + (a^2 - b^2) \tan(c + dx))}{6(a^2 + b^2)^2 d} + \frac{\cos^4(c + dx)(12ab(3a^2 + b^2) + (13a^4 - 13b^4))}{24(a^2 + b^2)^3 d} \\ &= -\frac{\cos^6(c + dx)(2ab + (a^2 - b^2) \tan(c + dx))}{6(a^2 + b^2)^2 d} + \frac{\cos^4(c + dx)(12ab(3a^2 + b^2) + (13a^4 - 13b^4))}{24(a^2 + b^2)^3 d} \\ &= \frac{2a^5 b (a^2 - 3b^2) \log(a + b \tan(c + dx))}{(a^2 + b^2)^5 d} - \frac{a^6 b}{(a^2 + b^2)^4 d (a + b \tan(c + dx))} - \frac{\cos^6(c + dx)}{(a^2 + b^2)^3 d} \\ &= \frac{2a^5 b (a^2 - 3b^2) \log(a + b \tan(c + dx))}{(a^2 + b^2)^5 d} - \frac{a^6 b}{(a^2 + b^2)^4 d (a + b \tan(c + dx))} - \frac{\cos^6(c + dx)}{(a^2 + b^2)^3 d} \\ &= \frac{(5a^8 - 80a^6 b^2 + 50a^4 b^4 + 8a^2 b^6 + b^8)x}{16(a^2 + b^2)^5} + \frac{2a^5 b (a^2 - 3b^2) \log(\cos(c + dx))}{(a^2 + b^2)^5 d} + \frac{2a^5 b (a^2 - 3b^2) \log(a + b \tan(c + dx))}{(a^2 + b^2)^5 d} \end{aligned}$$

Mathematica [A] time = 6.47982, size = 526, normalized size = 1.77

$$b \left(\frac{12(a^2+b^2)(6a^4b^2+4a^2b^4-3a^6+b^6) \sin(2(c+dx))}{b} - 144a^5(a^2+b^2) \cos^2(c+dx) - 16a(a^2+b^2)^3 \cos^6(c+dx) + 24a(a^2+b^2)^2 (3a^2$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]

[Out] (b*((24*(a^2 + b^2)*(-3*a^6 + 6*a^4*b^2 + 4*a^2*b^4 + b^6)*ArcTan[Tan[c + d*x]])/b - 144*a^5*(a^2 + b^2)*Cos[c + d*x]^2 + 24*a*(a^2 + b^2)^2*(3*a^2 + b^2)*Cos[c + d*x]^4 - 16*a*(a^2 + b^2)^3*Cos[c + d*x]^6 - 24*a^5*(2*a^2 - 6*b^2 + (-a^3 + 7*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 96*a^5*(a^2 - 3*b^2)*Log[a + b*Tan[c + d*x]] - 24*a^5*(2*a^2 - 6*b^2 + (a^3 - 7*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - (12*(a^2 + b^2)^2*(-3*a^4 + 3*a^2*b^2 + 2*b^4)*Cos[c + d*x]^3*Sin[c + d*x])/b + (8*(-a^2 + b^2)*(a^2 + b^2)^3*Cos[c + d*x]^5*Sin[c + d*x])/b + (12*(a^2 + b^2)*(-3*a^6 + 6*a^4*b^2 + 4*a^2*b^4 + b^6)*Sin[2*(c + d*x)])/b - (9*(a^2 + b^2)^2*(-3*a^4 + 3*a^2*b^2 + 2*b^4)*(2*ArcTan[Tan[c + d*x]] + Sin[2*(c + d*x)]))/b + (5*(-a^2 + b^2)*(a^2 + b^2)^3*(12*ArcTan[Tan[c + d*x]] + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(4*b) - (48*a^6*(a^2 + b^2))/(a + b*Tan[c + d*x]))/(48*(a^2 + b^2)^5*d)

Maple [B] time = 0.103, size = 1211, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x)

[Out] -1/d/(a^2+b^2)^5*ln(1+tan(d*x+c)^2)*a^7*b+3/d/(a^2+b^2)^5*ln(1+tan(d*x+c)^2)*a^5*b^3-5/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^6*b^2+25/8/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^4*b^4+1/2/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^2*b^6+2/d*b*a^7/(a^2+b^2)^5*ln(a+b*tan(d*x+c))-6/d*b^3*a^5/(a^2+b^2)^5*ln(a+b*tan(d*x+c))-1/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*b^3*a^5-11/16/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^8+2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a^6*b^2+15/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a^4*b^4-1/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a^2*b^6-3/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^4*a^7*b-3/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^4*a^5*b^3+2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^6*b^2-1/3/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^2*b^6-9/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^2*a^7*b-5/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^2*b^3*a^5+5/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^2*a^3*b^5+1/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^2*a*b^7+1/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^2*b^6+13/3/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^6*b^2+5/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^4*b^4+25/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*a^4*b^4+5/16/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^8+1/16/d/(a^2+b^2)^5*arctan(tan(d*x+c))*b^8-11/6/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*a^7*b-a^6*b/(a^2+b^2)^4/d/(a+b*tan(d*x+c))+1/16/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^5*b^8-5/6/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*a^8-1/6/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)^3*b^8-5/16/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)*a^8-1/16/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3*tan(d*x+c)*b^8+3/2/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^3

$$*a^3*b^5+1/6/d/(a^2+b^2)^5/(1+\tan(dx+c)^2)^3*a*b^7$$

Maxima [B] time = 1.79554, size = 1079, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)^6/(a+b*tan(dx+c))^2,x, algorithm="maxima")

[Out] $\frac{1}{48} \cdot (3 \cdot (5a^8 - 80a^6b^2 + 50a^4b^4 + 8a^2b^6 + b^8) \cdot (dx + c) / (a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) + 96 \cdot (a^7b - 3a^5b^3) \cdot \log(b \cdot \tan(dx + c) + a) / (a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) - 48 \cdot (a^7b - 3a^5b^3) \cdot \log(\tan(dx + c)^2 + 1) / (a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) - (136a^6b - 64a^4b^3 - 8a^2b^5 + 3 \cdot (27a^6b - 43a^4b^3 - 7a^2b^5 - b^7) \cdot \tan(dx + c)^6 + 3 \cdot (11a^7 + 5a^5b^2 - 7a^3b^4 - ab^6) \cdot \tan(dx + c)^5 + 8 \cdot (41a^6b - 31a^4b^3 + a^2b^5 + b^7) \cdot \tan(dx + c)^4 + 8 \cdot (5a^7 - 4a^5b^2 - 11a^3b^4 - 2ab^6) \cdot \tan(dx + c)^3 + 3 \cdot (125a^6b - 69a^4b^3 - a^2b^5 + b^7) \cdot \tan(dx + c)^2 + (15a^7 - 23a^5b^2 - 43a^3b^4 - 5ab^6) \cdot \tan(dx + c)) / (a^9 + 4a^7b^2 + 6a^5b^4 + 4a^3b^6 + ab^8 + (a^8b + 4a^6b^3 + 6a^4b^5 + 4a^2b^7 + b^9) \cdot \tan(dx + c)^7 + (a^9 + 4a^7b^2 + 6a^5b^4 + 4a^3b^6 + ab^8) \cdot \tan(dx + c)^6 + 3 \cdot (a^8b + 4a^6b^3 + 6a^4b^5 + 4a^2b^7 + b^9) \cdot \tan(dx + c)^5 + 3 \cdot (a^9 + 4a^7b^2 + 6a^5b^4 + 4a^3b^6 + ab^8) \cdot \tan(dx + c)^4 + 3 \cdot (a^8b + 4a^6b^3 + 6a^4b^5 + 4a^2b^7 + b^9) \cdot \tan(dx + c)^3 + 3 \cdot (a^9 + 4a^7b^2 + 6a^5b^4 + 4a^3b^6 + ab^8) \cdot \tan(dx + c)^2 + (a^8b + 4a^6b^3 + 6a^4b^5 + 4a^2b^7 + b^9) \cdot \tan(dx + c)) / d$

Fricas [B] time = 2.89634, size = 1388, normalized size = 4.67

$$\frac{8(a^8b + 4a^6b^3 + 6a^4b^5 + 4a^2b^7 + b^9) \cos(dx + c)^7 - 2(19a^8b + 68a^6b^3 + 90a^4b^5 + 52a^2b^7 + 11b^9) \cos(dx + c)^5}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)^6/(a+b*tan(dx+c))^2,x, algorithm="fricas")

[Out] $-\frac{1}{48} \cdot (8 \cdot (a^8b + 4a^6b^3 + 6a^4b^5 + 4a^2b^7 + b^9) \cdot \cos(dx + c)^7 - 2 \cdot (19a^8b + 68a^6b^3 + 90a^4b^5 + 52a^2b^7 + 11b^9) \cdot \cos(dx + c)^5 + (85a^8b + 224a^6b^3 + 210a^4b^5 + 88a^2b^7 + 17b^9) \cdot \cos(dx + c)^3 - (17a^8b + 72a^6b^3 + 120a^4b^5 + 20a^2b^7 + 3b^9 + 3 \cdot (5a^9 - 80a^7b^2 + 50a^5b^4 + 8a^3b^6 + ab^8) \cdot dx) \cdot \cos(dx + c) - 48 \cdot ((a^8b - 3a^6b^3) \cdot \cos(dx + c) + (a^7b^2 - 3a^5b^4) \cdot \sin(dx + c)) \cdot \log(2ab \cdot \cos(dx + c) \cdot \sin(dx + c) + (a^2 - b^2) \cdot \cos(dx + c)^2 + b^2) - (98a^7b^2 + 24a^5b^4 - 30a^3b^6 - 4ab^8 - 8 \cdot (a^9 + 4a^7b^2 + 6a^5b^4 + 4a^3b^6 + ab^8) \cdot \cos(dx + c)^6 + 2 \cdot (13a^9 + 44a^7b^2 + 54a^5b^4 + 28a^3b^6 + 5ab^8) \cdot \cos(dx + c)^4 + 3 \cdot (5a^8b - 80a^6b^3 + 50a^4b^5 + 8a^2b^7 + b^9) \cdot dx - 3 \cdot (11a^9 + 16a^7b^2 - 2a^5b^4 - 8a^3b^6 - ab^8) \cdot \cos(dx + c)^2) \cdot \sin(dx + c)) / ((a^{11} + 5a^9b^2 + 10a^7b^4 + 10a^5b^6 + 5a^3b^8 + ab^{10}) \cdot d \cdot \cos(dx + c) + (a^{10}b + 5a^8b^3 + 10a^6b^5 + 10a^4b^7 + 5a^2b^9 + b^{11}) \cdot d \cdot \sin(dx + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**6/(a+b*tan(d*x+c))**2,x)

[Out] Timed out

Giac [B] time = 1.23314, size = 992, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out]
$$\frac{1}{48} \cdot \frac{(3 \cdot (5a^8 - 80a^6b^2 + 50a^4b^4 + 8a^2b^6 + b^8) \cdot (dx + c) / (a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) - 48(a^7b - 3a^5b^3) \cdot \log(\tan(dx + c)^2 + 1) / (a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) + 96(a^7b^2 - 3a^5b^4) \cdot \log(\text{abs}(b \cdot \tan(dx + c) + a)) / (a^{10}b + 5a^8b^3 + 10a^6b^5 + 10a^4b^7 + 5a^2b^9 + b^{11}) - 48(2a^7b^2 \cdot \tan(dx + c) - 6a^5b^4 \cdot \tan(dx + c) + 3a^8b - 5a^6b^3) / ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) \cdot (b \cdot \tan(dx + c) + a)) + (88a^7b \cdot \tan(dx + c)^6 - 264a^5b^3 \cdot \tan(dx + c)^6 - 33a^8 \cdot \tan(dx + c)^5 + 96a^6b^2 \cdot \tan(dx + c)^5 + 150a^4b^4 \cdot \tan(dx + c)^5 + 24a^2b^6 \cdot \tan(dx + c)^5 + 3b^8 \cdot \tan(dx + c)^5 + 120a^7b \cdot \tan(dx + c)^4 - 936a^5b^3 \cdot \tan(dx + c)^4 - 40a^8 \cdot \tan(dx + c)^3 + 208a^6b^2 \cdot \tan(dx + c)^3 + 240a^4b^4 \cdot \tan(dx + c)^3 - 16a^2b^6 \cdot \tan(dx + c)^3 - 8b^8 \cdot \tan(dx + c)^3 + 48a^7b \cdot \tan(dx + c)^2 - 912a^5b^3 \cdot \tan(dx + c)^2 + 120a^3b^5 \cdot \tan(dx + c)^2 + 24ab^7 \cdot \tan(dx + c)^2 - 15a^8 \cdot \tan(dx + c) + 96a^6b^2 \cdot \tan(dx + c) + 90a^4b^4 \cdot \tan(dx + c) - 24a^2b^6 \cdot \tan(dx + c) - 3b^8 \cdot \tan(dx + c) - 288a^5b^3 + 72a^3b^5 + 8ab^7) / ((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) \cdot (\tan(dx + c)^2 + 1)^3) / d$$

$$3.62 \quad \int \frac{\sin^4(c+dx)}{(a+b \tan(c+dx))^2} dx$$

Optimal. Leaf size=217

$$-\frac{a^4 b}{d(a^2 + b^2)^3 (a + b \tan(c + dx))} + \frac{\cos^4(c + dx) ((a^2 - b^2) \tan(c + dx) + 2ab)}{4d(a^2 + b^2)^2} - \frac{\cos^2(c + dx) ((-12a^2 b^2 + 5a^4 - b^4))}{8d(a^2 + b^2)^3}$$

[Out] $((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*x)/(8*(a^2 + b^2)^4) + (2*a^3*b*(a^2 - 2*b^2)*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/((a^2 + b^2)^4*d) - (a^4*b)/((a^2 + b^2)^3*d*(a + b*\text{Tan}[c + d*x])) + (\text{Cos}[c + d*x]^4*(2*a*b + (a^2 - b^2)*\text{Tan}[c + d*x]))/(4*(a^2 + b^2)^2*d) - (\text{Cos}[c + d*x]^2*(16*a^3*b + (5*a^4 - 12*a^2*b^2 - b^4)*\text{Tan}[c + d*x]))/(8*(a^2 + b^2)^3*d)$

Rubi [A] time = 0.561945, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$-\frac{a^4 b}{d(a^2 + b^2)^3 (a + b \tan(c + dx))} + \frac{\cos^4(c + dx) ((a^2 - b^2) \tan(c + dx) + 2ab)}{4d(a^2 + b^2)^2} - \frac{\cos^2(c + dx) ((-12a^2 b^2 + 5a^4 - b^4))}{8d(a^2 + b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]

[Out] $((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*x)/(8*(a^2 + b^2)^4) + (2*a^3*b*(a^2 - 2*b^2)*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/((a^2 + b^2)^4*d) - (a^4*b)/((a^2 + b^2)^3*d*(a + b*\text{Tan}[c + d*x])) + (\text{Cos}[c + d*x]^4*(2*a*b + (a^2 - b^2)*\text{Tan}[c + d*x]))/(4*(a^2 + b^2)^2*d) - (\text{Cos}[c + d*x]^2*(16*a^3*b + (5*a^4 - 12*a^2*b^2 - b^4)*\text{Tan}[c + d*x]))/(8*(a^2 + b^2)^3*d)$

Rule 3516

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 1647

Int[(Pq_)*((d_) + (e_.)*(x_.))^(m_.)*((a_.) + (c_.)*(x_.)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_.))^(m_.)*((a_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin^4(c + dx)}{(a + b \tan(c + dx))^2} dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^4}{(a+x)^2(b^2+x^2)^3} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{\cos^4(c + dx) (2ab + (a^2 - b^2) \tan(c + dx))}{4(a^2 + b^2)^2 d} - \operatorname{Subst}\left(\int \frac{\frac{a^2 b^4 (a^2 - b^2)}{(a^2 + b^2)^2} - \frac{2ab^4(3a^2 + b^2)x}{(a^2 + b^2)^2} - \frac{b^2(4a^4 + 11a^2 b^2 + b^4)x^2}{(a^2 + b^2)^2}}{(a+x)^2(b^2+x^2)^2} dx, x, b \tan(c + dx)\right) \\ &= \frac{\cos^4(c + dx) (2ab + (a^2 - b^2) \tan(c + dx))}{4(a^2 + b^2)^2 d} - \frac{\cos^2(c + dx) (16a^3 b + (5a^4 - 12a^2 b^2 - b^4) \tan(c + dx))}{8(a^2 + b^2)^3 d} \\ &= \frac{\cos^4(c + dx) (2ab + (a^2 - b^2) \tan(c + dx))}{4(a^2 + b^2)^2 d} - \frac{\cos^2(c + dx) (16a^3 b + (5a^4 - 12a^2 b^2 - b^4) \tan(c + dx))}{8(a^2 + b^2)^3 d} \\ &= \frac{2a^3 b (a^2 - 2b^2) \log(a + b \tan(c + dx))}{(a^2 + b^2)^4 d} - \frac{a^4 b}{(a^2 + b^2)^3 d (a + b \tan(c + dx))} + \frac{\cos^4(c + dx) (2ab + (a^2 - b^2) \tan(c + dx))}{4(a^2 + b^2)^2 d} \\ &= \frac{2a^3 b (a^2 - 2b^2) \log(a + b \tan(c + dx))}{(a^2 + b^2)^4 d} - \frac{a^4 b}{(a^2 + b^2)^3 d (a + b \tan(c + dx))} + \frac{\cos^4(c + dx) (2ab + (a^2 - b^2) \tan(c + dx))}{4(a^2 + b^2)^2 d} \\ &= \frac{(3a^6 - 33a^4 b^2 + 13a^2 b^4 + b^6) x}{8(a^2 + b^2)^4} + \frac{2a^3 b (a^2 - 2b^2) \log(\cos(c + dx))}{(a^2 + b^2)^4 d} + \frac{2a^3 b (a^2 - 2b^2) \log(a + b \tan(c + dx))}{(a^2 + b^2)^4} \end{aligned}$$

Mathematica [A] time = 3.80693, size = 373, normalized size = 1.72

$$b \left(\frac{2(a^2 + b^2)(3a^2 b^2 - 2a^4 + b^4) \sin(2(c + dx))}{b} - 16a^3 (a^2 + b^2) \cos^2(c + dx) + 4a (a^2 + b^2)^2 \cos^4(c + dx) + \frac{4(a^2 + b^2)(3a^2 b^2 - 2a^4 + b^4) \tan^{-1}(\tan(c + dx))}{b} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^2, x]
```

```
[Out] (b*((4*(a^2 + b^2)*(-2*a^4 + 3*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b - 16*
a^3*(a^2 + b^2)*Cos[c + d*x]^2 + 4*a*(a^2 + b^2)^2*Cos[c + d*x]^4 - 4*a^3*(
2*a^2 - 4*b^2 + (-a^3 + 5*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x
]] + 16*a^3*(a^2 - 2*b^2)*Log[a + b*Tan[c + d*x]] - 4*a^3*(2*a^2 - 4*b^2 +
(a^3 - 5*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 - b^
2)*(a^2 + b^2)^2*Cos[c + d*x]^3*Sin[c + d*x])/b + (2*(a^2 + b^2)*(-2*a^4 +
3*a^2*b^2 + b^4)*Sin[2*(c + d*x)])/b + (3*(a^2 - b^2)*(a^2 + b^2)^2*(2*ArcT
an[Tan[c + d*x]] + Sin[2*(c + d*x)]))/(2*b) - (8*a^4*(a^2 + b^2))/(a + b*Ta
n[c + d*x])))/(8*(a^2 + b^2)^4*d)
```

Maple [B] time = 0.099, size = 724, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -5/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^6+7/8/d/(a^2+b^2)^4/(1
+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^4*b^2+13/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2
*tan(d*x+c)^3*a^2*b^4+1/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*b^6
-2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a^5*b-2/d/(a^2+b^2)^4/(1+t
an(d*x+c)^2)^2*tan(d*x+c)^2*a^3*b^3-3/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*ta
n(d*x+c)*a^6+9/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^4*b^2+11/8/d
/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^2*b^4-1/8/d/(a^2+b^2)^4/(1+tan
(d*x+c)^2)^2*tan(d*x+c)*b^6-3/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*a^5*b-1/d/
(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*a^3*b^3+1/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2
*a*b^5-1/d/(a^2+b^2)^4*ln(1+tan(d*x+c)^2)*a^5*b+2/d/(a^2+b^2)^4*ln(1+tan(d*
x+c)^2)*a^3*b^3+3/8/d/(a^2+b^2)^4*arctan(tan(d*x+c))*a^6-33/8/d/(a^2+b^2)^4
*arctan(tan(d*x+c))*a^4*b^2+13/8/d/(a^2+b^2)^4*arctan(tan(d*x+c))*a^2*b^4+1
/8/d/(a^2+b^2)^4*arctan(tan(d*x+c))*b^6-a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*x+c)
)+2/d*a^5*b/(a^2+b^2)^4*ln(a+b*tan(d*x+c))-4/d*a^3*b^3/(a^2+b^2)^4*ln(a+b*t
an(d*x+c))
```

Maxima [B] time = 1.53015, size = 684, normalized size = 3.15

$$\frac{(3a^6 - 33a^4b^2 + 13a^2b^4 + b^6)(dx+c)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} + \frac{16(a^5b - 2a^3b^3) \log(b \tan(dx+c) + a)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} - \frac{8(a^5b - 2a^3b^3) \log(\tan(dx+c)^2 + 1)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} - \frac{20a^4b - 4a^2b^3 + (13a^4b - 12a^2b^3 - b^5) \tan(dx+c)^4 + (5a^5 + 4a^3b^2 - ab^4) \tan(dx+c)^3 + (35a^4b - 12a^2b^3 + b^5) \tan(dx+c)^2 + 3(a^5 - ab^4) \tan(dx+c)}{(a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) \log(b \tan(dx+c) + a) + (a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) \log(\tan(dx+c)^2 + 1) - (20a^4b - 4a^2b^3 + (13a^4b - 12a^2b^3 - b^5) \tan(dx+c)^4 + (5a^5 + 4a^3b^2 - ab^4) \tan(dx+c)^3 + (35a^4b - 12a^2b^3 + b^5) \tan(dx+c)^2 + 3(a^5 - ab^4) \tan(dx+c))}{a^7 + 3a^5b^2 + 3a^3b^4 + ab^6 + (a^6b + 3a^4b^3 + 3a^2b^5 + b^7) \tan(dx+c)^5 + (a^7 + 3a^5b^2 + 3a^3b^4 + ab^6) \tan(dx+c)^4 + 2(a^6b + 3a^4b^3 + 3a^2b^5 + b^7) \tan(dx+c)^3 + 2(a^7 + 3a^5b^2 + 3a^3b^4 + ab^6) \tan(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8) + 16*(a^5*b - 2*a^3*b^3)*log(b*tan(d*x + c) + a
)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(a^5*b - 2*a^3*b^3)*l
og(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (2
0*a^4*b - 4*a^2*b^3 + (13*a^4*b - 12*a^2*b^3 - b^5)*tan(d*x + c)^4 + (5*a^5
+ 4*a^3*b^2 - a*b^4)*tan(d*x + c)^3 + (35*a^4*b - 12*a^2*b^3 + b^5)*tan(d*
x + c)^2 + 3*(a^5 - a*b^4)*tan(d*x + c))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)^5 + (a^7 + 3*a^5*b^
2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^4 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7)*tan(d*x + c)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^
```

$$2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*\tan(dx + c))/d$$

Fricas [B] time = 2.63226, size = 986, normalized size = 4.54

$$4(a^6b + 3a^4b^3 + 3a^2b^5 + b^7)\cos(dx + c)^5 - 6(3a^6b + 7a^4b^3 + 5a^2b^5 + b^7)\cos(dx + c)^3 + (3a^6b + 8a^4b^3 + 23a^2b^5 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)^4/(a+b*tan(dx+c))^2,x, algorithm="fricas")

[Out] 1/16*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(dx + c)^5 - 6*(3*a^6*b + 7*a^4*b^3 + 5*a^2*b^5 + b^7)*cos(dx + c)^3 + (3*a^6*b + 8*a^4*b^3 + 23*a^2*b^5 + 2*b^7 + 2*(3*a^7 - 33*a^5*b^2 + 13*a^3*b^4 + a*b^6)*d*x)*cos(dx + c) + 16*((a^6*b - 2*a^4*b^3)*cos(dx + c) + (a^5*b^2 - 2*a^3*b^4)*sin(dx + c))*log(2*a*b*cos(dx + c)*sin(dx + c) + (a^2 - b^2)*cos(dx + c)^2 + b^2) + (29*a^5*b^2 + 10*a^3*b^4 - 3*a*b^6 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(dx + c)^4 + 2*(3*a^6*b - 33*a^4*b^3 + 13*a^2*b^5 + b^7)*d*x - 2*(5*a^7 + 9*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(dx + c)^2)*sin(dx + c))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*d*cos(dx + c) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d*sin(dx + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)**4/(a+b*tan(dx+c))**2,x)

[Out] Timed out

Giac [B] time = 1.21571, size = 693, normalized size = 3.19

$$\frac{(3a^6 - 33a^4b^2 + 13a^2b^4 + b^6)(dx+c)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} - \frac{8(a^5b - 2a^3b^3)\log(\tan(dx+c)^2 + 1)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} + \frac{16(a^5b^2 - 2a^3b^4)\log(|b\tan(dx+c)+a|)}{a^8b + 4a^6b^3 + 6a^4b^5 + 4a^2b^7 + b^9} - \frac{8(2a^5b^2\tan(dx+c) - 4a^3b^4\tan(dx+c) + 3a^6b - 3a^4b^3)}{(a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8)(b\tan(dx+c) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)^4/(a+b*tan(dx+c))^2,x, algorithm="giac")

[Out] 1/8*((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*(dx + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(a^5*b - 2*a^3*b^3)*log(tan(dx + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 16*(a^5*b^2 - 2*a^3*b^4)*log(abs(b*tan(dx + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) - 8*(2*a^5*b^2*tan(dx + c) - 4*a^3*b^4*tan(dx + c) + 3*a^6*b - 3*a^4*b^3)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(dx + c) + a)) + (12*a^5*b*tan(dx + c)^4 - 24*a^3*b^3*tan(dx + c)^4 - 5*a^6*tan(dx + c)^3 + 7*a^4*b^2*tan(dx + c)^3 + 13*a^2*b^4*tan(dx + c)^3 + b^6*tan(dx + c)^3 + 8*a^5*b*tan(dx + c)^2 - 64*a^3*b^3*tan(dx + c)^2 - 3*a^6*tan(dx + c)^2 + \dots)

$$\begin{aligned} &+ c) + 9a^4b^2 \tan(dx + c) + 11a^2b^4 \tan(dx + c) - b^6 \tan(dx + c) \\ &- 32a^3b^3 + 4ab^5) / ((a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) * (\tan(dx + c)^2 + 1)^2) / d \end{aligned}$$

3.63 $\int \frac{\sin^2(c+dx)}{(a+b \tan(c+dx))^2} dx$

Optimal. Leaf size=148

$$\frac{a^2 b}{d(a^2 + b^2)^2 (a + b \tan(c + dx))} - \frac{\cos^2(c + dx) ((a^2 - b^2) \tan(c + dx) + 2ab)}{2d(a^2 + b^2)^2} + \frac{2ab(a^2 - b^2) \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)^3}$$

[Out] $((a^4 - 6a^2b^2 + b^4)x)/(2*(a^2 + b^2)^3) + (2*a*b*(a^2 - b^2)*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/((a^2 + b^2)^3*d) - (a^2*b)/((a^2 + b^2)^2*d*(a + b*\text{Tan}[c + d*x])) - (\text{Cos}[c + d*x]^2*(2*a*b + (a^2 - b^2)*\text{Tan}[c + d*x]))/(2*(a^2 + b^2)^2*d)$

Rubi [A] time = 0.294904, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$\frac{a^2 b}{d(a^2 + b^2)^2 (a + b \tan(c + dx))} - \frac{\cos^2(c + dx) ((a^2 - b^2) \tan(c + dx) + 2ab)}{2d(a^2 + b^2)^2} + \frac{2ab(a^2 - b^2) \log(a \cos(c + dx) + b \sin(c + dx))}{d(a^2 + b^2)^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[c + d*x]^2/(a + b*\text{Tan}[c + d*x])^2, x]$

[Out] $((a^4 - 6a^2b^2 + b^4)x)/(2*(a^2 + b^2)^3) + (2*a*b*(a^2 - b^2)*\text{Log}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/((a^2 + b^2)^3*d) - (a^2*b)/((a^2 + b^2)^2*d*(a + b*\text{Tan}[c + d*x])) - (\text{Cos}[c + d*x]^2*(2*a*b + (a^2 - b^2)*\text{Tan}[c + d*x]))/(2*(a^2 + b^2)^2*d)$

Rule 3516

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[b/f, \text{Subst}[\text{Int}[(x^m*(a + x)^n)/(b^2 + x^2)^{(m/2 + 1)}], x], x, b*\text{Tan}[e + f*x], x] /;$ FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 1647

$\text{Int}[(Pq_)*((d_) + (e_.)*(x_.))^{(m_.)}*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{Q = \text{PolynomialQuotient}[(d + e*x)^m*Pq, a + c*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]\}, \text{Simp}[(a*g - c*f*x)*(a + c*x^2)^{(p + 1)}/(2*a*c*(p + 1)), x] + \text{Dist}[1/(2*a*c*(p + 1)), \text{Int}[(d + e*x)^m*(a + c*x^2)^{(p + 1)}*\text{ExpandToSum}[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /;$ FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1629

$\text{Int}[(Pq_)*((d_) + (e_.)*(x_.))^{(m_.)}*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /;$ FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 635

$\text{Int}[(d + (e_.)*(x_.))/((a_.) + (c_.)*(x_.)^2), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] /;$ FreeQ[{a, c, d, e}

}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2} dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^2}{(a+x)^2(b^2+x^2)^2} dx, x, b\tan(c+dx)\right)}{d} \\ &= -\frac{\cos^2(c+dx)(2ab+(a^2-b^2)\tan(c+dx))}{2(a^2+b^2)^2 d} - \frac{\operatorname{Subst}\left(\int \frac{-\frac{a^2b^2(a^2-b^2)}{(a^2+b^2)^2} + \frac{2ab^2x}{a^2+b^2} + \frac{b^2(a^2-b^2)x^2}{(a^2+b^2)^2}}{(a+x)^2(b^2+x^2)} dx, x\right)}{2bd} \\ &= -\frac{\cos^2(c+dx)(2ab+(a^2-b^2)\tan(c+dx))}{2(a^2+b^2)^2 d} - \frac{\operatorname{Subst}\left(\int \left(-\frac{2a^2b^2}{(a^2+b^2)^2(a+x)^2} + \frac{4ab^2(-a^2+b^2)}{(a^2+b^2)^3(a+x)} + \frac{b^2(a^2-b^2)x}{(a^2+b^2)^2(b^2+x^2)}\right) dx, x\right)}{2bd} \\ &= \frac{2ab(a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} - \frac{a^2b}{(a^2+b^2)^2 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)(2ab+(a^2-b^2)\tan(c+dx))}{2(a^2+b^2)^2 d} \\ &= \frac{2ab(a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} - \frac{a^2b}{(a^2+b^2)^2 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)(2ab+(a^2-b^2)\tan(c+dx))}{2(a^2+b^2)^2 d} \\ &= \frac{(a^4-6a^2b^2+b^4)x}{2(a^2+b^2)^3} + \frac{2ab(a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^3 d} + \frac{2ab(a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} \end{aligned}$$

Mathematica [A] time = 3.25967, size = 246, normalized size = 1.66

$$b \left(\frac{(a-b)(a+b)(a^2+b^2)\sin(2(c+dx))}{2b} + 2a(a^2+b^2)\cos^2(c+dx) + \frac{(a^2-b^2)(a^2+b^2)\tan^{-1}(\tan(c+dx))}{b} + \frac{2a^2(a^2+b^2)}{a+b\tan(c+dx)} + a \left(\frac{3ab^2-a^3}{\sqrt{-b^2}} + 2a \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^2, x]

[Out] -(b*(((a^2 - b^2)*(a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + 2*a*(a^2 + b^2)*Cos[c + d*x]^2 + a*(2*a^2 - 2*b^2 + (-a^3 + 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 4*a*(a - b)*(a + b)*Log[a + b*Tan[c + d*x]] + a*(2*a^2 - 2*b^2 + (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + ((a - b)*(a + b)*(a^2 + b^2)*Sin[2*(c + d*x)])/(2*b) + (2*a^2*(a^2 + b^2))/(a + b*Tan[c + d*x])))/(2*(a^2 + b^2)^3*d)

Maple [B] time = 0.089, size = 352, normalized size = 2.4

$$\frac{\tan(dx+c)a^4}{2d(a^2+b^2)^3(1+(\tan(dx+c))^2)} + \frac{\tan(dx+c)b^4}{2d(a^2+b^2)^3(1+(\tan(dx+c))^2)} - \frac{ba^3}{d(a^2+b^2)^3(1+(\tan(dx+c))^2)} - \frac{1}{d(a^2+b^2)^3(1+(\tan(dx+c))^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x)

[Out] $-1/2/d/(a^2+b^2)^3/(1+\tan(dx+c)^2)*\tan(dx+c)*a^4+1/2/d/(a^2+b^2)^3/(1+\tan(dx+c)^2)*\tan(dx+c)*b^4-1/d/(a^2+b^2)^3/(1+\tan(dx+c)^2)*b*a^3-1/d/(a^2+b^2)^3/(1+\tan(dx+c)^2)*b^3*a-1/d/(a^2+b^2)^3*\ln(1+\tan(dx+c)^2)*a^3*b+1/d/(a^2+b^2)^3*b^3*a*\ln(1+\tan(dx+c)^2)-3/d/(a^2+b^2)^3*\arctan(\tan(dx+c))*a^2*b^2+1/2/d/(a^2+b^2)^3*\arctan(\tan(dx+c))*b^4+1/2/d/(a^2+b^2)^3*\arctan(\tan(dx+c))*a^4-a^2*b/(a^2+b^2)^2/d/(a+b*\tan(dx+c))+2/d*b*a^3/(a^2+b^2)^3*\ln(a+b*\tan(dx+c))-2/d*b^3/(a^2+b^2)^3*a*\ln(a+b*\tan(dx+c))$

Maxima [B] time = 1.55602, size = 396, normalized size = 2.68

$$\frac{(a^4-6a^2b^2+b^4)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{4(a^3b-ab^3)\log(b\tan(dx+c)+a)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{2(a^3b-ab^3)\log(\tan(dx+c)^2+1)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{4a^2b+(3a^2b-b^3)\tan(dx+c)^2+(a^3b-ab^3)\tan(dx+c)}{a^5+2a^3b^2+ab^4+(a^4b+2a^2b^3+b^5)\tan(dx+c)^3+(a^5+2a^3b^2+ab^4)\tan(dx+c)} \cdot \frac{1}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] $1/2*((a^4-6a^2b^2+b^4)*(dx+c)/(a^6+3a^4b^2+3a^2b^4+b^6)+4*(a^3b-ab^3)*\log(b*\tan(dx+c)+a)/(a^6+3a^4b^2+3a^2b^4+b^6)-2*(a^3b-ab^3)*\log(\tan(dx+c)^2+1)/(a^6+3a^4b^2+3a^2b^4+b^6)-(4*a^2*b+(3*a^2*b-b^3)*\tan(dx+c)^2+(a^3+a*b^2)*\tan(dx+c))/(a^5+2*a^3*b^2+a*b^4+(a^4*b+2*a^2*b^3+b^5)*\tan(dx+c)^3+(a^5+2*a^3*b^2+a*b^4)*\tan(dx+c)^2+(a^4*b+2*a^2*b^3+b^5)*\tan(dx+c)))/d$

Fricas [B] time = 2.2491, size = 640, normalized size = 4.32

$$\frac{(a^4b+2a^2b^3+b^5)\cos(dx+c)^3+(a^2b^3-b^5-(a^5-6a^3b^2+ab^4)dx)\cos(dx+c)-2((a^4b-a^2b^3)\cos(dx+c)+(a^3b-ab^3)\sin(dx+c))}{2((a^7+3a^5b^2+3a^3b^4+ab^6)\cos(dx+c)+(a^6b+3a^4b^3+3a^2b^5+b^7)d*\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] $-1/2*((a^4*b+2*a^2*b^3+b^5)*\cos(dx+c)^3+(a^2*b^3-b^5-(a^5-6*a^3*b^2+ab^4)*dx)*\cos(dx+c)-2*((a^4*b-a^2*b^3)*\cos(dx+c)+(a^3*b^2-ab^4)*\sin(dx+c))*\log(2*a*b*\cos(dx+c)*\sin(dx+c)+(a^2-b^2)*\cos(dx+c)^2+b^2)-(3*a^3*b^2+a*b^4+(a^4*b-6*a^2*b^3+b^5)*dx-(a^5+2*a^3*b^2+a*b^4)*\cos(dx+c)^2)*\sin(dx+c))/((a^7+3*a^5*b^2+3*a^3*b^4+ab^6)*d*\cos(dx+c)+(a^6*b+3*a^4*b^3+3*a^2*b^5+b^7)*d*\sin(dx+c))$

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**2,x)

[Out] Exception raised: AttributeError

Giac [A] time = 1.17682, size = 355, normalized size = 2.4

$$\frac{(a^4 - 6a^2b^2 + b^4)(dx+c)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} - \frac{2(a^3b - ab^3) \log(\tan(dx+c)^2 + 1)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} + \frac{4(a^3b^2 - ab^4) \log(|b \tan(dx+c) + a|)}{a^6b + 3a^4b^3 + 3a^2b^5 + b^7} - \frac{3a^2b \tan(dx+c)^2 - b^3 \tan(dx+c)^2 + a^3 \tan(dx+c) + ab^2 \tan(dx+c)}{(a^4 + 2a^2b^2 + b^4)(b \tan(dx+c)^3 + a \tan(dx+c)^2 + b \tan(dx+c) + a)}$$

$$2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{2} * ((a^4 - 6a^2b^2 + b^4) * (d*x + c) / (a^6 + 3a^4b^2 + 3a^2b^4 + b^6) - 2 * (a^3b - ab^3) * \log(\tan(d*x + c)^2 + 1) / (a^6 + 3a^4b^2 + 3a^2b^4 + b^6) + 4 * (a^3b^2 - ab^4) * \log(\text{abs}(b * \tan(d*x + c) + a)) / (a^6b + 3a^4b^3 + 3a^2b^5 + b^7) - (3a^2b * \tan(d*x + c)^2 - b^3 * \tan(d*x + c)^2 + a^3 * \tan(d*x + c) + a * b^2 * \tan(d*x + c) + 4 * a^2 * b) / ((a^4 + 2a^2b^2 + b^4) * (b * \tan(d*x + c)^3 + a * \tan(d*x + c)^2 + b * \tan(d*x + c) + a))) / d$

$$3.64 \quad \int \frac{\csc^2(c+dx)}{(a+b \tan(c+dx))^2} dx$$

Optimal. Leaf size=72

$$-\frac{b}{a^2d(a+b \tan(c+dx))} - \frac{2b \log(\tan(c+dx))}{a^3d} + \frac{2b \log(a+b \tan(c+dx))}{a^3d} - \frac{\cot(c+dx)}{a^2d}$$

[Out] $-(\text{Cot}[c + d*x]/(a^2*d)) - (2*b*\text{Log}[\text{Tan}[c + d*x]])/(a^3*d) + (2*b*\text{Log}[a + b*\text{Tan}[c + d*x]])/(a^3*d) - b/(a^2*d*(a + b*\text{Tan}[c + d*x]))$

Rubi [A] time = 0.0654904, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 44}

$$-\frac{b}{a^2d(a+b \tan(c+dx))} - \frac{2b \log(\tan(c+dx))}{a^3d} + \frac{2b \log(a+b \tan(c+dx))}{a^3d} - \frac{\cot(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]

[Out] $-(\text{Cot}[c + d*x]/(a^2*d)) - (2*b*\text{Log}[\text{Tan}[c + d*x]])/(a^3*d) + (2*b*\text{Log}[a + b*\text{Tan}[c + d*x]])/(a^3*d) - b/(a^2*d*(a + b*\text{Tan}[c + d*x]))$

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 44

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(c+dx)}{(a+b \tan(c+dx))^2} dx &= \frac{b \text{Subst}\left(\int \frac{1}{x^2(a+x)^2} dx, x, b \tan(c+dx)\right)}{d} \\ &= \frac{b \text{Subst}\left(\int \left(\frac{1}{a^2x^2} - \frac{2}{a^3x} + \frac{1}{a^2(a+x)^2} + \frac{2}{a^3(a+x)}\right) dx, x, b \tan(c+dx)\right)}{d} \\ &= -\frac{\cot(c+dx)}{a^2d} - \frac{2b \log(\tan(c+dx))}{a^3d} + \frac{2b \log(a+b \tan(c+dx))}{a^3d} - \frac{b}{a^2d(a+b \tan(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.372834, size = 109, normalized size = 1.51

$$\frac{-a^2 \cot^2(c+dx) + b^2(2 \log(a \cos(c+dx) + b \sin(c+dx)) - 2 \log(\sin(c+dx) + 1) - ab \cot(c+dx)(-2 \log(a \cos(c+dx) + b \sin(c+dx))))}{a^3d(a \cot(c+dx) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]

[Out] $(-(a^2 \cot[c + d*x]^2) - a*b \cot[c + d*x] * (1 + 2 \log[\sin[c + d*x]]) - 2 \log[a * \cos[c + d*x] + b * \sin[c + d*x]]) + b^2 * (1 - 2 \log[\sin[c + d*x]] + 2 \log[a * \cos[c + d*x] + b * \sin[c + d*x]]) / (a^3 * d * (b + a * \cot[c + d*x]))$

Maple [A] time = 0.092, size = 75, normalized size = 1.

$$-\frac{1}{a^2 d \tan(dx + c)} - 2 \frac{b \ln(\tan(dx + c))}{a^3 d} - \frac{b}{a^2 d (a + b \tan(dx + c))} + 2 \frac{b \ln(a + b \tan(dx + c))}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x)

[Out] $-1/d/a^2/\tan(dx+c) - 2*b*\ln(\tan(dx+c))/a^3/d - b/a^2/d/(a+b*\tan(dx+c)) + 2*b*\ln(a+b*\tan(dx+c))/a^3/d$

Maxima [A] time = 0.987045, size = 100, normalized size = 1.39

$$-\frac{\frac{2b \tan(dx+c)+a}{a^2 b \tan(dx+c)^2 + a^3 \tan(dx+c)} - \frac{2b \log(b \tan(dx+c)+a)}{a^3} + \frac{2b \log(\tan(dx+c))}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] $-((2*b*\tan(dx + c) + a)/(a^2*b*\tan(dx + c)^2 + a^3*\tan(dx + c)) - 2*b*\log(b*\tan(dx + c) + a)/a^3 + 2*b*\log(\tan(dx + c))/a^3)/d$

Fricas [B] time = 2.19866, size = 663, normalized size = 9.21

$$a^2 b^2 - (a^4 + 2 a^2 b^2) \cos(dx + c)^2 - (a^3 b + 2 a b^3) \cos(dx + c) \sin(dx + c) + (a^2 b^2 + b^4 - (a^2 b^2 + b^4) \cos(dx + c))^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] $-(a^2 * b^2 - (a^4 + 2 * a^2 * b^2) * \cos(dx + c)^2 - (a^3 * b + 2 * a * b^3) * \cos(dx + c) * \sin(dx + c) + (a^2 * b^2 + b^4 - (a^2 * b^2 + b^4) * \cos(dx + c))^2 + (a^2 * b^2 + b^4 - (a^2 * b^2 + b^4) * \cos(dx + c))^2 + (a^3 * b + a * b^3) * \cos(dx + c) * \sin(dx + c)) * \log(2 * a * b * \cos(dx + c) * \sin(dx + c) + (a^2 - b^2) * \cos(dx + c)^2 + b^2) - (a^2 * b^2 + b^4 - (a^2 * b^2 + b^4) * \cos(dx + c)^2 + (a^3 * b + a * b^3) * \cos(dx + c) * \sin(dx + c)) * \log(-1/4 * \cos(dx + c)^2 + 1/4)) / ((a^5 * b + a^3 * b^3) * d * \cos(dx + c)^2 - (a^6 + a^4 * b^2) * d * \cos(dx + c) * \sin(dx + c) - (a^5 * b + a^3 * b^3) * d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(c + dx)}{(a + b \tan(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**2,x)

[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**2, x)

Giac [A] time = 1.21864, size = 100, normalized size = 1.39

$$\frac{\frac{2b \log(|b \tan(dx+c)+a|)}{a^3} - \frac{2b \log(|\tan(dx+c)|)}{a^3} - \frac{2b \tan(dx+c)+a}{(b \tan(dx+c)^2+a \tan(dx+c))a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] (2*b*log(abs(b*tan(d*x + c) + a))/a^3 - 2*b*log(abs(tan(d*x + c)))/a^3 - (2*b*tan(d*x + c) + a)/((b*tan(d*x + c)^2 + a*tan(d*x + c))*a^2))/d

$$3.65 \quad \int \frac{\csc^4(c+dx)}{(a+b \tan(c+dx))^2} dx$$

Optimal. Leaf size=140

$$\frac{b(a^2+b^2)}{a^4d(a+b \tan(c+dx))} - \frac{(a^2+3b^2) \cot(c+dx)}{a^4d} - \frac{2b(a^2+2b^2) \log(\tan(c+dx))}{a^5d} + \frac{2b(a^2+2b^2) \log(a+b \tan(c+dx))}{a^5d}$$

[Out] -(((a^2 + 3*b^2)*Cot[c + d*x])/(a^4*d)) + (b*Cot[c + d*x]^2)/(a^3*d) - Cot[c + d*x]^3/(3*a^2*d) - (2*b*(a^2 + 2*b^2)*Log[Tan[c + d*x]])/(a^5*d) + (2*b*(a^2 + 2*b^2)*Log[a + b*Tan[c + d*x]])/(a^5*d) - (b*(a^2 + b^2))/(a^4*d*(a + b*Tan[c + d*x]))

Rubi [A] time = 0.122497, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 894}

$$\frac{b(a^2+b^2)}{a^4d(a+b \tan(c+dx))} - \frac{(a^2+3b^2) \cot(c+dx)}{a^4d} - \frac{2b(a^2+2b^2) \log(\tan(c+dx))}{a^5d} + \frac{2b(a^2+2b^2) \log(a+b \tan(c+dx))}{a^5d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]

[Out] -(((a^2 + 3*b^2)*Cot[c + d*x])/(a^4*d)) + (b*Cot[c + d*x]^2)/(a^3*d) - Cot[c + d*x]^3/(3*a^2*d) - (2*b*(a^2 + 2*b^2)*Log[Tan[c + d*x]])/(a^5*d) + (2*b*(a^2 + 2*b^2)*Log[a + b*Tan[c + d*x]])/(a^5*d) - (b*(a^2 + b^2))/(a^4*d*(a + b*Tan[c + d*x]))

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 894

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{\csc^4(c+dx)}{(a+b \tan(c+dx))^2} dx &= \frac{b \operatorname{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^2} dx, x, b \tan(c+dx)\right)}{d} \\ &= \frac{b \operatorname{Subst}\left(\int \left(\frac{b^2}{a^2x^4} - \frac{2b^2}{a^3x^3} + \frac{a^2+3b^2}{a^4x^2} - \frac{2(a^2+2b^2)}{a^5x} + \frac{a^2+b^2}{a^4(a+x)^2} + \frac{2(a^2+2b^2)}{a^5(a+x)}\right) dx, x, b \tan(c+dx)\right)}{d} \\ &= -\frac{(a^2+3b^2) \cot(c+dx)}{a^4d} + \frac{b \cot^2(c+dx)}{a^3d} - \frac{\cot^3(c+dx)}{3a^2d} - \frac{2b(a^2+2b^2) \log(\tan(c+dx))}{a^5d} \end{aligned}$$

Mathematica [A] time = 2.50639, size = 244, normalized size = 1.74

$$-\cot^2(c+dx)\left(9a^2b^2+a^4\csc^2(c+dx)+2a^4\right)+3b^2\left(-2\left(a^2+2b^2\right)\log(\sin(c+dx))+2a^2\log(a\cos(c+dx))+b\sin(c+dx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]

[Out]
$$\frac{-\left(\cot[c+d*x]^2(2a^4+9a^2b^2+a^4\csc[c+d*x]^2)\right)+3b^2(a^2+b^2+a^2\csc[c+d*x]^2-2(a^2+2b^2)\log[\sin[c+d*x]]+2a^2\log[a\cos[c+d*x]+b\sin[c+d*x]]+4b^2\log[a\cos[c+d*x]+b\sin[c+d*x]])+a*b*\cot[c+d*x]*(-2a^2-9b^2+2a^2\csc[c+d*x]^2-6(a^2+2b^2)\log[\sin[c+d*x]]+6a^2\log[a\cos[c+d*x]+b\sin[c+d*x]]+12b^2\log[a\cos[c+d*x]+b\sin[c+d*x]])}{3a^5d(b+a\cot[c+d*x])}$$

Maple [A] time = 0.153, size = 189, normalized size = 1.4

$$-\frac{1}{3a^2d(\tan(dx+c))^3}-\frac{1}{a^2d\tan(dx+c)}-3\frac{b^2}{da^4\tan(dx+c)}+\frac{b}{da^3(\tan(dx+c))^2}-2\frac{b\ln(\tan(dx+c))}{da^3}-4\frac{b^3\ln(\tan(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x)

[Out]
$$-1/3/d/a^2/\tan(d*x+c)^3-1/d/a^2/\tan(d*x+c)-3/d/a^4/\tan(d*x+c)*b^2+1/d/a^3*b/\tan(d*x+c)^2-2*b*\ln(\tan(d*x+c))/a^3/d-4/d*b^3/a^5*\ln(\tan(d*x+c))-b/a^2/d/(a+b*\tan(d*x+c))-1/d*b^3/a^4/(a+b*\tan(d*x+c))+2*b*\ln(a+b*\tan(d*x+c))/a^3/d+4/d*b^3/a^5*\ln(a+b*\tan(d*x+c))$$

Maxima [A] time = 1.09184, size = 194, normalized size = 1.39

$$\frac{2a^2b\tan(dx+c)-6(a^2b+2b^3)\tan(dx+c)^3-a^3-3(a^3+2ab^2)\tan(dx+c)^2}{a^4b\tan(dx+c)^4+a^5\tan(dx+c)^3}+\frac{6(a^2b+2b^3)\log(b\tan(dx+c)+a)}{a^5}-\frac{6(a^2b+2b^3)\log(\tan(dx+c))}{a^5}$$

$3d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out]
$$1/3*((2a^2b*\tan(d*x+c)-6*(a^2b+2b^3)*\tan(d*x+c)^3-a^3-3*(a^3+2a*b^2)*\tan(d*x+c)^2)/(a^4*b*\tan(d*x+c)^4+a^5*\tan(d*x+c)^3)+6*(a^2b+2b^3)*\log(b*\tan(d*x+c)+a)/a^5-6*(a^2b+2b^3)*\log(\tan(d*x+c))/a^5)/d$$

Fricas [B] time = 2.37406, size = 1004, normalized size = 7.17

$$2(a^4+6a^2b^2)\cos(dx+c)^4+6a^2b^2-3(a^4+6a^2b^2)\cos(dx+c)^2+3((a^2b^2+2b^4)\cos(dx+c)^4+a^2b^2+2b^4-2(a^2b^2+2b^4)\cos(dx+c)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] $\frac{1}{3}*(2*(a^4 + 6*a^2*b^2)*\cos(d*x + c)^4 + 6*a^2*b^2 - 3*(a^4 + 6*a^2*b^2)*\cos(d*x + c)^2 + 3*((a^2*b^2 + 2*b^4)*\cos(d*x + c)^4 + a^2*b^2 + 2*b^4 - 2*(a^2*b^2 + 2*b^4)*\cos(d*x + c)^2 - ((a^3*b + 2*a*b^3)*\cos(d*x + c)^3 - (a^3*b + 2*a*b^3)*\cos(d*x + c))*\sin(d*x + c))*\log(2*a*b*\cos(d*x + c)*\sin(d*x + c) + (a^2 - b^2)*\cos(d*x + c)^2 + b^2) - 3*((a^2*b^2 + 2*b^4)*\cos(d*x + c)^4 + a^2*b^2 + 2*b^4 - 2*(a^2*b^2 + 2*b^4)*\cos(d*x + c)^2 - ((a^3*b + 2*a*b^3)*\cos(d*x + c)^3 - (a^3*b + 2*a*b^3)*\cos(d*x + c))*\sin(d*x + c))*\log(-1/4*\cos(d*x + c)^2 + 1/4) - 2*(6*a*b^3*\cos(d*x + c) - (a^3*b + 6*a*b^3)*\cos(d*x + c)^3)*\sin(d*x + c))/(a^5*b*d*\cos(d*x + c)^4 - 2*a^5*b*d*\cos(d*x + c)^2 + a^5*b*d - (a^6*d*\cos(d*x + c)^3 - a^6*d*\cos(d*x + c))*\sin(d*x + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**2,x)

[Out] Timed out

Giac [A] time = 1.29818, size = 274, normalized size = 1.96

$$\frac{6(a^2b+2b^3)\log(|\tan(dx+c)|)}{a^5} - \frac{6(a^2b^2+2b^4)\log(|b\tan(dx+c)+a|)}{a^5b} + \frac{3(2a^2b^2\tan(dx+c)+4b^4\tan(dx+c)+3a^3b+5ab^3)}{(b\tan(dx+c)+a)a^5} - \frac{11a^2b\tan(dx+c)^3+22b^3\tan(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] $-\frac{1}{3}*(6*(a^2*b + 2*b^3)*\log(\text{abs}(\tan(d*x + c)))/a^5 - 6*(a^2*b^2 + 2*b^4)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^5*b) + 3*(2*a^2*b^2*\tan(d*x + c) + 4*b^4*\tan(d*x + c) + 3*a^3*b + 5*a*b^3)/((b*\tan(d*x + c) + a)*a^5) - (11*a^2*b*\tan(d*x + c)^3 + 22*b^3*\tan(d*x + c)^3 - 3*a^3*\tan(d*x + c)^2 - 9*a*b^2*\tan(d*x + c)^2 + 3*a^2*b*\tan(d*x + c) - a^3)/(a^5*\tan(d*x + c)^3))/d$

$$3.66 \quad \int \frac{\csc^6(c+dx)}{(a+b \tan(c+dx))^2} dx$$

Optimal. Leaf size=219

$$\frac{b(a^2 + b^2)^2}{a^6 d(a + b \tan(c + dx))} - \frac{(2a^2 + 3b^2) \cot^3(c + dx)}{3a^4 d} + \frac{2b(a^2 + b^2) \cot^2(c + dx)}{a^5 d} - \frac{(a^2 + b^2)(a^2 + 5b^2) \cot(c + dx)}{a^6 d} - \frac{2b}{a^6 d}$$

[Out] -(((a^2 + b^2)*(a^2 + 5*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*(a^2 + b^2)*Cot[c + d*x]^2)/(a^5*d) - ((2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^4*d) + (b*Cot[c + d*x]^4)/(2*a^3*d) - Cot[c + d*x]^5/(5*a^2*d) - (2*b*(a^2 + b^2)*(a^2 + 3*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (2*b*(a^2 + b^2)*(a^2 + 3*b^2)*Log[a + b*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2)^2)/(a^6*d*(a + b*Tan[c + d*x]))

Rubi [A] time = 0.198334, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 894}

$$\frac{b(a^2 + b^2)^2}{a^6 d(a + b \tan(c + dx))} - \frac{(2a^2 + 3b^2) \cot^3(c + dx)}{3a^4 d} + \frac{2b(a^2 + b^2) \cot^2(c + dx)}{a^5 d} - \frac{(a^2 + b^2)(a^2 + 5b^2) \cot(c + dx)}{a^6 d} - \frac{2b}{a^6 d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]

[Out] -(((a^2 + b^2)*(a^2 + 5*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*(a^2 + b^2)*Cot[c + d*x]^2)/(a^5*d) - ((2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^4*d) + (b*Cot[c + d*x]^4)/(2*a^3*d) - Cot[c + d*x]^5/(5*a^2*d) - (2*b*(a^2 + b^2)*(a^2 + 3*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (2*b*(a^2 + b^2)*(a^2 + 3*b^2)*Log[a + b*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2)^2)/(a^6*d*(a + b*Tan[c + d*x]))

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 894

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2} dx = \frac{b \operatorname{Subst}\left(\int \frac{(b^2+x^2)^2}{x^6(a+x)^2} dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b \operatorname{Subst}\left(\int \left(\frac{b^4}{a^2x^6} - \frac{2b^4}{a^3x^5} + \frac{2a^2b^2+3b^4}{a^4x^4} - \frac{4b^2(a^2+b^2)}{a^5x^3} + \frac{a^4+6a^2b^2+5b^4}{a^6x^2} - \frac{2(a^4+4a^2b^2+3b^4)}{a^7x} + \frac{(a^2+b^2)^2}{a^6(a+x)^2}\right) dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{(a^2+b^2)(a^2+5b^2)\cot(c+dx)}{a^6d} + \frac{2b(a^2+b^2)\cot^2(c+dx)}{a^5d} - \frac{(2a^2+3b^2)\cot^3(c+dx)}{3a^4d}$$

Mathematica [B] time = 6.23138, size = 589, normalized size = 2.69

$$\frac{\sec^2(c+dx)(2a^2b^4\sin(c+dx) + a^4b^2\sin(c+dx) + b^6\sin(c+dx))(a\cos(c+dx) + b\sin(c+dx))}{a^7d(a+b\tan(c+dx))^2} - \frac{2(4a^2b^3 + a^4b + b^5)}{3a^4d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]

[Out] $-(\operatorname{Csc}[c+d*x]^5 \operatorname{Sec}[c+d*x] (a \operatorname{Cos}[c+d*x] + b \operatorname{Sin}[c+d*x])^2) / (5 a^2 d (a + b \operatorname{Tan}[c+d*x])^2) + ((-8 a^4 \operatorname{Cos}[c+d*x] - 75 a^2 b^2 \operatorname{Cos}[c+d*x] - 75 b^4 \operatorname{Cos}[c+d*x]) \operatorname{Csc}[c+d*x] \operatorname{Sec}[c+d*x]^2 (a \operatorname{Cos}[c+d*x] + b \operatorname{Sin}[c+d*x])^2) / (15 a^6 d (a + b \operatorname{Tan}[c+d*x])^2) + (b (a^2 + 2 b^2) \operatorname{Csc}[c+d*x]^2 \operatorname{Sec}[c+d*x]^2 (a \operatorname{Cos}[c+d*x] + b \operatorname{Sin}[c+d*x])^2) / (a^5 d (a + b \operatorname{Tan}[c+d*x])^2) + ((-4 a^2 \operatorname{Cos}[c+d*x] - 15 b^2 \operatorname{Cos}[c+d*x]) \operatorname{Csc}[c+d*x]^3 \operatorname{Sec}[c+d*x]^2 (a \operatorname{Cos}[c+d*x] + b \operatorname{Sin}[c+d*x])^2) / (15 a^4 d (a + b \operatorname{Tan}[c+d*x])^2) + (b \operatorname{Csc}[c+d*x]^4 \operatorname{Sec}[c+d*x]^2 (a \operatorname{Cos}[c+d*x] + b \operatorname{Sin}[c+d*x])^2) / (2 a^3 d (a + b \operatorname{Tan}[c+d*x])^2) - (2 (a^4 b + 4 a^2 b^3 + 3 b^5) \operatorname{Log}[\operatorname{Sin}[c+d*x]] \operatorname{Sec}[c+d*x]^2 (a \operatorname{Cos}[c+d*x] + b \operatorname{Sin}[c+d*x])^2) / (a^7 d (a + b \operatorname{Tan}[c+d*x])^2) + (2 (a^4 b + 4 a^2 b^3 + 3 b^5) \operatorname{Log}[a \operatorname{Cos}[c+d*x] + b \operatorname{Sin}[c+d*x]] \operatorname{Sec}[c+d*x]^2 (a \operatorname{Cos}[c+d*x] + b \operatorname{Sin}[c+d*x])^2) / (a^7 d (a + b \operatorname{Tan}[c+d*x])^2) + (\operatorname{Sec}[c+d*x]^2 (a \operatorname{Cos}[c+d*x] + b \operatorname{Sin}[c+d*x]) (a^4 b^2 \operatorname{Sin}[c+d*x] + 2 a^2 b^4 \operatorname{Sin}[c+d*x] + b^6 \operatorname{Sin}[c+d*x])) / (a^7 d (a + b \operatorname{Tan}[c+d*x])^2)$

Maple [A] time = 0.122, size = 343, normalized size = 1.6

$$\frac{1}{5 a^2 d (\tan(dx+c))^5} - \frac{2}{3 a^2 d (\tan(dx+c))^3} - \frac{b^2}{d a^4 (\tan(dx+c))^3} - \frac{1}{a^2 d \tan(dx+c)} - 6 \frac{b^2}{d a^4 \tan(dx+c)} - 5 \frac{b^2}{d a^6 \tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x)

[Out] $-1/5/d/a^2/\tan(d*x+c)^5 - 2/3/d/a^2/\tan(d*x+c)^3 - 1/d/a^4/\tan(d*x+c)^3 b^2 - 1/d/a^2/\tan(d*x+c) - 6/d/a^4/\tan(d*x+c) * b^2 - 5/d/a^6/\tan(d*x+c) * b^4 + 1/2/d/a^3 * b/\tan(d*x+c)^4 + 2/d/a^3 * b/\tan(d*x+c)^2 + 2/d * b^3/a^5/\tan(d*x+c)^2 - 2 * b * \ln(\tan(d*x+c))/a^3/d - 8/d * b^3/a^5 * \ln(\tan(d*x+c)) - 6/d * b^5/a^7 * \ln(\tan(d*x+c)) - b/a^2/d/(a+b*\tan(d*x+c)) - 2/d * b^3/a^4/(a+b*\tan(d*x+c)) - 1/d * b^5/a^6/(a+b*\tan(d*x+c)) + 2 * b * \ln(a+b*\tan(d*x+c))/a^3/d + 8/d * b^3/a^5 * \ln(a+b*\tan(d*x+c)) + 6/d * b^5/a^7 * \ln(a+b*\tan(d*x+c))$

Maxima [A] time = 1.22476, size = 304, normalized size = 1.39

$$\frac{9a^4b \tan(dx+c) - 60(a^4b + 4a^2b^3 + 3b^5) \tan(dx+c)^5 - 6a^5 - 30(a^5 + 4a^3b^2 + 3ab^4) \tan(dx+c)^4 + 10(4a^4b + 3a^2b^3) \tan(dx+c)^3 - 5(4a^5 + 3a^3b^2) \tan(dx+c)^2 + \frac{60(a^6b \tan(dx+c)^6 + a^7 \tan(dx+c)^5)}{30d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] 1/30*((9*a^4*b*tan(d*x + c) - 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*tan(d*x + c)^5 - 6*a^5 - 30*(a^5 + 4*a^3*b^2 + 3*a*b^4)*tan(d*x + c)^4 + 10*(4*a^4*b + 3*a^2*b^3)*tan(d*x + c)^3 - 5*(4*a^5 + 3*a^3*b^2)*tan(d*x + c)^2)/(a^6*b*tan(d*x + c)^6 + a^7*tan(d*x + c)^5) + 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(b*tan(d*x + c) + a)/a^7 - 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(tan(d*x + c))/a^7)/d

Fricas [B] time = 2.59028, size = 1770, normalized size = 8.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] 1/30*(4*(4*a^6 + 45*a^4*b^2 + 45*a^2*b^4)*cos(d*x + c)^6 - 75*a^4*b^2 - 90*a^2*b^4 - 10*(4*a^6 + 45*a^4*b^2 + 45*a^2*b^4)*cos(d*x + c)^4 + 15*(2*a^6 + 23*a^4*b^2 + 24*a^2*b^4)*cos(d*x + c)^2 + 30*((a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^6 - a^4*b^2 - 4*a^2*b^4 - 3*b^6 - 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 - ((a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^5 - 2*(a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 + (a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*((a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^6 - a^4*b^2 - 4*a^2*b^4 - 3*b^6 - 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 - ((a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^5 - 2*(a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 + (a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) + (4*(4*a^5*b + 45*a^3*b^3 + 45*a*b^5)*cos(d*x + c)^5 - 10*(a^5*b + 33*a^3*b^3 + 36*a*b^5)*cos(d*x + c)^3 - 15*(a^5*b - 10*a^3*b^3 - 12*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*cos(d*x + c)^2 - a^7*b*d - (a^8*d*cos(d*x + c)^5 - 2*a^8*d*cos(d*x + c)^3 + a^8*d*cos(d*x + c))*sin(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**2,x)

[Out] Timed out

Giac [A] time = 1.23916, size = 448, normalized size = 2.05

$$\frac{60(a^4b+4a^2b^3+3b^5)\log(|\tan(dx+c)|)}{a^7} - \frac{60(a^4b^2+4a^2b^4+3b^6)\log(|b\tan(dx+c)+a|)}{a^7b} + \frac{30(2a^4b^2\tan(dx+c)+8a^2b^4\tan(dx+c)+6b^6\tan(dx+c)+3a^5b+10a^3b^3+7ab^5)}{(b\tan(dx+c)+a)a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/30*(60*(a^4*b + 4*a^2*b^3 + 3*b^5)*\log(\text{abs}(\tan(d*x + c))))/a^7 - 60*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^7*b) + 30*(2*a^4*b^2*\tan(d*x + c) + 8*a^2*b^4*\tan(d*x + c) + 6*b^6*\tan(d*x + c) + 3*a^5*b + 10*a^3*b^3 + 7*a*b^5)/((b*\tan(d*x + c) + a)*a^7) - (137*a^4*b*\tan(d*x + c)^5 + 548*a^2*b^3*\tan(d*x + c)^5 + 411*b^5*\tan(d*x + c)^5 - 30*a^5*\tan(d*x + c)^4 - 180*a^3*b^2*\tan(d*x + c)^4 - 150*a*b^4*\tan(d*x + c)^4 + 60*a^4*b*\tan(d*x + c)^3 + 60*a^2*b^3*\tan(d*x + c)^3 - 20*a^5*\tan(d*x + c)^2 - 30*a^3*b^2*\tan(d*x + c)^2 + 15*a^4*b*\tan(d*x + c) - 6*a^5)/(a^7*\tan(d*x + c)^5)/d \end{aligned}$$

$$3.67 \quad \int \frac{\sin^6(c+dx)}{(a+b \tan(c+dx))^3} dx$$

Optimal. Leaf size=382

$$\frac{a^6 b}{2d(a^2 + b^2)^4 (a + b \tan(c + dx))^2} - \frac{2a^5 b (a^2 - 3b^2)}{d(a^2 + b^2)^5 (a + b \tan(c + dx))} - \frac{a \cos^2(c + dx) \left((-119a^4 b^2 + 65a^2 b^4 + 11a^6 + 3b^6) \right)}{16d(a^2 + b^2)^5}$$

[Out] (a*(5*a^8 - 180*a^6*b^2 + 390*a^4*b^4 - 68*a^2*b^6 - 3*b^8)*x)/(16*(a^2 + b^2)^6) + (a^4*b*(3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^6*d) - (a^6*b)/(2*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x])^2) - (2*a^5*b*(a^2 - 3*b^2))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^6*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(6*(a^2 + b^2)^3*d) + (Cos[c + d*x]^4*(6*b*(9*a^4 - 4*a^2*b^2 - b^4) + a*(13*a^4 - 62*a^2*b^2 - 3*b^4)*Tan[c + d*x]))/(24*(a^2 + b^2)^4*d) - (a*Cos[c + d*x]^2*(24*a^3*b*(3*a^2 - 5*b^2) + (11*a^6 - 119*a^4*b^2 + 65*a^2*b^4 + 3*b^6)*Tan[c + d*x]))/(16*(a^2 + b^2)^5*d)

Rubi [A] time = 1.43182, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$\frac{a^6 b}{2d(a^2 + b^2)^4 (a + b \tan(c + dx))^2} - \frac{2a^5 b (a^2 - 3b^2)}{d(a^2 + b^2)^5 (a + b \tan(c + dx))} - \frac{a \cos^2(c + dx) \left((-119a^4 b^2 + 65a^2 b^4 + 11a^6 + 3b^6) \right)}{16d(a^2 + b^2)^5}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]

[Out] (a*(5*a^8 - 180*a^6*b^2 + 390*a^4*b^4 - 68*a^2*b^6 - 3*b^8)*x)/(16*(a^2 + b^2)^6) + (a^4*b*(3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^6*d) - (a^6*b)/(2*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x])^2) - (2*a^5*b*(a^2 - 3*b^2))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^6*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(6*(a^2 + b^2)^3*d) + (Cos[c + d*x]^4*(6*b*(9*a^4 - 4*a^2*b^2 - b^4) + a*(13*a^4 - 62*a^2*b^2 - 3*b^4)*Tan[c + d*x]))/(24*(a^2 + b^2)^4*d) - (a*Cos[c + d*x]^2*(24*a^3*b*(3*a^2 - 5*b^2) + (11*a^6 - 119*a^4*b^2 + 65*a^2*b^4 + 3*b^6)*Tan[c + d*x]))/(16*(a^2 + b^2)^5*d)

Rule 3516

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 1647

Int[(Pq_)*((d_) + (e_.)*(x_.))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1629

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3} dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^6}{(a+x)^3(b^2+x^2)^4} dx, x, b \tan(c+dx)\right)}{d} \\
&= -\frac{\cos^6(c+dx)(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{6(a^2+b^2)^3 d} - \operatorname{Subst}\left(\int \frac{-\frac{a^4 b^6(a^2-3b^2)}{(a^2+b^2)^3} + \frac{3a^3 b^6(5a^2+b^2)x}{(a^2+b^2)^3}}{\dots} dx\right) \\
&= -\frac{\cos^6(c+dx)(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{6(a^2+b^2)^3 d} + \frac{\cos^4(c+dx)(6b(9a^4-4a^2b^2-b^4))}{24(a^2+b^2)^3 d} \\
&= -\frac{\cos^6(c+dx)(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{6(a^2+b^2)^3 d} + \frac{\cos^4(c+dx)(6b(9a^4-4a^2b^2-b^4))}{24(a^2+b^2)^3 d} \\
&= -\frac{\cos^6(c+dx)(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{6(a^2+b^2)^3 d} + \frac{\cos^4(c+dx)(6b(9a^4-4a^2b^2-b^4))}{24(a^2+b^2)^3 d} \\
&= \frac{a^4 b(3a^4-22a^2b^2+15b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^6 d} - \frac{a^6 b}{2(a^2+b^2)^4 d(a+b\tan(c+dx))^2} - \frac{a^6}{(a^2+b^2)^6} \\
&= \frac{a^4 b(3a^4-22a^2b^2+15b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^6 d} - \frac{a^6 b}{2(a^2+b^2)^4 d(a+b\tan(c+dx))^2} - \frac{a^6}{(a^2+b^2)^6} \\
&= \frac{a(5a^8-180a^6b^2+390a^4b^4-68a^2b^6-3b^8)x}{16(a^2+b^2)^6} + \frac{a^4 b(3a^4-22a^2b^2+15b^4)\log(\cos(c+dx))}{(a^2+b^2)^6 d}
\end{aligned}$$

Mathematica [A] time = 6.60369, size = 683, normalized size = 1.79

$$b \left(\frac{3a^4(3a^2-5b^2)\cos^2(c+dx)}{2(a^2+b^2)^5} - \frac{(3a^2-b^2)\cos^6(c+dx)}{6(a^2+b^2)^3} + \frac{(-4a^2b^2+9a^4-b^4)\cos^4(c+dx)}{4(a^2+b^2)^4} - \frac{3a^5(a^2-7b^2)\tan^{-1}(\tan(c+dx))}{2b(a^2+b^2)^5} - \frac{a^6}{2(a^2+b^2)^4(a+b\tan(c+dx))^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^3, x]

[Out] (b*((-3*a^5*(a^2 - 7*b^2)*ArcTan[Tan[c + d*x]])/(2*b*(a^2 + b^2)^5) - (3*a^4*(3*a^2 - 5*b^2)*Cos[c + d*x]^2)/(2*(a^2 + b^2)^5) + ((9*a^4 - 4*a^2*b^2 - b^4)*Cos[c + d*x]^4)/(4*(a^2 + b^2)^4) - ((3*a^2 - b^2)*Cos[c + d*x]^6)/(6*(a^2 + b^2)^3) - (a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4 - (a^5 - 18*a^3*b^2 + 21*a*b^4)/Sqrt[-b^2]))*Log[Sqrt[-b^2] - b*Tan[c + d*x]]/(2*(a^2 + b^2)^6) + (a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[a + b*Tan[c + d*x]]/(a^2 + b^2)^6 - (a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4 + (a^5 - 18*a^3*b^2 + 21*a*b^4)/Sqrt[-b^2]))*Log[Sqrt[-b^2] + b*Tan[c + d*x]]/(2*(a^2 + b^2)^6) - (3*a^5*(a^2 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*b*(a^2 + b^2)^5) + (3*a*(a^4 - 4*a^2*b^2 - b^4)*Cos[c + d*x]^3*Ssin[c + d*x])/(4*b*(a^2 + b^2)^4) - (a*(a^2 - 3*b^2)*

$$\frac{\cos[c + dx]^5 \sin[c + dx]}{(6b(a^2 + b^2)^3 + (9a(a^4 - 4a^2b^2 - b^4) \operatorname{ArcTan}[\tan[c + dx]]/b + (\cos[c + dx] \sin[c + dx])/b)) / (8(a^2 + b^2)^4 - (5a(a^2 - 3b^2) \operatorname{ArcTan}[\tan[c + dx]]/b^3 + (\cos[c + dx] \sin[c + dx])/b^3)) / (48(a^2 + b^2)^3 - a^6 / (2(a^2 + b^2)^4 (a + b \tan[c + dx])^2 - (2a^5(a^2 - 3b^2)) / ((a^2 + b^2)^5 (a + b \tan[c + dx])))) / d$$

Maple [B] time = 0.118, size = 1449, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(dx+c)^6/(a+b*tan(dx+c))^3,x)`

[Out]
$$\begin{aligned} & -15/2/d/(a^2+b^2)^6 \ln(1+\tan(dx+c)^2) * a^4 b^5 + 13/2/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * a^4 b^5 - 11/4/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * a^8 b + 31/6/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * a^6 b^3 - 3/2/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * a^2 b^7 - 11/16/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^5 * a^9 - 5/6/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^3 * a^9 - 1/4/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^2 * b^9 - 5/16/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c) * a^9 + 3/d * a^8 * b / (a^2+b^2)^6 * \ln(a+b \tan(dx+c)) - 22/d * a^6 * b^3 / (a^2+b^2)^6 * \ln(a+b \tan(dx+c)) + 15/d * a^4 * b^5 / (a^2+b^2)^6 * \ln(a+b \tan(dx+c)) - 2/d * b * a^7 / (a^2+b^2)^5 / (a+b \tan(dx+c)) + 6/d * b^3 * a^5 / (a^2+b^2)^5 / (a+b \tan(dx+c)) - 45/4/d/(a^2+b^2)^6 * \arctan(\tan(dx+c)) * a^7 * b^2 + 195/8/d/(a^2+b^2)^6 * \arctan(\tan(dx+c)) * a^5 * b^4 - 17/4/d/(a^2+b^2)^6 * \arctan(\tan(dx+c)) * a^3 * b^6 - 3/16/d/(a^2+b^2)^6 * \arctan(\tan(dx+c)) * a * b^8 - 3/2/d/(a^2+b^2)^6 * \ln(1+\tan(dx+c)^2) * a^8 * b + 11/d/(a^2+b^2)^6 * \ln(1+\tan(dx+c)^2) * a^6 * b^3 - 1/12/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * b^9 + 5/16/d/(a^2+b^2)^6 * \arctan(\tan(dx+c)) * a^9 + 27/4/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^5 * a^7 * b^2 + 27/8/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^5 * a^5 * b^4 - 17/4/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^5 * a^3 * b^6 - 3/16/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^5 * a * b^8 - 9/2/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^4 * a^8 * b + 3/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^4 * a^6 * b^3 + 15/2/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^4 * a^4 * b^5 + 12/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^3 * a^7 * b^2 + 2/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^3 * a^5 * b^4 - 34/3/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^3 * a^3 * b^6 - 1/2/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^3 * a * b^8 - 1/2 * a^6 * b / (a^2+b^2)^4 / (a+b \tan(dx+c))^2 - 27/4/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^2 * a^8 * b + 19/2/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^2 * a^6 * b^3 + 15/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^2 * a^4 * b^5 - 3/2/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c)^2 * a^2 * b^7 + 21/4/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c) * a^7 * b^2 - 3/8/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c) * a^5 * b^4 - 23/4/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c) * a^3 * b^6 + 3/16/d/(a^2+b^2)^6 / (1+\tan(dx+c)^2)^3 * \tan(dx+c) * a * b^8 \end{aligned}$$

Maxima [B] time = 1.93332, size = 1469, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(dx+c)^6/(a+b*tan(dx+c))^3,x, algorithm="maxima")`

[Out]
$$\frac{1}{48} * (3 * (5 * a^9 - 180 * a^7 * b^2 + 390 * a^5 * b^4 - 68 * a^3 * b^6 - 3 * a * b^8)) * (dx + c) / (a^{12} + 6 * a^{10} * b^2 + 15 * a^8 * b^4 + 20 * a^6 * b^6 + 15 * a^4 * b^8 + 6 * a^2 * b^{10} +$$

$$\begin{aligned}
& b^{12}) + 48*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*\log(b*\tan(d*x + c) + a)/(a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12}) \\
& - 24*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*\log(\tan(d*x + c)^2 + 1)/(a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12}) - (252 \\
& *a^8*b - 644*a^6*b^3 + 68*a^4*b^5 + 4*a^2*b^7 + 3*(43*a^7*b^2 - 215*a^5*b^4 \\
& + 65*a^3*b^6 + 3*a*b^8)*\tan(d*x + c)^7 + 6*(31*a^8*b - 127*a^6*b^3 + 5*a^4 \\
& *b^5 + 3*a^2*b^7)*\tan(d*x + c)^6 + (33*a^9 + 403*a^7*b^2 - 2005*a^5*b^4 + 5 \\
& 29*a^3*b^6 + 24*a*b^8)*\tan(d*x + c)^5 + 4*(164*a^8*b - 515*a^6*b^3 + 65*a^4 \\
& *b^5 + 27*a^2*b^7 + 3*b^9)*\tan(d*x + c)^4 + (40*a^9 + 335*a^7*b^2 - 2171*a^5 \\
& *b^4 + 429*a^3*b^6 + 15*a*b^8)*\tan(d*x + c)^3 + 2*(357*a^8*b - 987*a^6*b^3 \\
& + 125*a^4*b^5 + 31*a^2*b^7 + 2*b^9)*\tan(d*x + c)^2 + (15*a^9 + 93*a^7*b^2 \\
& - 763*a^5*b^4 + 127*a^3*b^6 + 8*a*b^8)*\tan(d*x + c))/(a^{12} + 5*a^{10}*b^2 + 1 \\
& 0*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8 + a^2*b^{10} + (a^{10}*b^2 + 5*a^8*b^4 + 10* \\
& a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^{10} + b^{12})*\tan(d*x + c)^8 + 2*(a^{11}*b + 5*a^9 \\
& *b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^{11})*\tan(d*x + c)^7 + (a^{12} \\
& + 8*a^{10}*b^2 + 25*a^8*b^4 + 40*a^6*b^6 + 35*a^4*b^8 + 16*a^2*b^{10} + 3*b^{12}) \\
& *\tan(d*x + c)^6 + 6*(a^{11}*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3 \\
& *b^9 + a*b^{11})*\tan(d*x + c)^5 + 3*(a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6* \\
& b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12})*\tan(d*x + c)^4 + 6*(a^{11}*b + 5*a^9*b^3 \\
& + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^{11})*\tan(d*x + c)^3 + (3*a^{12} \\
& + 16*a^{10}*b^2 + 35*a^8*b^4 + 40*a^6*b^6 + 25*a^4*b^8 + 8*a^2*b^{10} + b^{12})*\tan \\
& (d*x + c)^2 + 2*(a^{11}*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 \\
& + a*b^{11})*\tan(d*x + c))/d
\end{aligned}$$

Fricas [B] time = 3.58986, size = 2137, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] $1/48*(195*a^8*b^3 - 427*a^6*b^5 - 165*a^4*b^7 + 27*a^2*b^9 + 2*b^{11} - 8*(a^{10}*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^{11})*\cos(d*x + c)^8 + 20*(2*a^{10}*b + 9*a^8*b^3 + 16*a^6*b^5 + 14*a^4*b^7 + 6*a^2*b^9 + b^{11})*\cos(d*x + c)^6 - 2*(49*a^{10}*b + 162*a^8*b^3 + 198*a^6*b^5 + 112*a^4*b^7 + 33*a^2*b^9 + 6*b^{11})*\cos(d*x + c)^4 + 3*(5*a^9*b^2 - 180*a^7*b^4 + 390*a^5*b^6 - 68*a^3*b^8 - 3*a*b^{10})*d*x + (9*a^{10}*b - 46*a^8*b^3 + 994*a^6*b^5 + 144*a^4*b^7 - 43*a^2*b^9 - 2*b^{11} + 3*(5*a^{11} - 185*a^9*b^2 + 570*a^7*b^4 - 458*a^5*b^6 + 65*a^3*b^8 + 3*a*b^{10})*d*x)*\cos(d*x + c)^2 + 24*(3*a^8*b^3 - 22*a^6*b^5 + 15*a^4*b^7 + (3*a^{10}*b - 25*a^8*b^3 + 37*a^6*b^5 - 15*a^4*b^7)*\cos(d*x + c)^2 + 2*(3*a^9*b^2 - 22*a^7*b^4 + 15*a^5*b^6)*\cos(d*x + c)*\sin(d*x + c))*\log(2*a*b*\cos(d*x + c)*\sin(d*x + c) + (a^2 - b^2)*\cos(d*x + c)^2 + b^2) - (8*(a^{11} + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b^8 + a*b^{10})*\cos(d*x + c)^7 - 2*(13*a^{11} + 55*a^9*b^2 + 90*a^7*b^4 + 70*a^5*b^6 + 25*a^3*b^8 + 3*a*b^{10})*\cos(d*x + c)^5 + (33*a^{11} + 49*a^9*b^2 - 54*a^7*b^4 - 126*a^5*b^6 - 59*a^3*b^8 - 3*a*b^{10})*\cos(d*x + c)^3 - (261*a^9*b^2 - 338*a^7*b^4 + 120*a^5*b^6 - 150*a^3*b^8 - 5*a*b^{10} + 6*(5*a^{10}*b - 180*a^8*b^3 + 390*a^6*b^5 - 68*a^4*b^7 - 3*a^2*b^9)*d*x)*\cos(d*x + c))*\sin(d*x + c))/((a^{14} + 5*a^{12}*b^2 + 9*a^{10}*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^{10} - 5*a^2*b^{12} - b^{14})*d*\cos(d*x + c)^2 + 2*(a^{13}*b + 6*a^{11}*b^3 + 15*a^9*b^5 + 20*a^7*b^7 + 15*a^5*b^9 + 6*a^3*b^{11} + a*b^{13})*d*\cos(d*x + c)*\sin(d*x + c) + (a^{12}*b^2 + 6*a^{10}*b^4 + 15*a^8*b^6 + 20*a^6*b^8 + 15*a^4*b^{10} + 6*a^2*b^{12} + b^{14})*d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**6/(a+b*tan(d*x+c))**3,x)

[Out] Timed out

Giac [B] time = 1.28551, size = 1246, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out]
$$\frac{1}{48} \cdot (3 \cdot (5a^9 - 180a^7b^2 + 390a^5b^4 - 68a^3b^6 - 3ab^8) \cdot (dx + c) / (a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) - 24 \cdot (3a^8b - 22a^6b^3 + 15a^4b^5) \cdot \log(\tan(dx + c)^2 + 1) / (a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) + 48 \cdot (3a^8b^2 - 22a^6b^4 + 15a^4b^6) \cdot \log(\text{abs}(b \cdot \tan(dx + c) + a)) / (a^{12}b + 6a^{10}b^3 + 15a^8b^5 + 20a^6b^7 + 15a^4b^9 + 6a^2b^{11} + b^{13}) - 24 \cdot (9a^8b^3 \tan(dx + c)^2 - 66a^6b^5 \tan(dx + c)^2 + 45a^4b^7 \tan(dx + c)^2 + 22a^9b^2 \tan(dx + c) - 140a^7b^4 \tan(dx + c) + 78a^5b^6 \tan(dx + c) + 14a^{10}b - 72a^8b^3 + 34a^6b^5) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) \cdot (b \cdot \tan(dx + c) + a)^2) + (132a^8b \tan(dx + c)^6 - 968a^6b^3 \tan(dx + c)^6 + 660a^4b^5 \tan(dx + c)^6 - 33a^9 \tan(dx + c)^5 + 324a^7b^2 \tan(dx + c)^5 + 162a^5b^4 \tan(dx + c)^5 - 204a^3b^6 \tan(dx + c)^5 - 9ab^8 \tan(dx + c)^5 + 180a^8b \tan(dx + c)^4 - 2760a^6b^3 \tan(dx + c)^4 + 2340a^4b^5 \tan(dx + c)^4 - 40a^9 \tan(dx + c)^3 + 576a^7b^2 \tan(dx + c)^3 + 96a^5b^4 \tan(dx + c)^3 - 544a^3b^6 \tan(dx + c)^3 - 24ab^8 \tan(dx + c)^3 + 72a^8b \tan(dx + c)^2 - 2448a^6b^3 \tan(dx + c)^2 + 2700a^4b^5 \tan(dx + c)^2 - 72a^2b^7 \tan(dx + c)^2 - 12b^9 \tan(dx + c)^2 - 15a^9 \tan(dx + c) + 252a^7b^2 \tan(dx + c) - 18a^5b^4 \tan(dx + c) - 276a^3b^6 \tan(dx + c) + 9ab^8 \tan(dx + c) - 720a^6b^3 + 972a^4b^5 - 72a^2b^7 - 4b^9) / ((a^{12} + 6a^{10}b^2 + 15a^8b^4 + 20a^6b^6 + 15a^4b^8 + 6a^2b^{10} + b^{12}) \cdot (\tan(dx + c)^2 + 1)^3) / d$$

$$3.68 \quad \int \frac{\sin^4(c+dx)}{(a+b \tan(c+dx))^3} dx$$

Optimal. Leaf size=285

$$\frac{a^4 b}{2d(a^2 + b^2)^3 (a + b \tan(c + dx))^2} - \frac{2a^3 b (a^2 - 2b^2)}{d(a^2 + b^2)^4 (a + b \tan(c + dx))} - \frac{a \cos^2(c + dx) ((-34a^2 b^2 + 5a^4 + 9b^4) \tan(c + dx))}{8d(a^2 + b^2)^4}$$

[Out] (3*a*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*x)/(8*(a^2 + b^2)^5) + (3*a^2*b*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^5*d) - (a^4*b)/(2*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) - (2*a^3*b*(a^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(4*(a^2 + b^2)^3*d) - (a*Cos[c + d*x]^2*(24*a*b*(a^2 - b^2) + (5*a^4 - 34*a^2*b^2 + 9*b^4)*Tan[c + d*x]))/(8*(a^2 + b^2)^4*d)

Rubi [A] time = 0.849748, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$\frac{a^4 b}{2d(a^2 + b^2)^3 (a + b \tan(c + dx))^2} - \frac{2a^3 b (a^2 - 2b^2)}{d(a^2 + b^2)^4 (a + b \tan(c + dx))} - \frac{a \cos^2(c + dx) ((-34a^2 b^2 + 5a^4 + 9b^4) \tan(c + dx))}{8d(a^2 + b^2)^4}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]

[Out] (3*a*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*x)/(8*(a^2 + b^2)^5) + (3*a^2*b*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^5*d) - (a^4*b)/(2*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) - (2*a^3*b*(a^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(4*(a^2 + b^2)^3*d) - (a*Cos[c + d*x]^2*(24*a*b*(a^2 - b^2) + (5*a^4 - 34*a^2*b^2 + 9*b^4)*Tan[c + d*x]))/(8*(a^2 + b^2)^4*d)

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 1647

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,

d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{\sin^4(c+dx)}{(a+b \tan(c+dx))^3} dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^4}{(a+x)^3(b^2+x^2)^3} dx, x, b \tan(c+dx)\right)}{d} \\ &= \frac{\cos^4(c+dx) (b(3a^2-b^2) + a(a^2-3b^2) \tan(c+dx))}{4(a^2+b^2)^3 d} - \operatorname{Subst}\left(\int \frac{\frac{a^4 b^4 (a^2-3b^2)}{(a^2+b^2)^3} - \frac{a^3 b^4 (9a^2+5b^2)x}{(a^2+b^2)^3}}{(a^2+b^2)^3} dx, x, b \tan(c+dx)\right) \\ &= \frac{\cos^4(c+dx) (b(3a^2-b^2) + a(a^2-3b^2) \tan(c+dx))}{4(a^2+b^2)^3 d} - \frac{a \cos^2(c+dx) (24ab(a^2-b^2) + 8(a^4-5a^2b^2+2b^4) \log(a+b \tan(c+dx)))}{8(a^2+b^2)^3 d} \\ &= \frac{\cos^4(c+dx) (b(3a^2-b^2) + a(a^2-3b^2) \tan(c+dx))}{4(a^2+b^2)^3 d} - \frac{a \cos^2(c+dx) (24ab(a^2-b^2) + 8(a^4-5a^2b^2+2b^4) \log(a+b \tan(c+dx)))}{8(a^2+b^2)^3 d} \\ &= \frac{3a^2b(a^4-5a^2b^2+2b^4) \log(a+b \tan(c+dx))}{(a^2+b^2)^5 d} - \frac{a^4b}{2(a^2+b^2)^3 d(a+b \tan(c+dx))^2} - \frac{a^4b}{2(a^2+b^2)^3 d(a+b \tan(c+dx))^2} \\ &= \frac{3a^2b(a^4-5a^2b^2+2b^4) \log(a+b \tan(c+dx))}{(a^2+b^2)^5 d} - \frac{a^4b}{2(a^2+b^2)^3 d(a+b \tan(c+dx))^2} - \frac{a^4b}{2(a^2+b^2)^3 d(a+b \tan(c+dx))^2} \\ &= \frac{3a(a^6-25a^4b^2+35a^2b^4-3b^6)x}{8(a^2+b^2)^5} + \frac{3a^2b(a^4-5a^2b^2+2b^4) \log(\cos(c+dx))}{(a^2+b^2)^5 d} + \frac{3a^2b(a^4-5a^2b^2+2b^4) \log(a+b \tan(c+dx))}{(a^2+b^2)^5 d} \end{aligned}$$

Mathematica [A] time = 6.43835, size = 501, normalized size = 1.76

$$b \left(-\frac{3a^2(a-b)(a+b) \cos^2(c+dx)}{(a^2+b^2)^4} + \frac{(3a^2-b^2) \cos^4(c+dx)}{4(a^2+b^2)^3} - \frac{a^3(a^2-5b^2) \tan^{-1}(\tan(c+dx))}{b(a^2+b^2)^4} - \frac{a^4}{2(a^2+b^2)^3(a+b \tan(c+dx))^2} - \frac{2a^3(a^2-2b^2)}{(a^2+b^2)^4(a+b \tan(c+dx))} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]

[Out] $(b * (-((a^3 * (a^2 - 5 * b^2) * \text{ArcTan}[\text{Tan}[c + d * x]])) / (b * (a^2 + b^2)^4)) - (3 * a^2 * (a - b) * (a + b) * \text{Cos}[c + d * x]^2) / (a^2 + b^2)^4 + ((3 * a^2 - b^2) * \text{Cos}[c + d * x]^4) / (4 * (a^2 + b^2)^3) - (a^2 * (3 * a^4 - 15 * a^2 * b^2 + 6 * b^4 - (a^5 - 13 * a^3 * b^2 + 10 * a * b^4) / \text{Sqrt}[-b^2]) * \text{Log}[\text{Sqrt}[-b^2] - b * \text{Tan}[c + d * x]]) / (2 * (a^2 + b^2)^5) + (3 * a^2 * (a^4 - 5 * a^2 * b^2 + 2 * b^4) * \text{Log}[a + b * \text{Tan}[c + d * x]]) / (a^2 + b^2)^5 - (a^2 * (3 * a^4 - 15 * a^2 * b^2 + 6 * b^4 + (a^5 - 13 * a^3 * b^2 + 10 * a * b^4) / \text{Sqrt}[-b^2]) * \text{Log}[\text{Sqrt}[-b^2] + b * \text{Tan}[c + d * x]]) / (2 * (a^2 + b^2)^5) - (a^3 * (a^2 - 5 * b^2) * \text{Cos}[c + d * x] * \text{Sin}[c + d * x]) / (b * (a^2 + b^2)^4) + (a * (a^2 - 3 * b^2) * \text{Cos}[c + d * x]^3 * \text{Sin}[c + d * x]) / (4 * b * (a^2 + b^2)^3) + (3 * a * (a^2 - 3 * b^2) * (\text{ArcTan}[\text{Tan}[c + d * x]] / b + (\text{Cos}[c + d * x] * \text{Sin}[c + d * x]) / b)) / (8 * (a^2 + b^2)^3) - a^4 / (2 * (a^2 + b^2)^3 * (a + b * \text{Tan}[c + d * x])^2) - (2 * a^3 * (a^2 - 2 * b^2)) / ((a^2 + b^2)^4 * (a + b * \text{Tan}[c + d * x])))) / d$

Maple [B] time = 0.109, size = 882, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x)

[Out] $3/8/d/(a^2+b^2)^5 * \arctan(\tan(d*x+c)) * a^7 + 3/d/(a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * \tan(d*x+c)^2 * a^2 * b^5 + 5/4/d/(a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * a^4 * b^3 + 13/4/d/(a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * a^2 * b^5 - 3/d/(a^2+b^2)^5 * \ln(1+\tan(d*x+c)^2) * a^2 * b^5 - 5/8/d/(a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * \tan(d*x+c)^3 * a^7 - 3/8/d/(a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * \tan(d*x+c) * a^7 - 9/8/d/(a^2+b^2)^5 * \arctan(\tan(d*x+c)) * a * b^6 - 75/8/d/(a^2+b^2)^5 * \arctan(\tan(d*x+c)) * a^5 * b^2 + 6/d * b^5 / (a^2+b^2)^5 * \ln(a + b * \tan(d*x+c)) * a^2 - 9/4/d/(a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * a^6 * b + 105/8/d/(a^2+b^2)^5 * \arctan(\tan(d*x+c)) * a^3 * b^4 - 3/2/d/(a^2+b^2)^5 * \ln(1+\tan(d*x+c)^2) * a^6 * b + 15/2/d/(a^2+b^2)^5 * \ln(1+\tan(d*x+c)^2) * a^4 * b^3 - 2/d * a^5 * b / (a^2+b^2)^4 / (a + b * \tan(d*x+c)) + 4/d * a^3 * b^3 / (a^2+b^2)^4 / (a + b * \tan(d*x+c)) + 3/d * b * a^6 / (a^2+b^2)^5 * \ln(a + b * \tan(d*x+c)) - 15/d * b^3 * a^4 / (a^2+b^2)^5 * \ln(a + b * \tan(d*x+c)) - 3/d / (a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * \tan(d*x+c)^2 * a^6 * b + 29/8/d/(a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * \tan(d*x+c)^3 * a^5 * b^2 - 1/2 * a^4 * b / (a^2+b^2)^3 / d / (a + b * \tan(d*x+c))^2 + 27/8/d / (a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * \tan(d*x+c) * a^5 * b^2 + 25/8/d/(a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * \tan(d*x+c)^3 * a^3 * b^4 - 9/8/d/(a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * \tan(d*x+c)^3 * a * b^6 - 1/4/d/(a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * b^7 - 15/8/d/(a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * \tan(d*x+c) * a * b^6 + 15/8/d/(a^2+b^2)^5 / (1+\tan(d*x+c)^2)^2 * \tan(d*x+c) * a^3 * b^4$

Maxima [B] time = 1.81091, size = 1004, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] $1/8 * (3 * (a^7 - 25 * a^5 * b^2 + 35 * a^3 * b^4 - 3 * a * b^6) * (d * x + c) / (a^{10} + 5 * a^8 * b^2 + 10 * a^6 * b^4 + 10 * a^4 * b^6 + 5 * a^2 * b^8 + b^{10}) + 24 * (a^6 * b - 5 * a^4 * b^3 + 2$

$$\begin{aligned} & *a^2*b^5*\log(b*\tan(d*x + c) + a)/(a^{10} + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^{10}) - 12*(a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*\log(\tan(d*x + c)^2 + 1)/(a^{10} + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^{10}) - \\ & (38*a^6*b - 56*a^4*b^3 + 2*a^2*b^5 + 3*(7*a^5*b^2 - 22*a^3*b^4 + 3*a*b^6))*\tan(d*x + c)^5 + 6*(5*a^6*b - 12*a^4*b^3 - a^2*b^5)*\tan(d*x + c)^4 + (5*a^7 + 49*a^5*b^2 - 133*a^3*b^4 + 15*a*b^6)*\tan(d*x + c)^3 + 2*(35*a^6*b - 61*a^4*b^3 + a^2*b^5 + b^7)*\tan(d*x + c)^2 + (3*a^7 + 22*a^5*b^2 - 73*a^3*b^4 + 4*a*b^6)*\tan(d*x + c))/(a^{10} + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8 + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^{10})*\tan(d*x + c)^6 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*\tan(d*x + c)^5 + (a^{10} + 6*a^8*b^2 + 14*a^6*b^4 + 16*a^4*b^6 + 9*a^2*b^8 + 2*b^{10})*\tan(d*x + c)^4 + 4*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*\tan(d*x + c)^3 + (2*a^{10} + 9*a^8*b^2 + 16*a^6*b^4 + 14*a^4*b^6 + 6*a^2*b^8 + b^{10})*\tan(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*\tan(d*x + c))/d \end{aligned}$$

Fricas [B] time = 3.09478, size = 1569, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] $\frac{1}{32}*(119*a^6*b^3 - 159*a^4*b^5 - 51*a^2*b^7 + 3*b^9 + 8*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*\cos(d*x + c)^6 - 8*(5*a^8*b + 16*a^6*b^3 + 18*a^4*b^5 + 8*a^2*b^7 + b^9)*\cos(d*x + c)^4 + 12*(a^7*b^2 - 25*a^5*b^4 + 35*a^3*b^6 - 3*a*b^8)*d*x - (a^8*b + 110*a^6*b^3 - 420*a^4*b^5 - 78*a^2*b^7 + 3*b^9 - 12*(a^9 - 26*a^7*b^2 + 60*a^5*b^4 - 38*a^3*b^6 + 3*a*b^8)*d*x)*\cos(d*x + c)^2 + 48*(a^6*b^3 - 5*a^4*b^5 + 2*a^2*b^7 + (a^8*b - 6*a^6*b^3 + 7*a^4*b^5 - 2*a^2*b^7)*\cos(d*x + c)^2 + 2*(a^7*b^2 - 5*a^5*b^4 + 2*a^3*b^6)*\cos(d*x + c)*\sin(d*x + c))*\log(2*a*b*\cos(d*x + c)*\sin(d*x + c) + (a^2 - b^2)*\cos(d*x + c)^2 + b^2) + 2*(4*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*\cos(d*x + c)^5 - 2*(5*a^9 + 12*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 - 3*a*b^8)*\cos(d*x + c)^3 + (77*a^7*b^2 - 69*a^5*b^4 + 63*a^3*b^6 - 15*a*b^8 + 12*(a^8*b - 25*a^6*b^3 + 35*a^4*b^5 - 3*a^2*b^7)*d*x)*\cos(d*x + c))*\sin(d*x + c))/((a^{12} + 4*a^{10}*b^2 + 5*a^8*b^4 - 5*a^4*b^8 - 4*a^2*b^{10} - b^{12})*d*\cos(d*x + c)^2 + 2*(a^{11}*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^{11})*d*\cos(d*x + c)*\sin(d*x + c) + (a^{10}*b^2 + 5*a^8*b^4 + 10*a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^{10} + b^{12})*d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**3,x)

[Out] Timed out

Giac [B] time = 1.27008, size = 794, normalized size = 2.79

$$\frac{3(a^7 - 25a^5b^2 + 35a^3b^4 - 3ab^6)(dx+c)}{a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}} - \frac{12(a^6b - 5a^4b^3 + 2a^2b^5)\log(\tan(dx+c)^2 + 1)}{a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}} + \frac{24(a^6b^2 - 5a^4b^4 + 2a^2b^6)\log(|b\tan(dx+c)+a|)}{a^{10}b + 5a^8b^3 + 10a^6b^5 + 10a^4b^7 + 5a^2b^9 + b^{11}} - \frac{21a^5b^2\tan(dx+c)}{a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] $\frac{1}{8} \cdot (3 \cdot (a^7 - 25a^5b^2 + 35a^3b^4 - 3ab^6) \cdot (dx + c) / (a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) - 12 \cdot (a^6b - 5a^4b^3 + 2a^2b^5) \cdot \log(\tan(dx + c)^2 + 1) / (a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}) + 24 \cdot (a^6b^2 - 5a^4b^4 + 2a^2b^6) \cdot \log(\text{abs}(b \cdot \tan(dx + c) + a)) / (a^{10}b + 5a^8b^3 + 10a^6b^5 + 10a^4b^7 + 5a^2b^9 + b^{11}) - (21a^5b^2 \cdot \tan(dx + c)^5 - 66a^3b^4 \cdot \tan(dx + c)^5 + 9a^2b^6 \cdot \tan(dx + c)^5 + 30a^6b \cdot \tan(dx + c)^4 - 72a^4b^3 \cdot \tan(dx + c)^4 - 6a^2b^5 \cdot \tan(dx + c)^4 + 5a^7 \cdot \tan(dx + c)^3 + 49a^5b^2 \cdot \tan(dx + c)^3 - 13a^3b^4 \cdot \tan(dx + c)^3 + 15a^2b^6 \cdot \tan(dx + c)^3 + 70a^6b \cdot \tan(dx + c)^2 - 122a^4b^3 \cdot \tan(dx + c)^2 + 2a^2b^5 \cdot \tan(dx + c)^2 + 2b^7 \cdot \tan(dx + c)^2 + 3a^7 \cdot \tan(dx + c) + 22a^5b^2 \cdot \tan(dx + c) - 73a^3b^4 \cdot \tan(dx + c) + 4a^2b^6 \cdot \tan(dx + c) + 38a^6b - 56a^4b^3 + 2a^2b^5) / ((a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) \cdot (b \cdot \tan(dx + c))^3 + a \cdot \tan(dx + c)^2 + b \cdot \tan(dx + c) + a)^2) / d$

$$3.69 \quad \int \frac{\sin^2(c+dx)}{(a+b \tan(c+dx))^3} dx$$

Optimal. Leaf size=206

$$\frac{a^2 b}{2d(a^2 + b^2)^2 (a + b \tan(c + dx))^2} - \frac{2ab(a^2 - b^2)}{d(a^2 + b^2)^3 (a + b \tan(c + dx))} - \frac{\cos^2(c + dx)(a(a^2 - 3b^2) \tan(c + dx) + b(3a^2 - b^2))}{2d(a^2 + b^2)^3}$$

[Out] (a*(a^4 - 14*a^2*b^2 + 9*b^4)*x)/(2*(a^2 + b^2)^4) + (b*(3*a^4 - 8*a^2*b^2 + b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^2*b)/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) - (2*a*b*(a^2 - b^2))/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(2*(a^2 + b^2)^3*d)

Rubi [A] time = 0.396246, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$\frac{a^2 b}{2d(a^2 + b^2)^2 (a + b \tan(c + dx))^2} - \frac{2ab(a^2 - b^2)}{d(a^2 + b^2)^3 (a + b \tan(c + dx))} - \frac{\cos^2(c + dx)(a(a^2 - 3b^2) \tan(c + dx) + b(3a^2 - b^2))}{2d(a^2 + b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]

[Out] (a*(a^4 - 14*a^2*b^2 + 9*b^4)*x)/(2*(a^2 + b^2)^4) + (b*(3*a^4 - 8*a^2*b^2 + b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^2*b)/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) - (2*a*b*(a^2 - b^2))/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(2*(a^2 + b^2)^3*d)

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 1647

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt
[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sin^2(c + dx)}{(a + b \tan(c + dx))^3} dx = \frac{b \operatorname{Subst}\left(\int \frac{x^2}{(a+x)^3(b^2+x^2)^2} dx, x, b \tan(c + dx)\right)}{d}$$

$$= -\frac{\cos^2(c + dx) (b(3a^2 - b^2) + a(a^2 - 3b^2) \tan(c + dx))}{2(a^2 + b^2)^3 d} - \operatorname{Subst}\left(\int \frac{-\frac{a^4 b^2 (a^2 - 3b^2)}{(a^2 + b^2)^3} + \frac{a^3 b^2 (3a^2 + 7b^2)x}{(a^2 + b^2)^3}}{(a+x)^3} dx, x, b \tan(c + dx)\right)$$

$$= -\frac{\cos^2(c + dx) (b(3a^2 - b^2) + a(a^2 - 3b^2) \tan(c + dx))}{2(a^2 + b^2)^3 d} - \operatorname{Subst}\left(\int \left(-\frac{2a^2 b^2}{(a^2 + b^2)^2 (a+x)^3} + \frac{4ab^2}{(a^2 + b^2)^2 (a+x)^3}\right) dx, x, b \tan(c + dx)\right)$$

$$= \frac{b(3a^4 - 8a^2 b^2 + b^4) \log(a + b \tan(c + dx))}{(a^2 + b^2)^4 d} - \frac{a^2 b}{2(a^2 + b^2)^2 d (a + b \tan(c + dx))^2} - \frac{a^2 b}{(a^2 + b^2)^2 d (a + b \tan(c + dx))^2}$$

$$= \frac{b(3a^4 - 8a^2 b^2 + b^4) \log(a + b \tan(c + dx))}{(a^2 + b^2)^4 d} - \frac{a^2 b}{2(a^2 + b^2)^2 d (a + b \tan(c + dx))^2} - \frac{a^2 b}{(a^2 + b^2)^2 d (a + b \tan(c + dx))^2}$$

$$= \frac{a(a^4 - 14a^2 b^2 + 9b^4)x}{2(a^2 + b^2)^4} + \frac{b(3a^4 - 8a^2 b^2 + b^4) \log(\cos(c + dx))}{(a^2 + b^2)^4 d} + \frac{b(3a^4 - 8a^2 b^2 + b^4) \log(a + b \tan(c + dx))}{(a^2 + b^2)^4 d}$$

Mathematica [A] time = 3.95863, size = 316, normalized size = 1.53

$$b \left(\frac{a(a^2 - 3b^2)(a^2 + b^2) \sin(2(c + dx))}{2b} + (3a^2 - b^2)(a^2 + b^2) \cos^2(c + dx) + \frac{a(a^2 - 3b^2)(a^2 + b^2) \tan^{-1}(\tan(c + dx))}{b} + \frac{a^2(a^2 + b^2)^2}{(a + b \tan(c + dx))^2} + \frac{4(a^5 - b^5)}{a + b \tan(c + dx)} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^3, x]
```

```
[Out] -(b*((a*(a^2 - 3*b^2)*(a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + (3*a^2 - b^2)*(
a^2 + b^2)*Cos[c + d*x]^2 + (3*a^4 - 8*a^2*b^2 + b^4 - (a^5 - 8*a^3*b^2 + 3
*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*(3*a^4 - 8*a^2*b^2
+ b^4)*Log[a + b*Tan[c + d*x]] + (3*a^4 - 8*a^2*b^2 + b^4 + (a^5 - 8*a^3*b
```

$$\sqrt{-2 + 3ab^4}/\sqrt{-b^2})\cdot\text{Log}[\sqrt{-b^2} + b\cdot\tan[c + dx]] + (a(a^2 - 3b^2)(a^2 + b^2)\sin[2(c + dx)])/(2b) + (a^2(a^2 + b^2)^2)/(a + b\cdot\tan[c + dx])^2 + (4(a^5 - ab^4))/(a + b\cdot\tan[c + dx]))/(2(a^2 + b^2)^4d)$$

Maple [B] time = 0.115, size = 542, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(dx+c)^2/(a+b*tan(dx+c))^3,x)

[Out]
$$-1/2d/(a^2+b^2)^4/(1+\tan(dx+c)^2)\cdot\tan(dx+c)\cdot a^5+1/d/(a^2+b^2)^4/(1+\tan(dx+c)^2)\cdot\tan(dx+c)\cdot a^3b^2+3/2d/(a^2+b^2)^4/(1+\tan(dx+c)^2)\cdot\tan(dx+c)\cdot ab^4-3/2d/(a^2+b^2)^4/(1+\tan(dx+c)^2)\cdot a^4b-1/d/(a^2+b^2)^4/(1+\tan(dx+c)^2)\cdot a^2b^3+1/2d/(a^2+b^2)^4/(1+\tan(dx+c)^2)\cdot b^5-3/2d/(a^2+b^2)^4\cdot\ln(1+\tan(dx+c)^2)\cdot a^4b+4/d/(a^2+b^2)^4\cdot\ln(1+\tan(dx+c)^2)\cdot a^2b^3-1/2d/(a^2+b^2)^4\cdot\ln(1+\tan(dx+c)^2)\cdot b^5-7/d/(a^2+b^2)^4\cdot\arctan(\tan(dx+c))\cdot a^3b^2+9/2d/(a^2+b^2)^4\cdot\arctan(\tan(dx+c))\cdot ab^4+1/2d/(a^2+b^2)^4\cdot\arctan(\tan(dx+c))\cdot a^5-1/2\cdot a^2b/(a^2+b^2)^2d/(a+b\cdot\tan(dx+c))^2+3db/(a^2+b^2)^4\cdot\ln(a+b\cdot\tan(dx+c))\cdot a^4-8/db^3/(a^2+b^2)^4\cdot\ln(a+b\cdot\tan(dx+c))\cdot a^2+1/db^5/(a^2+b^2)^4\cdot\ln(a+b\cdot\tan(dx+c))-2/db\cdot a^3/(a^2+b^2)^3/(a+b\cdot\tan(dx+c))+2/db^3/(a^2+b^2)^3\cdot a/(a+b\cdot\tan(dx+c))$$

Maxima [B] time = 1.84901, size = 625, normalized size = 3.03

$$\frac{(a^5-14a^3b^2+9ab^4)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} + \frac{2(3a^4b-8a^2b^3+b^5)\log(b\tan(dx+c)+a)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{(3a^4b-8a^2b^3+b^5)\log(\tan(dx+c)^2+1)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{2a^8+3a^6b^2+3a^4b^4+a^2b^6+(a^6b^2+3a^4b^4+3a^2b^6+b^8)\tan(dx+c)}{2a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(dx+c)^2/(a+b*tan(dx+c))^3,x, algorithm="maxima")

[Out]
$$1/2\cdot((a^5 - 14a^3b^2 + 9a^2b^4)\cdot(dx + c)/(a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) + 2\cdot(3a^4b - 8a^2b^3 + b^5)\cdot\log(b\cdot\tan(dx + c) + a)/(a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) - (3a^4b - 8a^2b^3 + b^5)\cdot\log(\tan(dx + c)^2 + 1)/(a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8) - (8a^4b - 4a^2b^3 + (5a^3b^2 - 7a^2b^4)\cdot\tan(dx + c)^3 + (7a^4b - 6a^2b^3 - b^5)\cdot\tan(dx + c)^2 + (a^5 + 7a^3b^2 - 6a^2b^4)\cdot\tan(dx + c)))/(a^8 + 3a^6b^2 + 3a^4b^4 + a^2b^6 + (a^6b^2 + 3a^4b^4 + 3a^2b^6 + b^8)\cdot\tan(dx + c)^4 + 2\cdot(a^7b + 3a^5b^3 + 3a^3b^5 + a^2b^7)\cdot\tan(dx + c)^3 + (a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8)\cdot\tan(dx + c)^2 + 2\cdot(a^7b + 3a^5b^3 + 3a^3b^5 + a^2b^7)\cdot\tan(dx + c)))/d$$

Fricas [B] time = 2.48319, size = 1152, normalized size = 5.59

$$13a^4b^3 - 8a^2b^5 - b^7 - 2(a^6b + 3a^4b^3 + 3a^2b^5 + b^7)\cos(dx + c)^4 + 2(a^5b^2 - 14a^3b^4 + 9ab^6)dx - (a^6b + 23a^4b^3 - 2a^2b^5 - b^7)\sin(dx + c)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] $\frac{1}{4}*(13*a^4*b^3 - 8*a^2*b^5 - b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) * \cos(d*x + c)^4 + 2*(a^5*b^2 - 14*a^3*b^4 + 9*a*b^6)*d*x - (a^6*b + 23*a^4*b^3 - 21*a^2*b^5 - 3*b^7 - 2*(a^7 - 15*a^5*b^2 + 23*a^3*b^4 - 9*a*b^6)*d*x) * \cos(d*x + c)^2 + 2*(3*a^4*b^3 - 8*a^2*b^5 + b^7 + (3*a^6*b - 11*a^4*b^3 + 9*a^2*b^5 - b^7)*\cos(d*x + c)^2 + 2*(3*a^5*b^2 - 8*a^3*b^4 + a*b^6)*\cos(d*x + c)*\sin(d*x + c))*\log(2*a*b*\cos(d*x + c)*\sin(d*x + c) + (a^2 - b^2)*\cos(d*x + c)^2 + b^2) - 2*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*\cos(d*x + c)^3 - 2*(4*a^5*b^2 - 3*a^3*b^4 + 3*a*b^6 + (a^6*b - 14*a^4*b^3 + 9*a^2*b^5)*d*x) * \cos(d*x + c))*\sin(d*x + c))/((a^{10} + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^{10})*d*\cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*\cos(d*x + c)*\sin(d*x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^{10})*d)$

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**3,x)

[Out] Exception raised: AttributeError

Giac [B] time = 1.23699, size = 651, normalized size = 3.16

$$\frac{(a^5 - 14a^3b^2 + 9ab^4)(dx+c)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} - \frac{(3a^4b - 8a^2b^3 + b^5) \log(\tan(dx+c)^2 + 1)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} + \frac{2(3a^4b^2 - 8a^2b^4 + b^6) \log(|b \tan(dx+c) + a|)}{a^8b + 4a^6b^3 + 6a^4b^5 + 4a^2b^7 + b^9} + \frac{3a^4b \tan(dx+c)^2 - 8a^2b^3 \tan(dx+c)^2}{(a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] $\frac{1}{2}*((a^5 - 14*a^3*b^2 + 9*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (3*a^4*b - 8*a^2*b^3 + b^5)*\log(\tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 2*(3*a^4*b^2 - 8*a^2*b^4 + b^6)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) + (3*a^4*b*\tan(d*x + c)^2 - 8*a^2*b^3*\tan(d*x + c)^2 + b^5*\tan(d*x + c)^2 - a^5*\tan(d*x + c) + 2*a^3*b^2*\tan(d*x + c) + 3*a*b^4*\tan(d*x + c) - 10*a^2*b^3 + 2*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(\tan(d*x + c)^2 + 1)) - (9*a^4*b^3*\tan(d*x + c)^2 - 24*a^2*b^5*\tan(d*x + c)^2 + 3*b^7*\tan(d*x + c)^2 + 22*a^5*b^2*\tan(d*x + c) - 48*a^3*b^4*\tan(d*x + c) + 2*a*b^6*\tan(d*x + c) + 14*a^6*b - 22*a^4*b^3)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*\tan(d*x + c) + a)^2)/d$

$$3.70 \quad \int \frac{\csc^2(c+dx)}{(a+b \tan(c+dx))^3} dx$$

Optimal. Leaf size=95

$$\frac{2b}{a^3d(a+b \tan(c+dx))} - \frac{b}{2a^2d(a+b \tan(c+dx))^2} - \frac{3b \log(\tan(c+dx))}{a^4d} + \frac{3b \log(a+b \tan(c+dx))}{a^4d} - \frac{\cot(c+dx)}{a^3d}$$

[Out] $-(\text{Cot}[c + d*x]/(a^3*d)) - (3*b*\text{Log}[\text{Tan}[c + d*x]])/(a^4*d) + (3*b*\text{Log}[a + b*\text{Tan}[c + d*x]])/(a^4*d) - b/(2*a^2*d*(a + b*\text{Tan}[c + d*x])^2) - (2*b)/(a^3*d*(a + b*\text{Tan}[c + d*x]))$

Rubi [A] time = 0.0774468, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 44}

$$\frac{2b}{a^3d(a+b \tan(c+dx))} - \frac{b}{2a^2d(a+b \tan(c+dx))^2} - \frac{3b \log(\tan(c+dx))}{a^4d} + \frac{3b \log(a+b \tan(c+dx))}{a^4d} - \frac{\cot(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c + d*x]^2/(a + b*\text{Tan}[c + d*x])^3, x]$

[Out] $-(\text{Cot}[c + d*x]/(a^3*d)) - (3*b*\text{Log}[\text{Tan}[c + d*x]])/(a^4*d) + (3*b*\text{Log}[a + b*\text{Tan}[c + d*x]])/(a^4*d) - b/(2*a^2*d*(a + b*\text{Tan}[c + d*x])^2) - (2*b)/(a^3*d*(a + b*\text{Tan}[c + d*x]))$

Rule 3516

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] := \text{Dist}[b/f, \text{Subst}[\text{Int}[(x^m*(a+x)^n)/(b^2+x^2)^{(m/2+1)}, x], x, b*\text{Tan}[e+f*x]], x] /;$ FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 44

$\text{Int}[((a_.) + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a+b*x)^m*(c+d*x)^n, x], x] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(c+dx)}{(a+b \tan(c+dx))^3} dx &= \frac{b \text{Subst}\left(\int \frac{1}{x^2(a+x)^3} dx, x, b \tan(c+dx)\right)}{d} \\ &= \frac{b \text{Subst}\left(\int \left(\frac{1}{a^3x^2} - \frac{3}{a^4x} + \frac{1}{a^2(a+x)^3} + \frac{2}{a^3(a+x)^2} + \frac{3}{a^4(a+x)}\right) dx, x, b \tan(c+dx)\right)}{d} \\ &= -\frac{\cot(c+dx)}{a^3d} - \frac{3b \log(\tan(c+dx))}{a^4d} + \frac{3b \log(a+b \tan(c+dx))}{a^4d} - \frac{b}{2a^2d(a+b \tan(c+dx))} \end{aligned}$$

Mathematica [B] time = 2.58893, size = 241, normalized size = 2.54

$$b(a^2(-b^2)\sec^2(c+dx) - 2a^2(a^2+b^2)(-3\log(a\cos(c+dx)+b\sin(c+dx))+3\log(\sin(c+dx))+2) - 2b^2\tan^2(c+dx))$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]

[Out] $(-2*a^3*(a^2 + b^2)*\cot[c + d*x] + b*(-2*a^2*(a^2 + b^2)*(2 + 3*\log[\sin[c + d*x]] - 3*\log[a*\cos[c + d*x] + b*\sin[c + d*x]]) - a^2*b^2*\sec[c + d*x]^2 + 2*a*b*(2*a^2 + b^2 - 6*(a^2 + b^2)*\log[\sin[c + d*x]] + 6*(a^2 + b^2)*\log[a*\cos[c + d*x] + b*\sin[c + d*x]])*tan[c + d*x] - 2*b^2*(-3*a^2 - 2*b^2 + 3*(a^2 + b^2)*\log[\sin[c + d*x]] - 3*(a^2 + b^2)*\log[a*\cos[c + d*x] + b*\sin[c + d*x]])*tan[c + d*x]^2)/(2*a^4*(a^2 + b^2)*d*(a + b*\tan[c + d*x])^2)$

Maple [A] time = 0.119, size = 96, normalized size = 1.

$$-\frac{1}{da^3 \tan(dx+c)} - 3 \frac{b \ln(\tan(dx+c))}{a^4 d} - \frac{b}{2 a^2 d (a + b \tan(dx+c))^2} + 3 \frac{b \ln(a + b \tan(dx+c))}{a^4 d} - 2 \frac{b}{da^3 (a + b \tan(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x)

[Out] $-1/d/a^3/\tan(d*x+c) - 3*b*\ln(\tan(d*x+c))/a^4/d - 1/2*b/a^2/d/(a+b*\tan(d*x+c))^2 + 3*b*\ln(a+b*\tan(d*x+c))/a^4/d - 2*b/a^3/d/(a+b*\tan(d*x+c))$

Maxima [A] time = 1.14572, size = 146, normalized size = 1.54

$$-\frac{\frac{6b^2 \tan(dx+c)^2 + 9ab \tan(dx+c) + 2a^2}{a^3 b^2 \tan(dx+c)^3 + 2a^4 b \tan(dx+c)^2 + a^5 \tan(dx+c)} - \frac{6b \log(b \tan(dx+c) + a)}{a^4} + \frac{6b \log(\tan(dx+c))}{a^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] $-1/2*((6*b^2*\tan(d*x + c)^2 + 9*a*b*\tan(d*x + c) + 2*a^2)/(a^3*b^2*\tan(d*x + c)^3 + 2*a^4*b*\tan(d*x + c)^2 + a^5*\tan(d*x + c)) - 6*b*\log(b*\tan(d*x + c) + a)/a^4 + 6*b*\log(\tan(d*x + c))/a^4)/d$

Fricas [B] time = 2.4596, size = 1219, normalized size = 12.83

$$2(a^7 + 4a^5b^2 - 2a^3b^4 - 3ab^6) \cos(dx+c)^3 - 2(2a^5b^2 - 3a^3b^4 - 3ab^6) \cos(dx+c) + 3(2(a^5b^2 + 2a^3b^4 + ab^6) \cos(dx+c) - 2(a^5b^2 + 2a^3b^4 + ab^6) \cos(dx+c)^3 - 2(a^5b^2 + 2a^3b^4 + ab^6) \cos(dx+c) - (a^4b^3 + 2a^2b^5 + b^7 + (a^6b + a^4b^3 - a^2b^5 - b^7) \cos(dx+c)^2) \sin(dx+c)) \log(2a*b*\cos(dx+c)*\sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 + b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] $1/2*(2*(a^7 + 4*a^5*b^2 - 2*a^3*b^4 - 3*a*b^6)*\cos(d*x + c)^3 - 2*(2*a^5*b^2 - 3*a^3*b^4 - 3*a*b^6)*\cos(d*x + c) + 3*(2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*\cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*\cos(d*x + c) - (a^4*b^3 + 2*a^2*b^5 + b^7 + (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(2*a*b*\cos(d*x + c)*\sin(d*x + c) + (a^2 - b^2)*\cos(d*x + c)^2 + b$

$$\begin{aligned} &^2) - 3*(2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*\cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*\cos(d*x + c) - (a^4*b^3 + 2*a^2*b^5 + b^7 + (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(-1/4*\cos(d*x + c)^2 + 1/4) - (5*a^4*b^3 + 3*a^2*b^5 - 4*(a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*\cos(d*x + c)^2)*\sin(d*x + c))/(2*(a^9*b + 2*a^7*b^3 + a^5*b^5)*d*\cos(d*x + c)^3 - 2*(a^9*b + 2*a^7*b^3 + a^5*b^5)*d*\cos(d*x + c) - ((a^10 + a^8*b^2 - a^6*b^4 - a^4*b^6)*d*\cos(d*x + c)^2 + (a^8*b^2 + 2*a^6*b^4 + a^4*b^6)*d)*\sin(d*x + c)) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**3,x)

[Out] Timed out

Giac [A] time = 1.26987, size = 153, normalized size = 1.61

$$\frac{\frac{6b \log(b \tan(dx+c)+a)}{a^4} - \frac{6b \log(|\tan(dx+c)|)}{a^4} + \frac{2(3b \tan(dx+c)-a)}{a^4 \tan(dx+c)} - \frac{9b^3 \tan(dx+c)^2 + 22ab^2 \tan(dx+c) + 14a^2b}{(b \tan(dx+c)+a)^2 a^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] 1/2*(6*b*log(abs(b*tan(d*x + c) + a))/a^4 - 6*b*log(abs(tan(d*x + c)))/a^4 + 2*(3*b*tan(d*x + c) - a)/(a^4*tan(d*x + c)) - (9*b^3*tan(d*x + c)^2 + 22*a*b^2*tan(d*x + c) + 14*a^2*b)/((b*tan(d*x + c) + a)^2*a^4))/d

$$3.71 \quad \int \frac{\csc^4(c+dx)}{(a+b \tan(c+dx))^3} dx$$

Optimal. Leaf size=178

$$\frac{2b(a^2+2b^2)}{a^5d(a+b \tan(c+dx))} - \frac{b(a^2+b^2)}{2a^4d(a+b \tan(c+dx))^2} - \frac{(a^2+6b^2) \cot(c+dx)}{a^5d} - \frac{b(3a^2+10b^2) \log(\tan(c+dx))}{a^6d} + \frac{b(3a^2+10b^2)}{a^6d}$$

[Out] -(((a^2 + 6*b^2)*Cot[c + d*x])/(a^5*d)) + (3*b*Cot[c + d*x]^2)/(2*a^4*d) - Cot[c + d*x]^3/(3*a^3*d) - (b*(3*a^2 + 10*b^2)*Log[Tan[c + d*x]])/(a^6*d) + (b*(3*a^2 + 10*b^2)*Log[a + b*Tan[c + d*x]])/(a^6*d) - (b*(a^2 + b^2))/(2*a^4*d*(a + b*Tan[c + d*x])^2) - (2*b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d*x]))

Rubi [A] time = 0.152575, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 894}

$$\frac{2b(a^2+2b^2)}{a^5d(a+b \tan(c+dx))} - \frac{b(a^2+b^2)}{2a^4d(a+b \tan(c+dx))^2} - \frac{(a^2+6b^2) \cot(c+dx)}{a^5d} - \frac{b(3a^2+10b^2) \log(\tan(c+dx))}{a^6d} + \frac{b(3a^2+10b^2)}{a^6d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]

[Out] -(((a^2 + 6*b^2)*Cot[c + d*x])/(a^5*d)) + (3*b*Cot[c + d*x]^2)/(2*a^4*d) - Cot[c + d*x]^3/(3*a^3*d) - (b*(3*a^2 + 10*b^2)*Log[Tan[c + d*x]])/(a^6*d) + (b*(3*a^2 + 10*b^2)*Log[a + b*Tan[c + d*x]])/(a^6*d) - (b*(a^2 + b^2))/(2*a^4*d*(a + b*Tan[c + d*x])^2) - (2*b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d*x]))

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 894

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{\csc^4(c+dx)}{(a+b \tan(c+dx))^3} dx &= \frac{b \operatorname{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^3} dx, x, b \tan(c+dx)\right)}{d} \\ &= \frac{b \operatorname{Subst}\left(\int \left(\frac{b^2}{a^3x^4} - \frac{3b^2}{a^4x^3} + \frac{a^2+6b^2}{a^5x^2} + \frac{-3a^2-10b^2}{a^6x} + \frac{a^2+b^2}{a^4(a+x)^3} + \frac{2(a^2+2b^2)}{a^5(a+x)^2} + \frac{3a^2+10b^2}{a^6(a+x)}\right) dx, x, b \tan(c+dx)\right)}{d} \\ &= -\frac{(a^2+6b^2) \cot(c+dx)}{a^5d} + \frac{3b \cot^2(c+dx)}{2a^4d} - \frac{\cot^3(c+dx)}{3a^3d} - \frac{b(3a^2+10b^2) \log(\tan(c+dx))}{a^6d} \end{aligned}$$

Mathematica [B] time = 3.22664, size = 456, normalized size = 2.56

$$\frac{b^3 \sec^3(c + dx)(a \cos(c + dx) + b \sin(c + dx))}{2a^4 d(a + b \tan(c + dx))^3} + \frac{\sec^3(c + dx)(3a^2 b^2 \sin(c + dx) + 4b^4 \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))}{a^6 d(a + b \tan(c + dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]

[Out] $-(b^3 \operatorname{Sec}[c + d*x]^3(a \operatorname{Cos}[c + d*x] + b \operatorname{Sin}[c + d*x]))/(2*a^4*d*(a + b \operatorname{Tan}[c + d*x])^3) - (\operatorname{Csc}[c + d*x]^3 \operatorname{Sec}[c + d*x]^2(a \operatorname{Cos}[c + d*x] + b \operatorname{Sin}[c + d*x])^3)/(3*a^3*d*(a + b \operatorname{Tan}[c + d*x])^3) - (2*(a^2 \operatorname{Cos}[c + d*x] + 9*b^2 \operatorname{Cos}[c + d*x]) \operatorname{Csc}[c + d*x] \operatorname{Sec}[c + d*x]^3(a \operatorname{Cos}[c + d*x] + b \operatorname{Sin}[c + d*x])^3)/(3*a^5*d*(a + b \operatorname{Tan}[c + d*x])^3) + (3*b \operatorname{Csc}[c + d*x]^2 \operatorname{Sec}[c + d*x]^3(a \operatorname{Cos}[c + d*x] + b \operatorname{Sin}[c + d*x])^3)/(2*a^4*d*(a + b \operatorname{Tan}[c + d*x])^3) + ((-3*a^2*b - 10*b^3) \operatorname{Log}[\operatorname{Sin}[c + d*x]] \operatorname{Sec}[c + d*x]^3(a \operatorname{Cos}[c + d*x] + b \operatorname{Sin}[c + d*x])^3)/(a^6*d*(a + b \operatorname{Tan}[c + d*x])^3) + ((3*a^2*b + 10*b^3) \operatorname{Log}[a \operatorname{Cos}[c + d*x] + b \operatorname{Sin}[c + d*x]] \operatorname{Sec}[c + d*x]^3(a \operatorname{Cos}[c + d*x] + b \operatorname{Sin}[c + d*x])^3)/(a^6*d*(a + b \operatorname{Tan}[c + d*x])^3) + (\operatorname{Sec}[c + d*x]^3(a \operatorname{Cos}[c + d*x] + b \operatorname{Sin}[c + d*x])^2(3*a^2*b^2 \operatorname{Sin}[c + d*x] + 4*b^4 \operatorname{Sin}[c + d*x]))/(a^6*d*(a + b \operatorname{Tan}[c + d*x])^3)$

Maple [A] time = 0.13, size = 234, normalized size = 1.3

$$\frac{1}{3da^3(\tan(dx+c))^3} - \frac{1}{da^3 \tan(dx+c)} - 6 \frac{b^2}{da^5 \tan(dx+c)} + \frac{3b}{2da^4(\tan(dx+c))^2} - 3 \frac{b \ln(\tan(dx+c))}{da^4} - 10 \frac{b^3 \ln(\tan(dx+c))}{da^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x)

[Out] $-1/3/d/a^3/\tan(d*x+c)^3 - 1/d/a^3/\tan(d*x+c) - 6/d/a^5/\tan(d*x+c)*b^2 + 3/2/d/a^4*b/\tan(d*x+c)^2 - 3*b*\ln(\tan(d*x+c))/a^4/d - 10/d*b^3/a^6*\ln(\tan(d*x+c)) + 3*b*\ln(a+b*\tan(d*x+c))/a^4/d + 10/d*b^3/a^6*\ln(a+b*\tan(d*x+c)) - 1/2*b/a^2/d/(a+b*\tan(d*x+c))^2 - 1/2/d*b^3/a^4/(a+b*\tan(d*x+c))^2 - 2*b/a^3/d/(a+b*\tan(d*x+c)) - 4/d*b^3/a^5/(a+b*\tan(d*x+c))$

Maxima [A] time = 1.20175, size = 259, normalized size = 1.46

$$\frac{5a^3b \tan(dx+c) - 6(3a^2b^2 + 10b^4) \tan(dx+c)^4 - 2a^4 - 9(3a^3b + 10ab^3) \tan(dx+c)^3 - 2(3a^4 + 10a^2b^2) \tan(dx+c)^2}{a^5b^2 \tan(dx+c)^5 + 2a^6b \tan(dx+c)^4 + a^7 \tan(dx+c)^3} + \frac{6(3a^2b + 10b^3) \log(b \tan(dx+c) + a)}{a^6} - \frac{6}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] $1/6*((5*a^3*b*\tan(d*x + c) - 6*(3*a^2*b^2 + 10*b^4)*\tan(d*x + c)^4 - 2*a^4 - 9*(3*a^3*b + 10*a*b^3)*\tan(d*x + c)^3 - 2*(3*a^4 + 10*a^2*b^2)*\tan(d*x + c)^2)/(a^5*b^2*\tan(d*x + c)^5 + 2*a^6*b*\tan(d*x + c)^4 + a^7*\tan(d*x + c)^3) + 6*(3*a^2*b + 10*b^3)*\log(b*\tan(d*x + c) + a)/a^6 - 6*(3*a^2*b + 10*b^3)*\log(\tan(d*x + c))/a^6/d$

Fricas [B] time = 2.59892, size = 1818, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] $\frac{1}{6} * (2 * (2 * a^7 + 27 * a^5 * b^2 + a^3 * b^4 - 30 * a * b^6) * \cos(d * x + c)^5 - 2 * (3 * a^7 + 43 * a^5 * b^2 - 8 * a^3 * b^4 - 60 * a * b^6) * \cos(d * x + c)^3 + 6 * (5 * a^5 * b^2 - 3 * a^3 * b^4 - 10 * a * b^6) * \cos(d * x + c) + 3 * (2 * (3 * a^5 * b^2 + 13 * a^3 * b^4 + 10 * a * b^6) * \cos(d * x + c)^5 - 4 * (3 * a^5 * b^2 + 13 * a^3 * b^4 + 10 * a * b^6) * \cos(d * x + c)^3 + 2 * (3 * a^5 * b^2 + 13 * a^3 * b^4 + 10 * a * b^6) * \cos(d * x + c) + (3 * a^4 * b^3 + 13 * a^2 * b^5 + 10 * b^7 - (3 * a^6 * b + 10 * a^4 * b^3 - 3 * a^2 * b^5 - 10 * b^7) * \cos(d * x + c)^4 + (3 * a^6 * b + 7 * a^4 * b^3 - 16 * a^2 * b^5 - 20 * b^7) * \cos(d * x + c)^2) * \sin(d * x + c)) * \log(2 * a * b * \cos(d * x + c) * \sin(d * x + c) + (a^2 - b^2) * \cos(d * x + c)^2 + b^2) - 3 * (2 * (3 * a^5 * b^2 + 13 * a^3 * b^4 + 10 * a * b^6) * \cos(d * x + c)^5 - 4 * (3 * a^5 * b^2 + 13 * a^3 * b^4 + 10 * a * b^6) * \cos(d * x + c)^3 + 2 * (3 * a^5 * b^2 + 13 * a^3 * b^4 + 10 * a * b^6) * \cos(d * x + c) + (3 * a^4 * b^3 + 13 * a^2 * b^5 + 10 * b^7 - (3 * a^6 * b + 10 * a^4 * b^3 - 3 * a^2 * b^5 - 10 * b^7) * \cos(d * x + c)^4 + (3 * a^6 * b + 7 * a^4 * b^3 - 16 * a^2 * b^5 - 20 * b^7) * \cos(d * x + c)^2) * \sin(d * x + c)) * \log(-1/4 * \cos(d * x + c)^2 + 1/4) + (24 * a^4 * b^3 + 30 * a^2 * b^5 + 4 * (2 * a^6 * b + 29 * a^4 * b^3 + 30 * a^2 * b^5) * \cos(d * x + c)^4 - 3 * (a^6 * b + 45 * a^4 * b^3 + 50 * a^2 * b^5) * \cos(d * x + c)^2) * \sin(d * x + c)) / (2 * (a^9 * b + a^7 * b^3) * d * \cos(d * x + c)^5 - 4 * (a^9 * b + a^7 * b^3) * d * \cos(d * x + c)^3 + 2 * (a^9 * b + a^7 * b^3) * d * \cos(d * x + c) - ((a^10 - a^6 * b^4) * d * \cos(d * x + c)^4 - (a^10 - a^8 * b^2 - 2 * a^6 * b^4) * d * \cos(d * x + c)^2 - (a^8 * b^2 + a^6 * b^4) * d) * \sin(d * x + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**3,x)

[Out] Timed out

Giac [A] time = 1.34649, size = 320, normalized size = 1.8

$$\frac{6(3a^2b+10b^3)\log(|\tan(dx+c)|)}{a^6} - \frac{6(3a^2b^2+10b^4)\log(|b\tan(dx+c)+a|)}{a^6b} + \frac{3(9a^2b^3\tan(dx+c)^2+30b^5\tan(dx+c)^2+22a^3b^2\tan(dx+c)+68ab^4\tan(dx+c)+30b^6)}{(b\tan(dx+c)+a)^2a^6}$$

6d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] $-\frac{1}{6} * (6 * (3 * a^2 * b + 10 * b^3) * \log(\text{abs}(\tan(d * x + c)))) / a^6 - 6 * (3 * a^2 * b^2 + 10 * b^4) * \log(\text{abs}(b * \tan(d * x + c) + a)) / (a^6 * b) + 3 * (9 * a^2 * b^3 * \tan(d * x + c)^2 + 30 * b^5 * \tan(d * x + c)^2 + 22 * a^3 * b^2 * \tan(d * x + c) + 68 * a * b^4 * \tan(d * x + c) + 14 * a^4 * b + 39 * a^2 * b^3) / ((b * \tan(d * x + c) + a)^2 * a^6) - (33 * a^2 * b * \tan(d * x + c)^3 + 110 * b^3 * \tan(d * x + c)^3 - 6 * a^3 * \tan(d * x + c)^2 - 36 * a * b^2 * \tan(d * x + c)^2 + 9 * a^2 * b * \tan(d * x + c) - 2 * a^3) / (a^6 * \tan(d * x + c)^3) / d$

$$3.72 \quad \int \frac{\csc^6(c+dx)}{(a+b \tan(c+dx))^3} dx$$

Optimal. Leaf size=265

$$\frac{2b(a^2+b^2)(a^2+3b^2)}{a^7d(a+b \tan(c+dx))} - \frac{b(a^2+b^2)^2}{2a^6d(a+b \tan(c+dx))^2} - \frac{2(a^2+3b^2)\cot^3(c+dx)}{3a^5d} + \frac{b(3a^2+5b^2)\cot^2(c+dx)}{a^6d} - \frac{(12a^2+5b^2)\cot(c+dx)}{a^6d}$$

[Out] -(((a^4 + 12*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(a^7*d)) + (b*(3*a^2 + 5*b^2)*Cot[c + d*x]^2)/(a^6*d) - (2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^5*d) + (3*b*Cot[c + d*x]^4)/(4*a^4*d) - Cot[c + d*x]^5/(5*a^3*d) - (b*(3*a^4 + 20*a^2*b^2 + 21*b^4)*Log[Tan[c + d*x]])/(a^8*d) + (b*(3*a^4 + 20*a^2*b^2 + 21*b^4)*Log[a + b*Tan[c + d*x]])/(a^8*d) - (b*(a^2 + b^2)^2)/(2*a^6*d*(a + b*Tan[c + d*x])^2) - (2*b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x]))

Rubi [A] time = 0.239401, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 894}

$$\frac{2b(a^2+b^2)(a^2+3b^2)}{a^7d(a+b \tan(c+dx))} - \frac{b(a^2+b^2)^2}{2a^6d(a+b \tan(c+dx))^2} - \frac{2(a^2+3b^2)\cot^3(c+dx)}{3a^5d} + \frac{b(3a^2+5b^2)\cot^2(c+dx)}{a^6d} - \frac{(12a^2+5b^2)\cot(c+dx)}{a^6d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]

[Out] -(((a^4 + 12*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(a^7*d)) + (b*(3*a^2 + 5*b^2)*Cot[c + d*x]^2)/(a^6*d) - (2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^5*d) + (3*b*Cot[c + d*x]^4)/(4*a^4*d) - Cot[c + d*x]^5/(5*a^3*d) - (b*(3*a^4 + 20*a^2*b^2 + 21*b^4)*Log[Tan[c + d*x]])/(a^8*d) + (b*(3*a^4 + 20*a^2*b^2 + 21*b^4)*Log[a + b*Tan[c + d*x]])/(a^8*d) - (b*(a^2 + b^2)^2)/(2*a^6*d*(a + b*Tan[c + d*x])^2) - (2*b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x]))

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 894

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3} dx = \frac{b \operatorname{Subst}\left(\int \frac{(b^2+x^2)^2}{x^6(a+x)^3} dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b \operatorname{Subst}\left(\int \left(\frac{b^4}{a^3x^6} - \frac{3b^4}{a^4x^5} + \frac{2b^2(a^2+3b^2)}{a^5x^4} - \frac{2(3a^2b^2+5b^4)}{a^6x^3} + \frac{a^4+12a^2b^2+15b^4}{a^7x^2} + \frac{-3a^4-20a^2b^2-21b^4}{a^8x} + \frac{(a^2+b^2)}{a^6(a+x)}\right) dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{(a^4+12a^2b^2+15b^4)\cot(c+dx)}{a^7d} + \frac{b(3a^2+5b^2)\cot^2(c+dx)}{a^6d} - \frac{2(a^2+3b^2)\cot^3(c+dx)}{3a^5d}$$

Mathematica [A] time = 4.67453, size = 494, normalized size = 1.86

$$\frac{\csc^5(c+dx)(5\sec(c+dx)(-3b(89a^4b^2+345a^2b^4+8a^6+210b^6)\tan(c+dx)-27a^5b^2-42a^3b^4+40a^7+135ab^6)+\dots}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^3, x]

[Out] $-(\operatorname{Csc}[c+d*x]^5(\operatorname{Sec}[c+d*x]^2((8*a^7+567*a^5*b^2+630*a^3*b^4-1215*a*b^6)\operatorname{Cos}[3*(c+d*x)]-(24*a^7+619*a^5*b^2+630*a^3*b^4-675*a*b^6)\operatorname{Cos}[5*(c+d*x)]+8*a^7\operatorname{Cos}[7*(c+d*x)]+187*a^5*b^2\operatorname{Cos}[7*(c+d*x)]+210*a^3*b^4\operatorname{Cos}[7*(c+d*x)]-135*a*b^6\operatorname{Cos}[7*(c+d*x)]-126*a^6*b\operatorname{Sin}[3*(c+d*x)]+1665*a^4*b^3\operatorname{Sin}[3*(c+d*x)]+4635*a^2*b^5\operatorname{Sin}[3*(c+d*x)]+1890*b^7\operatorname{Sin}[3*(c+d*x)]+10*a^6*b\operatorname{Sin}[5*(c+d*x)]-1215*a^4*b^3\operatorname{Sin}[5*(c+d*x)]-2565*a^2*b^5\operatorname{Sin}[5*(c+d*x)]-630*b^7\operatorname{Sin}[5*(c+d*x)]+16*a^6*b\operatorname{Sin}[7*(c+d*x)]+345*a^4*b^3\operatorname{Sin}[7*(c+d*x)]+585*a^2*b^5\operatorname{Sin}[7*(c+d*x)]+90*b^7\operatorname{Sin}[7*(c+d*x)])+960*b*(3*a^4+20*a^2*b^2+21*b^4)(\operatorname{Log}[\operatorname{Sin}[c+d*x]]-\operatorname{Log}[a\operatorname{Cos}[c+d*x]+b\operatorname{Sin}[c+d*x]])\operatorname{Sin}[c+d*x]^5(a+b\tan[c+d*x])^2+5\operatorname{Sec}[c+d*x](40*a^7-27*a^5*b^2-42*a^3*b^4+135*a*b^6-3*b(8*a^6+89*a^4*b^2+345*a^2*b^4+210*b^6)\tan[c+d*x])))/(960*a^8*d(a+b\tan[c+d*x])^2)$

Maple [A] time = 0.137, size = 410, normalized size = 1.6

$$-\frac{1}{5da^3(\tan(dx+c))^5} - \frac{2}{3da^3(\tan(dx+c))^3} - 2\frac{b^2}{da^5(\tan(dx+c))^3} - \frac{1}{da^3\tan(dx+c)} - 12\frac{b^2}{da^5\tan(dx+c)} - 15\frac{b^2}{da^7\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^3, x)

[Out] $-1/5/d/a^3/\tan(d*x+c)^5-2/3/d/a^3/\tan(d*x+c)^3-2/d/a^5/\tan(d*x+c)^3*b^2-1/d/a^3/\tan(d*x+c)-12/d/a^5/\tan(d*x+c)*b^2-15/d/a^7/\tan(d*x+c)*b^4+3/4/d/a^4*b/\tan(d*x+c)^4+3/d/a^4*b/\tan(d*x+c)^2+5/d*b^3/a^6/\tan(d*x+c)^2-3*b*\ln(\tan(d*x+c))/a^4/d-20/d*b^3/a^6*\ln(\tan(d*x+c))-21/d*b^5/a^8*\ln(\tan(d*x+c))+3*b*\ln(a+b*\tan(d*x+c))/a^4/d+20/d*b^3/a^6*\ln(a+b*\tan(d*x+c))+21/d*b^5/a^8*\ln(a+b*\tan(d*x+c))-1/2*b/a^2/d/(a+b*\tan(d*x+c))^2-1/d*b^3/a^4/(a+b*\tan(d*x+c))^2-1/2/d*b^5/a^6/(a+b*\tan(d*x+c))^2-2*b/a^3/d/(a+b*\tan(d*x+c))-8/d*b^3/a^5/(a+b*\tan(d*x+c))-6/d*b^5/a^7/(a+b*\tan(d*x+c))$

Maxima [A] time = 1.21861, size = 379, normalized size = 1.43

$$\frac{21 a^5 b \tan(dx+c) - 60 (3 a^4 b^2 + 20 a^2 b^4 + 21 b^6) \tan(dx+c)^6 - 12 a^6 - 90 (3 a^5 b + 20 a^3 b^3 + 21 a b^5) \tan(dx+c)^5 - 20 (3 a^6 + 20 a^4 b^2 + 21 a^2 b^4) \tan(dx+c)^4 + 5 (20 a^5 b + 21 a^3 b^3) \tan(dx+c)^3 - 2 (20 a^6 + 21 a^4 b^2) \tan(dx+c)^2}{a^7 b^2 \tan(dx+c)^7 + 2 a^8 b \tan(dx+c)^6 + a^9 \tan(dx+c)^5} + 60 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] $\frac{1}{60} * ((21 * a^5 * b * \tan(d * x + c) - 60 * (3 * a^4 * b^2 + 20 * a^2 * b^4 + 21 * b^6) * \tan(d * x + c)^6 - 12 * a^6 - 90 * (3 * a^5 * b + 20 * a^3 * b^3 + 21 * a * b^5) * \tan(d * x + c)^5 - 20 * (3 * a^6 + 20 * a^4 * b^2 + 21 * a^2 * b^4) * \tan(d * x + c)^4 + 5 * (20 * a^5 * b + 21 * a^3 * b^3) * \tan(d * x + c)^3 - 2 * (20 * a^6 + 21 * a^4 * b^2) * \tan(d * x + c)^2) / (a^7 * b^2 * \tan(d * x + c)^7 + 2 * a^8 * b * \tan(d * x + c)^6 + a^9 * \tan(d * x + c)^5) + 60 * (3 * a^4 * b + 20 * a^2 * b^3 + 21 * b^5) * \log(b * \tan(d * x + c) + a) / a^8 - 60 * (3 * a^4 * b + 20 * a^2 * b^3 + 21 * b^5) * \log(\tan(d * x + c)) / a^8) / d$

Fricas [B] time = 3.01524, size = 2344, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] $\frac{1}{60} * (4 * (8 * a^7 + 187 * a^5 * b^2 + 120 * a^3 * b^4 - 315 * a * b^6) * \cos(d * x + c)^7 - 4 * (20 * a^7 + 482 * a^5 * b^2 + 255 * a^3 * b^4 - 945 * a * b^6) * \cos(d * x + c)^5 + 10 * (6 * a^7 + 157 * a^5 * b^2 + 60 * a^3 * b^4 - 378 * a * b^6) * \cos(d * x + c)^3 - 30 * (13 * a^5 * b^2 + 2 * a^3 * b^4 - 42 * a * b^6) * \cos(d * x + c) + 30 * (2 * (3 * a^5 * b^2 + 20 * a^3 * b^4 + 21 * a * b^6) * \cos(d * x + c)^7 - 6 * (3 * a^5 * b^2 + 20 * a^3 * b^4 + 21 * a * b^6) * \cos(d * x + c)^5 + 6 * (3 * a^5 * b^2 + 20 * a^3 * b^4 + 21 * a * b^6) * \cos(d * x + c)^3 - 2 * (3 * a^5 * b^2 + 20 * a^3 * b^4 + 21 * a * b^6) * \cos(d * x + c) - (3 * a^4 * b^3 + 20 * a^2 * b^5 + 21 * b^7 + (3 * a^6 * b + 17 * a^4 * b^3 + a^2 * b^5 - 21 * b^7) * \cos(d * x + c)^6 - (6 * a^6 * b + 31 * a^4 * b^3 - 18 * a^2 * b^5 - 63 * b^7) * \cos(d * x + c)^4 + (3 * a^6 * b + 11 * a^4 * b^3 - 39 * a^2 * b^5 - 63 * b^7) * \cos(d * x + c)^2) * \sin(d * x + c)) * \log(2 * a * b * \cos(d * x + c) * \sin(d * x + c) + (a^2 - b^2) * \cos(d * x + c)^2 + b^2) - 30 * (2 * (3 * a^5 * b^2 + 20 * a^3 * b^4 + 21 * a * b^6) * \cos(d * x + c)^7 - 6 * (3 * a^5 * b^2 + 20 * a^3 * b^4 + 21 * a * b^6) * \cos(d * x + c)^5 + 6 * (3 * a^5 * b^2 + 20 * a^3 * b^4 + 21 * a * b^6) * \cos(d * x + c)^3 - 2 * (3 * a^5 * b^2 + 20 * a^3 * b^4 + 21 * a * b^6) * \cos(d * x + c) - (3 * a^4 * b^3 + 20 * a^2 * b^5 + 21 * b^7 + (3 * a^6 * b + 17 * a^4 * b^3 + a^2 * b^5 - 21 * b^7) * \cos(d * x + c)^6 - (6 * a^6 * b + 31 * a^4 * b^3 - 18 * a^2 * b^5 - 63 * b^7) * \cos(d * x + c)^4 + (3 * a^6 * b + 11 * a^4 * b^3 - 39 * a^2 * b^5 - 63 * b^7) * \cos(d * x + c)^2) * \sin(d * x + c)) * \log(-1/4 * \cos(d * x + c)^2 + 1/4) - (285 * a^4 * b^3 + 630 * a^2 * b^5 - 8 * (8 * a^6 * b + 195 * a^4 * b^3 + 315 * a^2 * b^5) * \cos(d * x + c)^6 + 10 * (7 * a^6 * b + 330 * a^4 * b^3 + 567 * a^2 * b^5) * \cos(d * x + c)^4 + 15 * (a^6 * b - 135 * a^4 * b^3 - 252 * a^2 * b^5) * \cos(d * x + c)^2) * \sin(d * x + c)) / (2 * a^9 * b * d * \cos(d * x + c)^7 - 6 * a^9 * b * d * \cos(d * x + c)^5 + 6 * a^9 * b * d * \cos(d * x + c)^3 - 2 * a^9 * b * d * \cos(d * x + c) - (a^8 * b^2 * d + (a^10 - a^8 * b^2) * d * \cos(d * x + c)^6 - (2 * a^10 - 3 * a^8 * b^2) * d * \cos(d * x + c)^4 + (a^10 - 3 * a^8 * b^2) * d * \cos(d * x + c)^2) * \sin(d * x + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**3,x)

[Out] Timed out

Giac [A] time = 1.23827, size = 516, normalized size = 1.95

$$\frac{60(3a^4b+20a^2b^3+21b^5)\log(|\tan(dx+c)|)}{a^8} - \frac{60(3a^4b^2+20a^2b^4+21b^6)\log(|b\tan(dx+c)+a|)}{a^8b} + \frac{30(9a^4b^3\tan(dx+c)^2+60a^2b^5\tan(dx+c)^2+63b^7\tan(dx+c)^2+22a^5b^2\tan(dx+c)+136a^3b^4\tan(dx+c)+138ab^6\tan(dx+c)+14a^6b+78a^4b^3+76a^2b^5)/((b\tan(dx+c)+a)^2a^8) - (411a^4b\tan(dx+c)^5+2740a^2b^3\tan(dx+c)^5+2877b^5\tan(dx+c)^5-60a^5\tan(dx+c)^4-720a^3b^2\tan(dx+c)^4-900ab^4\tan(dx+c)^4+180a^4b\tan(dx+c)^3+300a^2b^3\tan(dx+c)^3-40a^5\tan(dx+c)^2-120a^3b^2\tan(dx+c)^2+45a^4b\tan(dx+c)-12a^5)/(a^8\tan(dx+c)^5)/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] -1/60*(60*(3*a^4*b + 20*a^2*b^3 + 21*b^5)*log(abs(tan(d*x + c)))/a^8 - 60*(3*a^4*b^2 + 20*a^2*b^4 + 21*b^6)*log(abs(b*tan(d*x + c) + a))/(a^8*b) + 30*(9*a^4*b^3*tan(d*x + c)^2 + 60*a^2*b^5*tan(d*x + c)^2 + 63*b^7*tan(d*x + c)^2 + 22*a^5*b^2*tan(d*x + c) + 136*a^3*b^4*tan(d*x + c) + 138*a*b^6*tan(d*x + c) + 14*a^6*b + 78*a^4*b^3 + 76*a^2*b^5)/((b*tan(d*x + c) + a)^2*a^8) - (411*a^4*b*tan(d*x + c)^5 + 2740*a^2*b^3*tan(d*x + c)^5 + 2877*b^5*tan(d*x + c)^5 - 60*a^5*tan(d*x + c)^4 - 720*a^3*b^2*tan(d*x + c)^4 - 900*a*b^4*tan(d*x + c)^4 + 180*a^4*b*tan(d*x + c)^3 + 300*a^2*b^3*tan(d*x + c)^3 - 40*a^5*tan(d*x + c)^2 - 120*a^3*b^2*tan(d*x + c)^2 + 45*a^4*b*tan(d*x + c) - 12*a^5)/(a^8*tan(d*x + c)^5))/d

3.73 $\int \frac{\sin^4(c+dx)}{(a+b \tan(c+dx))^4} dx$

Optimal. Leaf size=366

$$\frac{a^4 b}{3d(a^2 + b^2)^3 (a + b \tan(c + dx))^3} - \frac{a^3 b(a^2 - 2b^2)}{d(a^2 + b^2)^4 (a + b \tan(c + dx))^2} - \frac{3a^2 b(-5a^2 b^2 + a^4 + 2b^4)}{d(a^2 + b^2)^5 (a + b \tan(c + dx))} + \frac{\cos^4(c + dx)}{d(a^2 + b^2)^5 (a + b \tan(c + dx))}$$

```
[Out] ((3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*x)/(8*(a^2 + b^2)^6) + (4*a*b*(a^2 - b^2)*(a^4 - 8*a^2*b^2 + b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^6*d) - (a^4*b)/(3*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^3) - (a^3*b*(a^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])^2) - (3*a^2*b*(a^4 - 5*a^2*b^2 + 2*b^4))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(4*a*b*(a^2 - b^2) + (a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x]))/(4*(a^2 + b^2)^4*d) - (Cos[c + d*x]^2*(16*a*b*(2*a^4 - 5*a^2*b^2 + b^4) + (5*a^6 - 65*a^4*b^2 + 55*a^2*b^4 - 3*b^6)*Tan[c + d*x]))/(8*(a^2 + b^2)^5*d)
```

Rubi [A] time = 1.37025, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$\frac{a^4 b}{3d(a^2 + b^2)^3 (a + b \tan(c + dx))^3} - \frac{a^3 b(a^2 - 2b^2)}{d(a^2 + b^2)^4 (a + b \tan(c + dx))^2} - \frac{3a^2 b(-5a^2 b^2 + a^4 + 2b^4)}{d(a^2 + b^2)^5 (a + b \tan(c + dx))} + \frac{\cos^4(c + dx)}{d(a^2 + b^2)^5 (a + b \tan(c + dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*x)/(8*(a^2 + b^2)^6) + (4*a*b*(a^2 - b^2)*(a^4 - 8*a^2*b^2 + b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^6*d) - (a^4*b)/(3*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^3) - (a^3*b*(a^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])^2) - (3*a^2*b*(a^4 - 5*a^2*b^2 + 2*b^4))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(4*a*b*(a^2 - b^2) + (a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x]))/(4*(a^2 + b^2)^4*d) - (Cos[c + d*x]^2*(16*a*b*(2*a^4 - 5*a^2*b^2 + b^4) + (5*a^6 - 65*a^4*b^2 + 55*a^2*b^4 - 3*b^6)*Tan[c + d*x]))/(8*(a^2 + b^2)^5*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sin^4(c + dx)}{(a + b \tan(c + dx))^4} dx = \frac{b \operatorname{Subst}\left(\int \frac{x^4}{(a+x)^4(b^2+x^2)^3} dx, x, b \tan(c + dx)\right)}{d}$$

$$= \frac{\cos^4(c + dx) (4ab(a^2 - b^2) + (a^4 - 6a^2b^2 + b^4) \tan(c + dx))}{4(a^2 + b^2)^4 d} - \operatorname{Subst}\left(\int \frac{\frac{a^4 b^4 (a^4 - 6a^2 b^2 + b^4) - 4a^3 b}{(a^2 + b^2)^4}}{(a^2 + b^2)^4} dx, x, b \tan(c + dx)\right)$$

$$= \frac{\cos^4(c + dx) (4ab(a^2 - b^2) + (a^4 - 6a^2b^2 + b^4) \tan(c + dx))}{4(a^2 + b^2)^4 d} - \frac{\cos^2(c + dx) (16ab(2a^4 - 5a^2b^2 + b^4))}{4(a^2 + b^2)^4 d}$$

$$= \frac{\cos^4(c + dx) (4ab(a^2 - b^2) + (a^4 - 6a^2b^2 + b^4) \tan(c + dx))}{4(a^2 + b^2)^4 d} - \frac{\cos^2(c + dx) (16ab(2a^4 - 5a^2b^2 + b^4))}{4(a^2 + b^2)^4 d}$$

$$= \frac{4ab(a^2 - b^2)(a^4 - 8a^2b^2 + b^4) \log(a + b \tan(c + dx))}{(a^2 + b^2)^6 d} - \frac{a^4 b}{3(a^2 + b^2)^3 d(a + b \tan(c + dx))^3}$$

$$= \frac{4ab(a^2 - b^2)(a^4 - 8a^2b^2 + b^4) \log(a + b \tan(c + dx))}{(a^2 + b^2)^6 d} - \frac{a^4 b}{3(a^2 + b^2)^3 d(a + b \tan(c + dx))^3}$$

$$= \frac{(3a^8 - 132a^6b^2 + 370a^4b^4 - 132a^2b^6 + 3b^8) x}{8(a^2 + b^2)^6} + \frac{4ab(a^2 - b^2)(a^4 - 8a^2b^2 + b^4) \log(\cos(c + dx))}{(a^2 + b^2)^6 d}$$

Mathematica [A] time = 5.41854, size = 564, normalized size = 1.54

$$b \left(\frac{12a^2(a^2+b^2)(-10a^2b^2+a^4+5b^4)\sin(2(c+dx))}{b} - 24a(a-b)(a+b)(a^2+b^2)^2 \cos^4(c+dx) + 48a(a^2+b^2)(-5a^2b^2+2a^4+b^4) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^4, x]

[Out]
$$\begin{aligned} & -(b*((24*a^2*(a^2 + b^2)*(a^4 - 10*a^2*b^2 + 5*b^4)*ArcTan[Tan[c + d*x]]))/b \\ & + 48*a*(a^2 + b^2)*(2*a^4 - 5*a^2*b^2 + b^4)*Cos[c + d*x]^2 - 24*a*(a - b) \\ & *(a + b)*(a^2 + b^2)^2*Cos[c + d*x]^4 + 12*a*(4*a^6 - 36*a^4*b^2 + 36*a^2*b^4 \\ & - 4*b^6 + (-a^7 + 24*a^5*b^2 - 45*a^3*b^4 + 10*a*b^6)/Sqrt[-b^2])*Log[Sq \\ & rt[-b^2] - b*Tan[c + d*x]] - 96*a*(a - b)*(a + b)*(a^4 - 8*a^2*b^2 + b^4)*L \\ & og[a + b*Tan[c + d*x]] + 12*a*(4*a^6 - 36*a^4*b^2 + 36*a^2*b^4 - 4*b^6 + (a \\ & ^7 - 24*a^5*b^2 + 45*a^3*b^4 - 10*a*b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan \\ & [c + d*x]] - (6*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x]^3*Sin[c \\ & + d*x])/b + (12*a^2*(a^2 + b^2)*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[2*(c + d*x)] \\ &)/b - (9*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*(2*ArcTan[Tan[c + d*x]] + Si \\ & n[2*(c + d*x)]))/(2*b) + (8*a^4*(a^2 + b^2)^3)/(a + b*Tan[c + d*x])^3 + (24 \\ & *a^3*(a^2 - 2*b^2)*(a^2 + b^2)^2)/(a + b*Tan[c + d*x])^2 + (72*a^2*(a^2 + b \\ & ^2)*(a^4 - 5*a^2*b^2 + 2*b^4))/(a + b*Tan[c + d*x])/(24*(a^2 + b^2)^6*d) \end{aligned}$$

Maple [B] time = 0.123, size = 1215, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^4, x)

[Out]
$$\begin{aligned} & -3/d/(a^2+b^2)^6/(1+\tan(d*x+c))^2*a*b^7+4/d*b*a^7/(a^2+b^2)^6*\ln(a+b*\tan(\\ & d*x+c))-36/d*b^3*a^5/(a^2+b^2)^6*\ln(a+b*\tan(d*x+c))+36/d*b^5*a^3/(a^2+b^2)^ \\ & 6*\ln(a+b*\tan(d*x+c))-4/d*b^7*a/(a^2+b^2)^6*\ln(a+b*\tan(d*x+c))-3/d*b*a^6/(a^ \\ & 2+b^2)^5/(a+b*\tan(d*x+c))-1/3*a^4*b/(a^2+b^2)^3/d/(a+b*\tan(d*x+c))^3-5/4/d/ \\ & (a^2+b^2)^6/(1+\tan(d*x+c))^2)^2*\tan(d*x+c)*a^4*b^4-3/8/d/(a^2+b^2)^6/(1+\tan(\\ & d*x+c))^2)^2*\tan(d*x+c)*a^8+15/d*b^3*a^4/(a^2+b^2)^5/(a+b*\tan(d*x+c))-6/d*b^ \\ & 5*a^2/(a^2+b^2)^5/(a+b*\tan(d*x+c))-1/d*a^5*b/(a^2+b^2)^4/(a+b*\tan(d*x+c))^2 \\ & +2/d*a^3*b^3/(a^2+b^2)^4/(a+b*\tan(d*x+c))^2-2/d/(a^2+b^2)^6*\ln(1+\tan(d*x+c) \\ & ^2)*a^7*b+18/d/(a^2+b^2)^6*\ln(1+\tan(d*x+c))^2)*a^5*b^3-18/d/(a^2+b^2)^6*\ln(1 \\ & +\tan(d*x+c))^2)*a^3*b^5+2/d/(a^2+b^2)^6*\ln(1+\tan(d*x+c))^2)*a*b^7-33/2/d/(a^2 \\ & +b^2)^6*arctan(\tan(d*x+c))*a^6*b^2+185/4/d/(a^2+b^2)^6*arctan(\tan(d*x+c))*a \\ & ^4*b^4-33/2/d/(a^2+b^2)^6*arctan(\tan(d*x+c))*a^2*b^6-5/8/d/(a^2+b^2)^6/(1+t \\ & an(d*x+c))^2)^2*\tan(d*x+c)^3*a^8+3/8/d/(a^2+b^2)^6/(1+\tan(d*x+c))^2)^2*\tan(d* \\ & x+c)^3*b^8+5/8/d/(a^2+b^2)^6/(1+\tan(d*x+c))^2)^2*\tan(d*x+c)*b^8-3/d/(a^2+b^2 \\ &)^6/(1+\tan(d*x+c))^2)^2*a^7*b+7/d/(a^2+b^2)^6/(1+\tan(d*x+c))^2)^2*b^3*a^5+7/d \\ & /(a^2+b^2)^6/(1+\tan(d*x+c))^2)^2*a^3*b^5+3/8/d/(a^2+b^2)^6*arctan(\tan(d*x+c) \\ &)*b^8+3/8/d/(a^2+b^2)^6*arctan(\tan(d*x+c))*a^8+15/2/d/(a^2+b^2)^6/(1+\tan(d* \\ & x+c))^2)^2*\tan(d*x+c)^3*a^6*b^2+5/4/d/(a^2+b^2)^6/(1+\tan(d*x+c))^2)^2*\tan(d*x \\ & +c)^3*a^4*b^4-13/2/d/(a^2+b^2)^6/(1+\tan(d*x+c))^2)^2*\tan(d*x+c)^3*a^2*b^6-4/ \\ & d/(a^2+b^2)^6/(1+\tan(d*x+c))^2)^2*\tan(d*x+c)^2*a^7*b+6/d/(a^2+b^2)^6/(1+\tan(\\ & d*x+c))^2)^2*\tan(d*x+c)^2*a^5*b^3+8/d/(a^2+b^2)^6/(1+\tan(d*x+c))^2)^2*\tan(d*x \\ & +c)^2*a^3*b^5-2/d/(a^2+b^2)^6/(1+\tan(d*x+c))^2)^2*\tan(d*x+c)^2*a*b^7+13/2/d/ \\ & (a^2+b^2)^6/(1+\tan(d*x+c))^2)^2*\tan(d*x+c)*a^6*b^2-15/2/d/(a^2+b^2)^6/(1+\tan \end{aligned}$$

$$(d*x+c)^2)^2*\tan(d*x+c)*a^2*b^6$$

Maxima [B] time = 1.89595, size = 1346, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="maxima")

[Out] $\frac{1}{24}*(3*(3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*(d*x + c) / (a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12}) + 96*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*\log(b*\tan(d*x + c) + a) / (a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12}) - 48*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*\log(\tan(d*x + c)^2 + 1) / (a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12}) - (176*a^8*b - 608*a^6*b^3 + 176*a^4*b^5 + 3*(29*a^6*b^3 - 185*a^4*b^5 + 103*a^2*b^7 - 3*b^9)*\tan(d*x + c)^6 + 3*(71*a^7*b^2 - 411*a^5*b^4 + 165*a^3*b^6 + 7*a*b^8)*\tan(d*x + c)^5 + (149*a^8*b - 512*a^6*b^3 - 1006*a^4*b^5 + 600*a^2*b^7 - 15*b^9)*\tan(d*x + c)^4 + 3*(5*a^9 + 152*a^7*b^2 - 822*a^5*b^4 + 320*a^3*b^6 + 9*a*b^8)*\tan(d*x + c)^3 + (331*a^8*b - 1183*a^6*b^3 - 239*a^4*b^5 + 315*a^2*b^7)*\tan(d*x + c)^2 + 3*(3*a^9 + 73*a^7*b^2 - 423*a^5*b^4 + 147*a^3*b^6)*\tan(d*x + c)) / (a^{13} + 5*a^{11}*b^2 + 10*a^9*b^4 + 10*a^7*b^6 + 5*a^5*b^8 + a^3*b^{10} + (a^{10}*b^3 + 5*a^8*b^5 + 10*a^6*b^7 + 10*a^4*b^9 + 5*a^2*b^{11} + b^{13})*\tan(d*x + c)^7 + 3*(a^{11}*b^2 + 5*a^9*b^4 + 10*a^7*b^6 + 10*a^5*b^8 + 5*a^3*b^{10} + a*b^{12})*\tan(d*x + c)^6 + (3*a^{12}*b + 17*a^{10}*b^3 + 40*a^8*b^5 + 50*a^6*b^7 + 35*a^4*b^9 + 13*a^2*b^{11} + 2*b^{13})*\tan(d*x + c)^5 + (a^{13} + 11*a^{11}*b^2 + 40*a^9*b^4 + 70*a^7*b^6 + 65*a^5*b^8 + 31*a^3*b^{10} + 6*a*b^{12})*\tan(d*x + c)^4 + (6*a^{12}*b + 31*a^{10}*b^3 + 65*a^8*b^5 + 70*a^6*b^7 + 40*a^4*b^9 + 11*a^2*b^{11} + b^{13})*\tan(d*x + c)^3 + (2*a^{13} + 13*a^{11}*b^2 + 35*a^9*b^4 + 50*a^7*b^6 + 40*a^5*b^8 + 17*a^3*b^{10} + 3*a*b^{12})*\tan(d*x + c)^2 + 3*(a^{12}*b + 5*a^{10}*b^3 + 10*a^8*b^5 + 10*a^6*b^7 + 5*a^4*b^9 + a^2*b^{11})*\tan(d*x + c)) / d$

Fricas [B] time = 4.00481, size = 2412, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="fricas")

[Out] $\frac{1}{24}*(6*(a^{10}*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^{11})*\cos(d*x + c)^7 - 3*(11*a^{10}*b + 45*a^8*b^3 + 70*a^6*b^5 + 50*a^4*b^7 + 15*a^2*b^9 + b^{11})*\cos(d*x + c)^5 - (6*a^{10}*b + 342*a^8*b^3 - 1830*a^6*b^5 + 614*a^4*b^7 - 216*a^2*b^9 + 12*b^{11} - 3*(3*a^{11} - 141*a^9*b^2 + 766*a^7*b^4 - 1242*a^5*b^6 + 399*a^3*b^8 - 9*a*b^{10})*d*x)*\cos(d*x + c)^3 + 3*(114*a^8*b^3 - 381*a^6*b^5 + 187*a^4*b^7 - 67*a^2*b^9 + 3*b^{11} + 3*(3*a^9*b^2 - 132*a^7*b^4 + 370*a^5*b^6 - 132*a^3*b^8 + 3*a*b^{10})*d*x)*\cos(d*x + c) + 48*((a^{10}*b - 12*a^8*b^3 + 36*a^6*b^5 - 28*a^4*b^7 + 3*a^2*b^9)*\cos(d*x + c)^3 + 3*(a^8*b^3 - 9*a^6*b^5 + 9*a^4*b^7 - a^2*b^9)*\cos(d*x + c) + (a^7*b^4 - 9*a^5*b^6 + 9*a^3*b^8 - a*b^{10} + (3*a^9*b^2 - 28*a^7*b^4 + 36*a^5*b^6 - 12*a^3*b^8 + a*b^{10})*\cos(d*x + c)^2)*\sin(d*x + c))*\log(2*a*b*\cos(d*x + c)*\sin(d*x + c) + (a^2 - b^2)*\cos(d*x + c)^2 + b^2) + (143*a^7*b^4 - 537*a^5*b^6 + 105*a^3*b^8 + 33*a*b^{10} + 6*(a^{11} + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b$

$$\begin{aligned} &^8 + a*b^{10})*\cos(d*x + c)^6 - 15*(a^{11} + 3*a^9*b^2 + 2*a^7*b^4 - 2*a^5*b^6 \\ &- 3*a^3*b^8 - a*b^{10})*\cos(d*x + c)^4 + 3*(3*a^8*b^3 - 132*a^6*b^5 + 370*a^4 \\ &*b^7 - 132*a^2*b^9 + 3*b^{11})*d*x + (216*a^9*b^2 - 734*a^7*b^4 + 1590*a^5*b^6 \\ &- 522*a^3*b^8 - 54*a*b^{10} + 3*(9*a^{10}*b - 399*a^8*b^3 + 1242*a^6*b^5 - 76 \\ &6*a^4*b^7 + 141*a^2*b^9 - 3*b^{11})*d*x)*\cos(d*x + c)^2*\sin(d*x + c))/((a^{15} \\ &+ 3*a^{13}*b^2 - 3*a^{11}*b^4 - 25*a^9*b^6 - 45*a^7*b^8 - 39*a^5*b^{10} - 17*a^3 \\ &*b^{12} - 3*a*b^{14})*d*\cos(d*x + c)^3 + 3*(a^{13}*b^2 + 6*a^{11}*b^4 + 15*a^9*b^6 \\ &+ 20*a^7*b^8 + 15*a^5*b^{10} + 6*a^3*b^{12} + a*b^{14})*d*\cos(d*x + c) + ((3*a^{14} \\ &*b + 17*a^{12}*b^3 + 39*a^{10}*b^5 + 45*a^8*b^7 + 25*a^6*b^9 + 3*a^4*b^{11} - 3*a \\ &^2*b^{13} - b^{15})*d*\cos(d*x + c)^2 + (a^{12}*b^3 + 6*a^{10}*b^5 + 15*a^8*b^7 + 20 \\ &*a^6*b^9 + 15*a^4*b^{11} + 6*a^2*b^{13} + b^{15})*d)*\sin(d*x + c)) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**4,x)

[Out] Timed out

Giac [B] time = 1.34998, size = 1218, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="giac")

[Out]
$$\frac{1}{24}*(3*(3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*(d*x + c) / (a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12}) - 48*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*\log(\tan(d*x + c)^2 + 1) / (a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12}) + 96*(a^7*b^2 - 9*a^5*b^4 + 9*a^3*b^6 - a*b^8)*\log(\text{abs}(b*\tan(d*x + c) + a)) / (a^{12}*b + 6*a^{10}*b^3 + 15*a^8*b^5 + 20*a^6*b^7 + 15*a^4*b^9 + 6*a^2*b^{11} + b^{13}) + 3*(24*a^7*b*\tan(d*x + c)^4 - 216*a^5*b^3*\tan(d*x + c)^4 + 216*a^3*b^5*\tan(d*x + c)^4 - 24*a*b^7*\tan(d*x + c)^4 - 5*a^8*\tan(d*x + c)^3 + 60*a^6*b^2*\tan(d*x + c)^3 + 10*a^4*b^4*\tan(d*x + c)^3 - 52*a^2*b^6*\tan(d*x + c)^3 + 3*b^8*\tan(d*x + c)^3 + 16*a^7*b*\tan(d*x + c)^2 - 384*a^5*b^3*\tan(d*x + c)^2 + 496*a^3*b^5*\tan(d*x + c)^2 - 64*a*b^7*\tan(d*x + c)^2 - 3*a^8*\tan(d*x + c) + 52*a^6*b^2*\tan(d*x + c) - 10*a^4*b^4*\tan(d*x + c) - 60*a^2*b^6*\tan(d*x + c) + 5*b^8*\tan(d*x + c) - 160*a^5*b^3 + 272*a^3*b^5 - 48*a*b^7) / ((a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12})*(\tan(d*x + c)^2 + 1)^2) - 8*(22*a^7*b^4*\tan(d*x + c)^3 - 198*a^5*b^6*\tan(d*x + c)^3 + 198*a^3*b^8*\tan(d*x + c)^3 - 22*a*b^{10}*\tan(d*x + c)^3 + 75*a^8*b^3*\tan(d*x + c)^2 - 630*a^6*b^5*\tan(d*x + c)^2 + 567*a^4*b^7*\tan(d*x + c)^2 - 48*a^2*b^9*\tan(d*x + c)^2 + 87*a^9*b^2*\tan(d*x + c) - 666*a^7*b^4*\tan(d*x + c) + 531*a^5*b^6*\tan(d*x + c) - 36*a^3*b^8*\tan(d*x + c) + 35*a^{10}*b - 231*a^8*b^3 + 165*a^6*b^5 - 9*a^4*b^7) / ((a^{12} + 6*a^{10}*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^{10} + b^{12})*(b*\tan(d*x + c) + a)^3) / d$$

$$3.74 \quad \int \frac{\sin^2(c+dx)}{(a+b \tan(c+dx))^4} dx$$

Optimal. Leaf size=264

$$\frac{a^2 b}{3d(a^2 + b^2)^2 (a + b \tan(c + dx))^3} - \frac{ab(a^2 - b^2)}{d(a^2 + b^2)^3 (a + b \tan(c + dx))^2} - \frac{b(-8a^2 b^2 + 3a^4 + b^4)}{d(a^2 + b^2)^4 (a + b \tan(c + dx))} - \frac{\cos^2(c + dx)}{d(a^2 + b^2)^4 (a + b \tan(c + dx))}$$

[Out] $((a^6 - 25a^4b^2 + 35a^2b^4 - 3b^6)x)/(2(a^2 + b^2)^5) + (4ab(a^4 - 5a^2b^2 + 2b^4) \operatorname{Log}[a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx]])/((a^2 + b^2)^5 d) - (a^2 b)/(3(a^2 + b^2)^2 d (a + b \operatorname{Tan}[c + dx])^3) - (ab(a^2 - b^2))/((a^2 + b^2)^3 d (a + b \operatorname{Tan}[c + dx])^2) - (b(3a^4 - 8a^2 b^2 + b^4))/(a^2 + b^2)^4 d (a + b \operatorname{Tan}[c + dx]) - (\operatorname{Cos}[c + dx]^2 (4ab(a^2 - b^2) + (a^4 - 6a^2 b^2 + b^4) \operatorname{Tan}[c + dx]))/(2(a^2 + b^2)^4 d)$

Rubi [A] time = 0.570884, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3516, 1647, 1629, 635, 203, 260}

$$\frac{a^2 b}{3d(a^2 + b^2)^2 (a + b \tan(c + dx))^3} - \frac{ab(a^2 - b^2)}{d(a^2 + b^2)^3 (a + b \tan(c + dx))^2} - \frac{b(-8a^2 b^2 + 3a^4 + b^4)}{d(a^2 + b^2)^4 (a + b \tan(c + dx))} - \frac{\cos^2(c + dx)}{d(a^2 + b^2)^4 (a + b \tan(c + dx))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sin}[c + dx]^2/(a + b \operatorname{Tan}[c + dx])^4, x]$

[Out] $((a^6 - 25a^4b^2 + 35a^2b^4 - 3b^6)x)/(2(a^2 + b^2)^5) + (4ab(a^4 - 5a^2b^2 + 2b^4) \operatorname{Log}[a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx]])/((a^2 + b^2)^5 d) - (a^2 b)/(3(a^2 + b^2)^2 d (a + b \operatorname{Tan}[c + dx])^3) - (ab(a^2 - b^2))/((a^2 + b^2)^3 d (a + b \operatorname{Tan}[c + dx])^2) - (b(3a^4 - 8a^2 b^2 + b^4))/(a^2 + b^2)^4 d (a + b \operatorname{Tan}[c + dx]) - (\operatorname{Cos}[c + dx]^2 (4ab(a^2 - b^2) + (a^4 - 6a^2 b^2 + b^4) \operatorname{Tan}[c + dx]))/(2(a^2 + b^2)^4 d)$

Rule 3516

$\operatorname{Int}[\sin[(e_.) + (f_.)x]^{(m_)}((a_.) + (b_.) \operatorname{tan}[(e_.) + (f_.)x])^{(n_)}], x_Symbol] \rightarrow \operatorname{Dist}[b/f, \operatorname{Subst}[\operatorname{Int}[(x^m(a + x)^n)/(b^2 + x^2)^{(m/2 + 1)}], x], x, b \operatorname{Tan}[e + fx], x] /;$ FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 1647

$\operatorname{Int}[(Pq_*)((d_.) + (e_.)x)^{(m_)}((a_.) + (c_.)x^2)^{(p_)}], x_Symbol] \rightarrow \operatorname{With}[\{Q = \operatorname{PolynomialQuotient}[(d + ex)^m Pq, a + cx^2, x], f = \operatorname{Coeff}[\operatorname{PolynomialRemainder}[(d + ex)^m Pq, a + cx^2, x], x, 0], g = \operatorname{Coeff}[\operatorname{PolynomialRemainder}[(d + ex)^m Pq, a + cx^2, x], x, 1]\}, \operatorname{Simp}[(a g - c f x)(a + cx^2)^{(p + 1)}/(2ac(p + 1)), x] + \operatorname{Dist}[1/(2ac(p + 1)), \operatorname{Int}[(d + ex)^m (a + cx^2)^{(p + 1)} \operatorname{ExpandToSum}[(2ac(p + 1)Q)/(d + ex)^m + (c f (2p + 3))/(d + ex)^m, x], x], x] /;$ FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c d^2 + a e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rule 1629

$\operatorname{Int}[(Pq_*)((d_.) + (e_.)x)^{(m_)}((a_.) + (c_.)x^2)^{(p_)}], x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(d + ex)^m Pq (a + cx^2)^p, x], x] /;$ FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(c + dx)}{(a + b \tan(c + dx))^4} dx &= \frac{b \operatorname{Subst} \left(\int \frac{x^2}{(a+x)^4(b^2+x^2)^2} dx, x, b \tan(c + dx) \right)}{d} \\ &= -\frac{\cos^2(c + dx) (4ab(a^2 - b^2) + (a^4 - 6a^2b^2 + b^4) \tan(c + dx))}{2(a^2 + b^2)^4 d} - \operatorname{Subst} \left(\int \frac{-\frac{a^4 b^2 (a^4 - 6a^2 b^2 + b^4)}{(a^2 + b^2)^4}}{(a^2 + b^2)^4} dx, x, b \tan(c + dx) \right) \\ &= -\frac{\cos^2(c + dx) (4ab(a^2 - b^2) + (a^4 - 6a^2b^2 + b^4) \tan(c + dx))}{2(a^2 + b^2)^4 d} - \operatorname{Subst} \left(\int \left(-\frac{2a^2 b^2}{(a^2 + b^2)^2 (a+x)^2} \right) dx, x, b \tan(c + dx) \right) \\ &= \frac{4ab(a^4 - 5a^2b^2 + 2b^4) \log(a + b \tan(c + dx))}{(a^2 + b^2)^5 d} - \frac{a^2 b}{3(a^2 + b^2)^2 d (a + b \tan(c + dx))^3} - \frac{a^2 b}{3(a^2 + b^2)^2 d (a + b \tan(c + dx))^3} \\ &= \frac{4ab(a^4 - 5a^2b^2 + 2b^4) \log(a + b \tan(c + dx))}{(a^2 + b^2)^5 d} - \frac{a^2 b}{3(a^2 + b^2)^2 d (a + b \tan(c + dx))^3} - \frac{a^2 b}{3(a^2 + b^2)^2 d (a + b \tan(c + dx))^3} \\ &= \frac{(a^6 - 25a^4b^2 + 35a^2b^4 - 3b^6)x}{2(a^2 + b^2)^5} + \frac{4ab(a^4 - 5a^2b^2 + 2b^4) \log(\cos(c + dx))}{(a^2 + b^2)^5 d} + \frac{4ab(a^4 - 5a^2b^2 + 2b^4) \log(\cos(c + dx))}{(a^2 + b^2)^5 d} \end{aligned}$$

Mathematica [A] time = 3.59427, size = 395, normalized size = 1.5

$$b \left(\frac{3(-6a^2b^2 + a^4 + b^4)(a^2 + b^2) \sin(2(c + dx))}{2b} + 12a(a - b)(a + b)(a^2 + b^2) \cos^2(c + dx) + \frac{3(-6a^2b^2 + a^4 + b^4)(a^2 + b^2) \tan^{-1}(\tan(c + dx))}{b} + \frac{4ab(a^4 - 5a^2b^2 + 2b^4) \log(\cos(c + dx))}{(a^2 + b^2)^5 d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^4, x]

[Out] -(b*((3*(a^2 + b^2)*(a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]]))/b + 12*a*(a - b)*(a + b)*(a^2 + b^2)*Cos[c + d*x]^2 + 3*(4*a^5 - 20*a^3*b^2 + 8*a*b^4 + (-a^6 + 15*a^4*b^2 - 15*a^2*b^4 + b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*T

$$\begin{aligned} & \text{an}[c + d*x]] - 24*a*(a^4 - 5*a^2*b^2 + 2*b^4)*\text{Log}[a + b*\text{Tan}[c + d*x]] + 3*(\\ & 4*a^5 - 20*a^3*b^2 + 8*a*b^4 + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/\text{Sqrt}[- \\ & b^2])*\text{Log}[\text{Sqrt}[-b^2] + b*\text{Tan}[c + d*x]] + (3*(a^2 + b^2)*(a^4 - 6*a^2*b^2 + \\ & b^4)*\text{Sin}[2*(c + d*x)])/(2*b) + (2*a^2*(a^2 + b^2)^3)/(a + b*\text{Tan}[c + d*x])^3 \\ & + (6*a*(a - b)*(a + b)*(a^2 + b^2)^2)/(a + b*\text{Tan}[c + d*x])^2 + (6*(a^2 + b \\ & ^2)*(3*a^4 - 8*a^2*b^2 + b^4))/(a + b*\text{Tan}[c + d*x]))/(6*(a^2 + b^2)^5*d) \end{aligned}$$

Maple [B] time = 0.12, size = 668, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x)`

[Out]
$$\begin{aligned} & -1/2/d/(a^2+b^2)^5/(1+\tan(d*x+c)^2)*\tan(d*x+c)*a^6+5/2/d/(a^2+b^2)^5/(1+\tan \\ & (d*x+c)^2)*\tan(d*x+c)*a^4*b^2+5/2/d/(a^2+b^2)^5/(1+\tan(d*x+c)^2)*\tan(d*x+c) \\ & *a^2*b^4-1/2/d/(a^2+b^2)^5/(1+\tan(d*x+c)^2)*\tan(d*x+c)*b^6-2/d/(a^2+b^2)^5/ \\ & (1+\tan(d*x+c)^2)*a^5*b+2/d/(a^2+b^2)^5/(1+\tan(d*x+c)^2)*a*b^5-2/d/(a^2+b^2) \\ & ^5*\ln(1+\tan(d*x+c)^2)*a^5*b+10/d/(a^2+b^2)^5*\ln(1+\tan(d*x+c)^2)*a^3*b^3-4/d \\ & /(a^2+b^2)^5*\ln(1+\tan(d*x+c)^2)*a*b^5-25/2/d/(a^2+b^2)^5*\arctan(\tan(d*x+c)) \\ & *a^4*b^2+35/2/d/(a^2+b^2)^5*\arctan(\tan(d*x+c))*a^2*b^4-3/2/d/(a^2+b^2)^5*\ar \\ & ctan(\tan(d*x+c))*b^6+1/2/d/(a^2+b^2)^5*\arctan(\tan(d*x+c))*a^6-1/3*a^2*b/(a^ \\ & 2+b^2)^2/d/(a+b*\tan(d*x+c))^3-3/d*b/(a^2+b^2)^4/(a+b*\tan(d*x+c))*a^4+8/d*b^ \\ & 3/(a^2+b^2)^4/(a+b*\tan(d*x+c))*a^2-1/d*b^5/(a^2+b^2)^4/(a+b*\tan(d*x+c))+4/d \\ & *a^5*b/(a^2+b^2)^5*\ln(a+b*\tan(d*x+c))-20/d*a^3*b^3/(a^2+b^2)^5*\ln(a+b*\tan(d \\ & *x+c))+8/d*a*b^5/(a^2+b^2)^5*\ln(a+b*\tan(d*x+c))-1/d*b*a^3/(a^2+b^2)^3/(a+b* \\ & \tan(d*x+c))^2+1/d*b^3*a/(a^2+b^2)^3/(a+b*\tan(d*x+c))^2 \end{aligned}$$

Maxima [B] time = 1.71198, size = 894, normalized size = 3.39

$$\frac{3(a^6 - 25a^4b^2 + 35a^2b^4 - 3b^6)(dx+c)}{a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}} + \frac{24(a^5b - 5a^3b^3 + 2ab^5)\log(b\tan(dx+c)+a)}{a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}} - \frac{12(a^5b - 5a^3b^3 + 2ab^5)\log(\tan(dx+c)^2+1)}{a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}} - \frac{1}{a^{11} + 4a^9b^2 + 6a^7b^4 + 4a^5b^6 + 4a^3b^8 + a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")`

[Out]
$$\begin{aligned} & 1/6*(3*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*(d*x + c)/(a^10 + 5*a^8*b^2 \\ & + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(a^5*b - 5*a^3*b^3 + 2*a \\ & *b^5)*\log(b*\text{tan}(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + \\ & 5*a^2*b^8 + b^10) - 12*(a^5*b - 5*a^3*b^3 + 2*a*b^5)*\log(\text{tan}(d*x + c)^2 + \\ & 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - (38*a^ \\ & 6*b - 56*a^4*b^3 + 2*a^2*b^5 + 3*(7*a^4*b^3 - 22*a^2*b^5 + 3*b^7)*\text{tan}(d*x + \\ & c)^4 + 3*(17*a^5*b^2 - 46*a^3*b^4 + a*b^6)*\text{tan}(d*x + c)^3 + (35*a^6*b - 44 \\ & *a^4*b^3 - 73*a^2*b^5 + 6*b^7)*\text{tan}(d*x + c)^2 + 3*(a^7 + 20*a^5*b^2 - 43*a^ \\ & 3*b^4 + 2*a*b^6)*\text{tan}(d*x + c))/(a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6 + \\ & a^3*b^8 + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*\text{tan}(d*x + c) \\ & ^5 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*\text{tan}(d*x + c)^ \\ & 4 + (3*a^10*b + 13*a^8*b^3 + 22*a^6*b^5 + 18*a^4*b^7 + 7*a^2*b^9 + b^11)*\text{ta} \\ & n(d*x + c)^3 + (a^11 + 7*a^9*b^2 + 18*a^7*b^4 + 22*a^5*b^6 + 13*a^3*b^8 + 3 \\ & *a*b^10)*\text{tan}(d*x + c)^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a \\ & ^2*b^9)*\text{tan}(d*x + c)))/d \end{aligned}$$

Fricas [B] time = 3.00205, size = 1789, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/6*(3*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*\cos(d*x + c)^5 + \\ & (3*a^8*b + 111*a^6*b^3 - 231*a^4*b^5 + 65*a^2*b^7 - 12*b^9 - 3*(a^9 - 28*a^7*b^2 + 110*a^5*b^4 - 108*a^3*b^6 + 9*a*b^8)*d*x)*\cos(d*x + c)^3 - 3*(25*a^6*b^3 - 51*a^4*b^5 + 25*a^2*b^7 - 3*b^9 + 3*(a^7*b^2 - 25*a^5*b^4 + 35*a^3*b^6 - 3*a*b^8)*d*x)*\cos(d*x + c) - 12*((a^8*b - 8*a^6*b^3 + 17*a^4*b^5 - 6*a^2*b^7)*\cos(d*x + c)^3 + 3*(a^6*b^3 - 5*a^4*b^5 + 2*a^2*b^7)*\cos(d*x + c) \\ & + (a^5*b^4 - 5*a^3*b^6 + 2*a*b^8 + (3*a^7*b^2 - 16*a^5*b^4 + 11*a^3*b^6 - 2*a*b^8)*\cos(d*x + c)^2)*\sin(d*x + c))*\log(2*a*b*\cos(d*x + c)*\sin(d*x + c) + \\ & (a^2 - b^2)*\cos(d*x + c)^2 + b^2) - (32*a^5*b^4 - 66*a^3*b^6 + 6*a*b^8 - 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*\cos(d*x + c)^4 + 3*(a^6*b^3 - 25*a^4*b^5 + 35*a^2*b^7 - 3*b^9)*d*x + (45*a^7*b^2 - 143*a^5*b^4 + 219*a^3*b^6 - 9*a*b^8 + 3*(3*a^8*b - 76*a^6*b^3 + 130*a^4*b^5 - 44*a^2*b^7 + 3*b^9)*d*x)*\cos(d*x + c)^2)*\sin(d*x + c))/((a^13 + 2*a^11*b^2 - 5*a^9*b^4 - 20*a^7*b^6 - 25*a^5*b^8 - 14*a^3*b^10 - 3*a*b^12)*d*\cos(d*x + c)^3 + 3*(a^11*b^2 + 5*a^9*b^4 + 10*a^7*b^6 + 10*a^5*b^8 + 5*a^3*b^10 + a*b^12)*d*\cos(d*x + c) + ((3*a^12*b + 14*a^10*b^3 + 25*a^8*b^5 + 20*a^6*b^7 + 5*a^4*b^9 - 2*a^2*b^11 - b^13)*d*\cos(d*x + c)^2 + (a^10*b^3 + 5*a^8*b^5 + 10*a^6*b^7 + 10*a^4*b^9 + 5*a^2*b^11 + b^13)*d)*\sin(d*x + c)) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**4,x)

[Out] Timed out

Giac [B] time = 1.32387, size = 867, normalized size = 3.28

$$\frac{3(a^6 - 25a^4b^2 + 35a^2b^4 - 3b^6)(dx+c)}{a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}} - \frac{12(a^5b - 5a^3b^3 + 2ab^5)\log(\tan(dx+c)^2+1)}{a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}} + \frac{24(a^5b^2 - 5a^3b^4 + 2ab^6)\log(|b \tan(dx+c)+a|)}{a^{10}b + 5a^8b^3 + 10a^6b^5 + 10a^4b^7 + 5a^2b^9 + b^{11}} + \frac{3(4a^5b \tan(dx+c) + a^6 \tan^2(dx+c) + a^7 \tan^3(dx+c) + a^8 \tan^4(dx+c) + a^9 \tan^5(dx+c) + a^{10} \tan^6(dx+c) + a^{11} \tan^7(dx+c))}{a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + 5a^2b^8 + b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/6*(3*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*(d*x + c)/(a^10 + 5*a^8*b^2 \\ & + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(a^5*b - 5*a^3*b^3 + 2*a \\ & *b^5)*\log(\tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + \\ & 5*a^2*b^8 + b^10) + 24*(a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*\log(\text{abs}(b*\tan(d*x + \\ & c) + a))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11) \end{aligned}$$

$$\begin{aligned}
& + 3(4a^5b\tan(dx + c)^2 - 20a^3b^3\tan(dx + c)^2 + 8ab^5\tan(dx \\
& + c)^2 - a^6\tan(dx + c) + 5a^4b^2\tan(dx + c) + 5a^2b^4\tan(dx + c) \\
& - b^6\tan(dx + c) - 20a^3b^3 + 12ab^5)/((a^{10} + 5a^8b^2 + 10a^6b^4 \\
& + 10a^4b^6 + 5a^2b^8 + b^{10})\cdot(\tan(dx + c)^2 + 1)) - 2(22a^5b^4\tan \\
& (dx + c)^3 - 110a^3b^6\tan(dx + c)^3 + 44ab^8\tan(dx + c)^3 + 75a^6 \\
& b^3\tan(dx + c)^2 - 345a^4b^5\tan(dx + c)^2 + 111a^2b^7\tan(dx + c) \\
&)^2 + 3b^9\tan(dx + c)^2 + 87a^7b^2\tan(dx + c) - 357a^5b^4\tan(dx \\
& + c) + 87a^3b^6\tan(dx + c) + 3ab^8\tan(dx + c) + 35a^8b - 119a^6b^3 \\
& + 23a^4b^5 + a^2b^7)/((a^{10} + 5a^8b^2 + 10a^6b^4 + 10a^4b^6 + \\
& 5a^2b^8 + b^{10})\cdot(b\tan(dx + c) + a)^3)/d
\end{aligned}$$

$$3.75 \quad \int \frac{\csc^2(c+dx)}{(a+b \tan(c+dx))^4} dx$$

Optimal. Leaf size=116

$$-\frac{3b}{a^4d(a+b \tan(c+dx))} - \frac{b}{a^3d(a+b \tan(c+dx))^2} - \frac{b}{3a^2d(a+b \tan(c+dx))^3} - \frac{4b \log(\tan(c+dx))}{a^5d} + \frac{4b \log(a+b \tan(c+dx))}{a^5d}$$

[Out] $-(\text{Cot}[c + d*x]/(a^4*d)) - (4*b*\text{Log}[\text{Tan}[c + d*x]])/(a^5*d) + (4*b*\text{Log}[a + b*\text{Tan}[c + d*x]])/(a^5*d) - b/(3*a^2*d*(a + b*\text{Tan}[c + d*x])^3) - b/(a^3*d*(a + b*\text{Tan}[c + d*x])^2) - (3*b)/(a^4*d*(a + b*\text{Tan}[c + d*x]))$

Rubi [A] time = 0.0884231, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 44}

$$-\frac{3b}{a^4d(a+b \tan(c+dx))} - \frac{b}{a^3d(a+b \tan(c+dx))^2} - \frac{b}{3a^2d(a+b \tan(c+dx))^3} - \frac{4b \log(\tan(c+dx))}{a^5d} + \frac{4b \log(a+b \tan(c+dx))}{a^5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c + d*x]^2/(a + b*\text{Tan}[c + d*x])^4, x]$

[Out] $-(\text{Cot}[c + d*x]/(a^4*d)) - (4*b*\text{Log}[\text{Tan}[c + d*x]])/(a^5*d) + (4*b*\text{Log}[a + b*\text{Tan}[c + d*x]])/(a^5*d) - b/(3*a^2*d*(a + b*\text{Tan}[c + d*x])^3) - b/(a^3*d*(a + b*\text{Tan}[c + d*x])^2) - (3*b)/(a^4*d*(a + b*\text{Tan}[c + d*x]))$

Rule 3516

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] :> \text{Dist}[b/f, \text{Subst}[\text{Int}[(x^m*(a+x)^n)/(b^2+x^2)^{(m/2+1)}], x], x, b*\text{Tan}[e+f*x], x] /;$ FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 44

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(c+dx)}{(a+b \tan(c+dx))^4} dx &= \frac{b \text{Subst}\left(\int \frac{1}{x^2(a+x)^4} dx, x, b \tan(c+dx)\right)}{d} \\ &= \frac{b \text{Subst}\left(\int \left(\frac{1}{a^4x^2} - \frac{4}{a^5x} + \frac{1}{a^2(a+x)^4} + \frac{2}{a^3(a+x)^3} + \frac{3}{a^4(a+x)^2} + \frac{4}{a^5(a+x)}\right) dx, x, b \tan(c+dx)\right)}{d} \\ &= -\frac{\cot(c+dx)}{a^4d} - \frac{4b \log(\tan(c+dx))}{a^5d} + \frac{4b \log(a+b \tan(c+dx))}{a^5d} - \frac{b}{3a^2d(a+b \tan(c+dx))} \end{aligned}$$

Mathematica [B] time = 2.06357, size = 259, normalized size = 2.23

$$\sec^3(c+dx)(a \cos(c+dx) + b \sin(c+dx)) \left(\frac{a^2b^4 \tan(c+dx)}{a^2+b^2} - \frac{2a^2b^3(3a^2+2b^2)(a+b \tan(c+dx))}{(a^2+b^2)^2} + \frac{b^2(23a^2b^2+18a^4+9b^4) \tan(c+dx)(a \cos(c+dx) + b \sin(c+dx))}{(a^2+b^2)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]

[Out] (Sec[c + d*x]^3*(a*cos[c + d*x] + b*sin[c + d*x])*(-3*a*(b + a*cot[c + d*x])^3*sin[c + d*x]^2 + (a^2*b^4*tan[c + d*x])/(a^2 + b^2) + (b^2*(18*a^4 + 23*a^2*b^2 + 9*b^4)*(a*cos[c + d*x] + b*sin[c + d*x])^2*tan[c + d*x])/(a^2 + b^2)^2 - (2*a^2*b^3*(3*a^2 + 2*b^2)*(a + b*tan[c + d*x]))/(a^2 + b^2)^2 - 12*b*cos[c + d*x]^2*Log[Sin[c + d*x]]*(a + b*tan[c + d*x])^3 + 12*b*cos[c + d*x]^2*Log[a*cos[c + d*x] + b*sin[c + d*x]]*(a + b*tan[c + d*x])^3))/(3*a^5*d*(a + b*tan[c + d*x])^4)

Maple [A] time = 0.131, size = 117, normalized size = 1.

$$-\frac{1}{da^4 \tan(dx+c)} - 4 \frac{b \ln(\tan(dx+c))}{a^5 d} - \frac{b}{3 a^2 d (a+b \tan(dx+c))^3} + 4 \frac{b \ln(a+b \tan(dx+c))}{a^5 d} - 3 \frac{b}{da^4 (a+b \tan(dx+c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x)

[Out] -1/d/a^4/tan(d*x+c)-4*b*ln(tan(d*x+c))/a^5/d-1/3*b/a^2/d/(a+b*tan(d*x+c))^3+4*b*ln(a+b*tan(d*x+c))/a^5/d-3*b/a^4/d/(a+b*tan(d*x+c))-b/a^3/d/(a+b*tan(d*x+c))^2

Maxima [A] time = 1.14022, size = 189, normalized size = 1.63

$$\frac{12b^3 \tan(dx+c)^3 + 30ab^2 \tan(dx+c)^2 + 22a^2b \tan(dx+c) + 3a^3}{a^4b^3 \tan(dx+c)^4 + 3a^5b^2 \tan(dx+c)^3 + 3a^6b \tan(dx+c)^2 + a^7 \tan(dx+c)} - \frac{12b \log(b \tan(dx+c) + a)}{a^5} + \frac{12b \log(\tan(dx+c))}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")

[Out] -1/3*((12*b^3*tan(d*x + c)^3 + 30*a*b^2*tan(d*x + c)^2 + 22*a^2*b*tan(d*x + c) + 3*a^3)/(a^4*b^3*tan(d*x + c)^4 + 3*a^5*b^2*tan(d*x + c)^3 + 3*a^6*b*tan(d*x + c)^2 + a^7*tan(d*x + c)) - 12*b*log(b*tan(d*x + c) + a)/a^5 + 12*b*log(tan(d*x + c))/a^5)/d

Fricas [B] time = 2.84577, size = 1901, normalized size = 16.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")

[Out] -1/3*(13*a^6*b^4 + 15*a^4*b^6 + 6*a^2*b^8 - (3*a^10 + 18*a^8*b^2 - 49*a^6*b^4 - 84*a^4*b^6 - 36*a^2*b^8)*cos(d*x + c)^4 + (9*a^8*b^2 - 71*a^6*b^4 - 102*a^4*b^6 - 42*a^2*b^8)*cos(d*x + c)^2 + 6*(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10 - (3*a^8*b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^10)*cos(d*x + c)^4 + (3*a^

$$8*b^2 + 7*a^6*b^4 + 3*a^4*b^6 - 3*a^2*b^8 - 2*b^{10})*\cos(d*x + c)^2 + ((a^9*b - 6*a^5*b^5 - 8*a^3*b^7 - 3*a*b^9)*\cos(d*x + c)^3 + 3*(a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*\cos(d*x + c))*\sin(d*x + c))*\log(2*a*b*\cos(d*x + c))*\sin(d*x + c) + (a^2 - b^2)*\cos(d*x + c)^2 + b^2) - 6*(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^{10} - (3*a^8*b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^{10})*\cos(d*x + c)^4 + (3*a^8*b^2 + 7*a^6*b^4 + 3*a^4*b^6 - 3*a^2*b^8 - 2*b^{10})*\cos(d*x + c)^2 + ((a^9*b - 6*a^5*b^5 - 8*a^3*b^7 - 3*a*b^9)*\cos(d*x + c)^3 + 3*(a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*\cos(d*x + c))*\sin(d*x + c))*\log(-1/4*\cos(d*x + c)^2 + 1/4) - ((9*a^9*b + 78*a^7*b^3 + 69*a^5*b^5 + 4*a^3*b^7 - 12*a*b^9)*\cos(d*x + c)^3 - 3*(9*a^7*b^3 + 3*a^5*b^5 - 6*a^3*b^7 - 4*a*b^9)*\cos(d*x + c))*\sin(d*x + c))/((3*a^{13}*b + 8*a^{11}*b^3 + 6*a^9*b^5 - a^5*b^9)*d*\cos(d*x + c)^4 - (3*a^{13}*b + 7*a^{11}*b^3 + 3*a^9*b^5 - 3*a^7*b^7 - 2*a^5*b^9)*d*\cos(d*x + c)^2 - (a^{11}*b^3 + 3*a^9*b^5 + 3*a^7*b^7 + a^5*b^9)*d - ((a^{14} - 6*a^{10}*b^4 - 8*a^8*b^6 - 3*a^6*b^8)*d*\cos(d*x + c)^3 + 3*(a^{12}*b^2 + 3*a^{10}*b^4 + 3*a^8*b^6 + a^6*b^8)*d*\cos(d*x + c))*\sin(d*x + c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**4,x)

[Out] Timed out

Giac [A] time = 1.2252, size = 174, normalized size = 1.5

$$\frac{\frac{12 b \log(|b \tan(dx+c)+a|)}{a^5} - \frac{12 b \log(|\tan(dx+c)|)}{a^5} + \frac{3(4 b \tan(dx+c)-a)}{a^5 \tan(dx+c)} - \frac{22 b^4 \tan(dx+c)^3 + 75 a b^3 \tan(dx+c)^2 + 87 a^2 b^2 \tan(dx+c) + 35 a^3 b}{(b \tan(dx+c)+a)^3 a^5}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")

[Out] 1/3*(12*b*log(abs(b*tan(d*x + c) + a))/a^5 - 12*b*log(abs(tan(d*x + c)))/a^5 + 3*(4*b*tan(d*x + c) - a)/(a^5*tan(d*x + c)) - (22*b^4*tan(d*x + c)^3 + 75*a*b^3*tan(d*x + c)^2 + 87*a^2*b^2*tan(d*x + c) + 35*a^3*b)/((b*tan(d*x + c) + a)^3*a^5))/d

$$3.76 \quad \int \frac{\csc^4(c+dx)}{(a+b \tan(c+dx))^4} dx$$

Optimal. Leaf size=205

$$\frac{b(3a^2 + 10b^2)}{a^6d(a + b \tan(c + dx))} - \frac{b(a^2 + 2b^2)}{a^5d(a + b \tan(c + dx))^2} - \frac{b(a^2 + b^2)}{3a^4d(a + b \tan(c + dx))^3} - \frac{(a^2 + 10b^2) \cot(c + dx)}{a^6d} - \frac{4b(a^2 + 5b^2)}{a^6d}$$

[Out] -(((a^2 + 10*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*Cot[c + d*x]^2)/(a^5*d) - Cot[c + d*x]^3/(3*a^4*d) - (4*b*(a^2 + 5*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (4*b*(a^2 + 5*b^2)*Log[a + b*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2))/(3*a^4*d*(a + b*Tan[c + d*x])^3) - (b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^2 + 10*b^2))/(a^6*d*(a + b*Tan[c + d*x]))

Rubi [A] time = 0.173254, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 894}

$$\frac{b(3a^2 + 10b^2)}{a^6d(a + b \tan(c + dx))} - \frac{b(a^2 + 2b^2)}{a^5d(a + b \tan(c + dx))^2} - \frac{b(a^2 + b^2)}{3a^4d(a + b \tan(c + dx))^3} - \frac{(a^2 + 10b^2) \cot(c + dx)}{a^6d} - \frac{4b(a^2 + 5b^2)}{a^6d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]

[Out] -(((a^2 + 10*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*Cot[c + d*x]^2)/(a^5*d) - Cot[c + d*x]^3/(3*a^4*d) - (4*b*(a^2 + 5*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (4*b*(a^2 + 5*b^2)*Log[a + b*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2))/(3*a^4*d*(a + b*Tan[c + d*x])^3) - (b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^2 + 10*b^2))/(a^6*d*(a + b*Tan[c + d*x]))

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 894

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{\csc^4(c + dx)}{(a + b \tan(c + dx))^4} dx &= \frac{b \operatorname{Subst} \left(\int \frac{b^2 + x^2}{x^4(a+x)^4} dx, x, b \tan(c + dx) \right)}{d} \\ &= \frac{b \operatorname{Subst} \left(\int \left(\frac{b^2}{a^4 x^4} - \frac{4b^2}{a^5 x^3} + \frac{a^2 + 10b^2}{a^6 x^2} - \frac{4(a^2 + 5b^2)}{a^7 x} + \frac{a^2 + b^2}{a^4(a+x)^4} + \frac{2(a^2 + 2b^2)}{a^5(a+x)^3} + \frac{3a^2 + 10b^2}{a^6(a+x)^2} + \frac{4(a^2 + 5b^2)}{a^7(a+x)} \right) dx, x, b \tan(c + dx) \right)}{d} \\ &= -\frac{(a^2 + 10b^2) \cot(c + dx)}{a^6d} + \frac{2b \cot^2(c + dx)}{a^5d} - \frac{\cot^3(c + dx)}{3a^4d} - \frac{4b(a^2 + 5b^2) \log(\tan(c + dx))}{a^7d} \end{aligned}$$

Mathematica [B] time = 2.03439, size = 528, normalized size = 2.58

$$\sec^4(c + dx)(a \cos(c + dx) + b \sin(c + dx)) \left(-192b(a^2 + 5b^2) \log(\sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))^3 + 192b \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^4, x]

[Out] (Sec[c + d*x]^4*(a*cos[c + d*x] + b*sin[c + d*x])*(-192*b*(a^2 + 5*b^2)*Log[Sin[c + d*x]]*(a*cos[c + d*x] + b*sin[c + d*x])^3 + 192*b*(a^2 + 5*b^2)*Log[a*cos[c + d*x] + b*sin[c + d*x]]*(a*cos[c + d*x] + b*sin[c + d*x])^3 - (Csc[c + d*x]^3*(8*a^8 - 4*a^6*b^2 - 50*a^4*b^4 - 190*a^2*b^6 - 150*b^8 + 3*(3*a^8 + 10*a^6*b^2 + 45*a^4*b^4 + 115*a^2*b^6 + 75*b^8)*Cos[2*(c + d*x)] + 6*(2*a^6*b^2 - 17*a^4*b^4 - 35*a^2*b^6 - 15*b^8)*Cos[4*(c + d*x)] - a^8*cos[6*(c + d*x)] - 22*a^6*b^2*cos[6*(c + d*x)] + 17*a^4*b^4*cos[6*(c + d*x)] + 55*a^2*b^6*cos[6*(c + d*x)] + 15*b^8*cos[6*(c + d*x)] - 3*a^7*b*sin[2*(c + d*x)] + 3*a^5*b^3*sin[2*(c + d*x)] - 75*a^3*b^5*sin[2*(c + d*x)] - 75*a*b^7*sin[2*(c + d*x)] - 6*a^7*b*sin[4*(c + d*x)] + 84*a^5*b^3*sin[4*(c + d*x)] + 156*a^3*b^5*sin[4*(c + d*x)] + 60*a*b^7*sin[4*(c + d*x)] - 3*a^7*b*sin[6*(c + d*x)] - 65*a^5*b^3*sin[6*(c + d*x)] - 79*a^3*b^5*sin[6*(c + d*x)] - 15*a*b^7*sin[6*(c + d*x)]))/(a^2 + b^2))/(48*a^7*d*(a + b*Tan[c + d*x])^4)

Maple [A] time = 0.198, size = 278, normalized size = 1.4

$$-\frac{1}{3da^4(\tan(dx+c))^3} - \frac{1}{da^4 \tan(dx+c)} - 10 \frac{b^2}{da^6 \tan(dx+c)} + 2 \frac{b}{da^5(\tan(dx+c))^2} - 4 \frac{b \ln(\tan(dx+c))}{da^5} - 20 \frac{b^3}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^4, x)

[Out] -1/3/d/a^4/tan(d*x+c)^3-1/d/a^4/tan(d*x+c)-10/d/a^6/tan(d*x+c)*b^2+2/d/a^5*b/tan(d*x+c)^2-4*b*ln(tan(d*x+c))/a^5/d-20/d*b^3/a^7*ln(tan(d*x+c))-3*b/a^4/d/(a+b*tan(d*x+c))-10/d*b^3/a^6/(a+b*tan(d*x+c))-1/3*b/a^2/d/(a+b*tan(d*x+c))^3-1/3/d*b^3/a^4/(a+b*tan(d*x+c))^3-b/a^3/d/(a+b*tan(d*x+c))^2-2/d*b^3/a^5/(a+b*tan(d*x+c))^2+4*b*ln(a+b*tan(d*x+c))/a^5/d+20/d*b^3/a^7*ln(a+b*tan(d*x+c))

Maxima [A] time = 1.12349, size = 308, normalized size = 1.5

$$\frac{3a^4b \tan(dx+c) - 12(a^2b^3 + 5b^5) \tan(dx+c)^5 - a^5 - 30(a^3b^2 + 5ab^4) \tan(dx+c)^4 - 22(a^4b + 5a^2b^3) \tan(dx+c)^3 - 3(a^5 + 5a^3b^2) \tan(dx+c)^2 + \frac{12(a^2b + 5b^3) \log(\tan(dx+c))}{a^7}}{a^6b^3 \tan(dx+c)^6 + 3a^7b^2 \tan(dx+c)^5 + 3a^8b \tan(dx+c)^4 + a^9 \tan(dx+c)^3} + \frac{12(a^2b + 5b^3) \log(\tan(dx+c))}{a^7}$$

3d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4, x, algorithm="maxima")

[Out] 1/3*((3*a^4*b*tan(d*x + c) - 12*(a^2*b^3 + 5*b^5)*tan(d*x + c)^5 - a^5 - 30*(a^3*b^2 + 5*a*b^4)*tan(d*x + c)^4 - 22*(a^4*b + 5*a^2*b^3)*tan(d*x + c)^3 - 3*(a^5 + 5*a^3*b^2)*tan(d*x + c)^2)/(a^6*b^3*tan(d*x + c)^6 + 3*a^7*b^2*tan(d*x + c)^5 + 3*a^8*b*tan(d*x + c)^4 + a^9*tan(d*x + c)^3) + 12*(a^2*b +

$$5*b^3)*\log(b*\tan(d*x + c) + a)/a^7 - 12*(a^2*b + 5*b^3)*\log(\tan(d*x + c))/a^7)/d$$

Fricas [B] time = 3.25469, size = 2732, normalized size = 13.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="fricas")

[Out]
$$\frac{1}{3}*(19*a^6*b^4 + 51*a^4*b^6 + 30*a^2*b^8 + 2*(a^{10} + 23*a^8*b^2 - 22*a^6*b^4 - 138*a^4*b^6 - 90*a^2*b^8)*\cos(d*x + c)^6 - 3*(a^{10} + 25*a^8*b^2 - 46*a^6*b^4 - 206*a^4*b^6 - 130*a^2*b^8)*\cos(d*x + c)^4 + 3*(9*a^8*b^2 - 38*a^6*b^4 - 131*a^4*b^6 - 80*a^2*b^8)*\cos(d*x + c)^2 + 6*(a^6*b^4 + 7*a^4*b^6 + 11*a^2*b^8 + 5*b^{10} + (3*a^8*b^2 + 20*a^6*b^4 + 26*a^4*b^6 + 4*a^2*b^8 - 5*b^{10})*\cos(d*x + c)^6 - 3*(2*a^8*b^2 + 13*a^6*b^4 + 15*a^4*b^6 - a^2*b^8 - 5*b^{10})*\cos(d*x + c)^4 + 3*(a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 - 6*a^2*b^8 - 5*b^{10})*\cos(d*x + c)^2 - ((a^9*b + 4*a^7*b^3 - 10*a^5*b^5 - 28*a^3*b^7 - 15*a*b^9)*\cos(d*x + c)^5 - (a^9*b + a^7*b^3 - 31*a^5*b^5 - 61*a^3*b^7 - 30*a*b^9)*\cos(d*x + c)^3 - 3*(a^7*b^3 + 7*a^5*b^5 + 11*a^3*b^7 + 5*a*b^9)*\cos(d*x + c))*\sin(d*x + c))*\log(2*a*b*\cos(d*x + c)*\sin(d*x + c) + (a^2 - b^2)*\cos(d*x + c)^2 + b^2) - 6*(a^6*b^4 + 7*a^4*b^6 + 11*a^2*b^8 + 5*b^{10} + (3*a^8*b^2 + 20*a^6*b^4 + 26*a^4*b^6 + 4*a^2*b^8 - 5*b^{10})*\cos(d*x + c)^6 - 3*(2*a^8*b^2 + 13*a^6*b^4 + 15*a^4*b^6 - a^2*b^8 - 5*b^{10})*\cos(d*x + c)^4 + 3*(a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 - 6*a^2*b^8 - 5*b^{10})*\cos(d*x + c)^2 - ((a^9*b + 4*a^7*b^3 - 10*a^5*b^5 - 28*a^3*b^7 - 15*a*b^9)*\cos(d*x + c)^5 - (a^9*b + a^7*b^3 - 31*a^5*b^5 - 61*a^3*b^7 - 30*a*b^9)*\cos(d*x + c)^3 - 3*(a^7*b^3 + 7*a^5*b^5 + 11*a^3*b^7 + 5*a*b^9)*\cos(d*x + c))*\sin(d*x + c))*\log(-1/4*\cos(d*x + c)^2 + 1/4) + (2*(3*a^9*b + 77*a^7*b^3 + 142*a^5*b^5 + 34*a^3*b^7 - 30*a*b^9)*\cos(d*x + c)^5 - (3*a^9*b + 193*a^7*b^3 + 350*a^5*b^5 + 26*a^3*b^7 - 120*a*b^9)*\cos(d*x + c)^3 + 3*(15*a^7*b^3 + 23*a^5*b^5 - 14*a^3*b^7 - 20*a*b^9)*\cos(d*x + c))*\sin(d*x + c))/((3*a^{13}*b + 5*a^{11}*b^3 + a^9*b^5 - a^7*b^7)*d*\cos(d*x + c)^6 - 3*(2*a^{13}*b + 3*a^{11}*b^3 - a^7*b^7)*d*\cos(d*x + c)^4 + 3*(a^{13}*b + a^{11}*b^3 - a^9*b^5 - a^7*b^7)*d*\cos(d*x + c)^2 + (a^{11}*b^3 + 2*a^9*b^5 + a^7*b^7)*d - ((a^{14} - a^{12}*b^2 - 5*a^{10}*b^4 - 3*a^8*b^6)*d*\cos(d*x + c)^5 - (a^{14} - 4*a^{12}*b^2 - 11*a^{10}*b^4 - 6*a^8*b^6)*d*\cos(d*x + c)^3 - 3*(a^{12}*b^2 + 2*a^{10}*b^4 + a^8*b^6)*d*\cos(d*x + c))*\sin(d*x + c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**4,x)

[Out] Timed out

Giac [A] time = 1.33433, size = 300, normalized size = 1.46

$$\frac{12(a^2b+5b^3)\log(|\tan(dx+c)|)}{a^7} - \frac{12(a^2b^2+5b^4)\log(|b\tan(dx+c)+a|)}{a^7b} + \frac{12a^2b^3\tan(dx+c)^5+60b^5\tan(dx+c)^5+30a^3b^2\tan(dx+c)^4+150ab^4\tan(dx+c)^4+2}{(b\tan(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^2*b + 5*b^3)*log(abs(tan(d*x + c)))/a^7 - 12*(a^2*b^2 + 5*b^4)*
log(abs(b*tan(d*x + c) + a))/(a^7*b) + (12*a^2*b^3*tan(d*x + c)^5 + 60*b^5*
tan(d*x + c)^5 + 30*a^3*b^2*tan(d*x + c)^4 + 150*a*b^4*tan(d*x + c)^4 + 22*
a^4*b*tan(d*x + c)^3 + 110*a^2*b^3*tan(d*x + c)^3 + 3*a^5*tan(d*x + c)^2 +
15*a^3*b^2*tan(d*x + c)^2 - 3*a^4*b*tan(d*x + c) + a^5)/((b*tan(d*x + c)^2
+ a*tan(d*x + c))^3*a^6))/d
```

$$3.77 \quad \int \frac{\csc^6(c+dx)}{(a+b \tan(c+dx))^4} dx$$

Optimal. Leaf size=300

$$\frac{b(20a^2b^2 + 3a^4 + 21b^4)}{a^8d(a + b \tan(c + dx))} - \frac{b(a^2 + b^2)(a^2 + 3b^2)}{a^7d(a + b \tan(c + dx))^2} - \frac{b(a^2 + b^2)^2}{3a^6d(a + b \tan(c + dx))^3} - \frac{2(a^2 + 5b^2) \cot^3(c + dx)}{3a^6d} + \frac{2b(2a^2 + 5b^2)}{3a^6d}$$

[Out] -(((a^4 + 20*a^2*b^2 + 35*b^4)*Cot[c + d*x])/(a^8*d)) + (2*b*(2*a^2 + 5*b^2)*Cot[c + d*x]^2)/(a^7*d) - (2*(a^2 + 5*b^2)*Cot[c + d*x]^3)/(3*a^6*d) + (b*Cot[c + d*x]^4)/(a^5*d) - Cot[c + d*x]^5/(5*a^4*d) - (4*b*(a^4 + 10*a^2*b^2 + 14*b^4)*Log[Tan[c + d*x]])/(a^9*d) + (4*b*(a^4 + 10*a^2*b^2 + 14*b^4)*Log[a + b*Tan[c + d*x]])/(a^9*d) - (b*(a^2 + b^2)^2)/(3*a^6*d*(a + b*Tan[c + d*x])^3) - (b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^4 + 20*a^2*b^2 + 21*b^4))/(a^8*d*(a + b*Tan[c + d*x]))

Rubi [A] time = 0.271243, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 894}

$$\frac{b(20a^2b^2 + 3a^4 + 21b^4)}{a^8d(a + b \tan(c + dx))} - \frac{b(a^2 + b^2)(a^2 + 3b^2)}{a^7d(a + b \tan(c + dx))^2} - \frac{b(a^2 + b^2)^2}{3a^6d(a + b \tan(c + dx))^3} - \frac{2(a^2 + 5b^2) \cot^3(c + dx)}{3a^6d} + \frac{2b(2a^2 + 5b^2)}{3a^6d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^4, x]

[Out] -(((a^4 + 20*a^2*b^2 + 35*b^4)*Cot[c + d*x])/(a^8*d)) + (2*b*(2*a^2 + 5*b^2)*Cot[c + d*x]^2)/(a^7*d) - (2*(a^2 + 5*b^2)*Cot[c + d*x]^3)/(3*a^6*d) + (b*Cot[c + d*x]^4)/(a^5*d) - Cot[c + d*x]^5/(5*a^4*d) - (4*b*(a^4 + 10*a^2*b^2 + 14*b^4)*Log[Tan[c + d*x]])/(a^9*d) + (4*b*(a^4 + 10*a^2*b^2 + 14*b^4)*Log[a + b*Tan[c + d*x]])/(a^9*d) - (b*(a^2 + b^2)^2)/(3*a^6*d*(a + b*Tan[c + d*x])^3) - (b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^4 + 20*a^2*b^2 + 21*b^4))/(a^8*d*(a + b*Tan[c + d*x]))

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 894

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4} dx = \frac{b \operatorname{Subst}\left(\int \frac{(b^2+x^2)^2}{x^6(a+x)^4} dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b \operatorname{Subst}\left(\int \left(\frac{b^4}{a^4x^6} - \frac{4b^4}{a^5x^5} + \frac{2b^2(a^2+5b^2)}{a^6x^4} - \frac{4(2a^2b^2+5b^4)}{a^7x^3} + \frac{a^4+20a^2b^2+35b^4}{a^8x^2} - \frac{4(a^4+10a^2b^2+14b^4)}{a^9x} + \frac{(a^4+10a^2b^2+14b^4)^2}{a^{10}}\right) dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{(a^4+20a^2b^2+35b^4)\cot(c+dx)}{a^8d} + \frac{2b(2a^2+5b^2)\cot^2(c+dx)}{a^7d} - \frac{2(a^2+5b^2)\cot^3(c+dx)}{3a^6d}$$

Mathematica [B] time = 1.63369, size = 673, normalized size = 2.24

$$\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-7680b(10a^2b^2+a^4+14b^4)\log(\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^4, x]

[Out] (Sec[c + d*x]^4*(a*cos[c + d*x] + b*sin[c + d*x])*(-7680*b*(a^4 + 10*a^2*b^2 + 14*b^4)*Log[Sin[c + d*x]]*(a*cos[c + d*x] + b*sin[c + d*x])^3 + 7680*b*(a^4 + 10*a^2*b^2 + 14*b^4)*Log[a*cos[c + d*x] + b*sin[c + d*x]]*(a*cos[c + d*x] + b*sin[c + d*x])^3 + Csc[c + d*x]^5*(-200*a^8 + 380*a^6*b^2 + 3070*a^4*b^4 + 11375*a^2*b^6 + 11025*b^8 - 4*(52*a^8 + 194*a^6*b^2 + 1510*a^4*b^4 + 5705*a^2*b^6 + 4410*b^8)*Cos[2*(c + d*x)] + 4*(4*a^8 - 16*a^6*b^2 + 1010*a^4*b^4 + 4585*a^2*b^6 + 2205*b^8)*Cos[4*(c + d*x)] + 16*a^8*Cos[6*(c + d*x)] + 776*a^6*b^2*Cos[6*(c + d*x)] - 1000*a^4*b^4*Cos[6*(c + d*x)] - 8540*a^2*b^6*Cos[6*(c + d*x)] - 2520*b^8*Cos[6*(c + d*x)] - 8*a^8*Cos[8*(c + d*x)] - 316*a^6*b^2*Cos[8*(c + d*x)] - 70*a^4*b^4*Cos[8*(c + d*x)] + 1645*a^2*b^6*Cos[8*(c + d*x)] + 315*b^8*Cos[8*(c + d*x)] + 264*a^7*b*sin[2*(c + d*x)] + 372*a^5*b^3*sin[2*(c + d*x)] + 4830*a^3*b^5*sin[2*(c + d*x)] + 1470*a*b^7*sin[2*(c + d*x)] + 144*a^7*b*sin[4*(c + d*x)] - 2476*a^5*b^3*sin[4*(c + d*x)] - 9730*a^3*b^5*sin[4*(c + d*x)] - 1470*a*b^7*sin[4*(c + d*x)] - 24*a^7*b*sin[6*(c + d*x)] + 2756*a^5*b^3*sin[6*(c + d*x)] + 7670*a^3*b^5*sin[6*(c + d*x)] + 630*a*b^7*sin[6*(c + d*x)] - 24*a^7*b*sin[8*(c + d*x)] - 922*a^5*b^3*sin[8*(c + d*x)] - 2095*a^3*b^5*sin[8*(c + d*x)] - 105*a*b^7*sin[8*(c + d*x)]))/(1920*a^9*d*(a + b*Tan[c + d*x])^4)

Maple [A] time = 0.165, size = 476, normalized size = 1.6

$$\frac{1}{5da^4(\tan(dx+c))^5} - \frac{2}{3da^4(\tan(dx+c))^3} - \frac{10b^2}{3da^6(\tan(dx+c))^3} - \frac{1}{da^4\tan(dx+c)} - 20\frac{b^2}{da^6\tan(dx+c)} - 35\frac{b^4}{da^8\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^4, x)

[Out] -1/5/d/a^4/tan(d*x+c)^5-2/3/d/a^4/tan(d*x+c)^3-10/3/d/a^6/tan(d*x+c)^3*b^2-1/d/a^4/tan(d*x+c)-20/d/a^6/tan(d*x+c)*b^2-35/d/a^8/tan(d*x+c)*b^4+1/d/a^5*b/tan(d*x+c)^4+4/d/a^5*b/tan(d*x+c)^2+10/d*b^3/a^7/tan(d*x+c)^2-4*b*ln(tan(d*x+c))/a^5/d-40/d*b^3/a^7*ln(tan(d*x+c))-56/d*b^5/a^9*ln(tan(d*x+c))-3*b/a^4/d/(a+b*tan(d*x+c))-20/d*b^3/a^6/(a+b*tan(d*x+c))-21/d*b^5/a^8/(a+b*tan(d*x+c))-1/3*b/a^2/d/(a+b*tan(d*x+c))^3-2/3/d*b^3/a^4/(a+b*tan(d*x+c))^3-1/3/

$$d^6b^5/a^6/(a+b*\tan(dx+c))^3-b/a^3/d/(a+b*\tan(dx+c))^2-4/d*b^3/a^5/(a+b*\tan(dx+c))^2-3/d*b^5/a^7/(a+b*\tan(dx+c))^2+4*b*\ln(a+b*\tan(dx+c))/a^5/d+40/d*b^3/a^7*\ln(a+b*\tan(dx+c))+56/d*b^5/a^9*\ln(a+b*\tan(dx+c))$$

Maxima [A] time = 1.23988, size = 439, normalized size = 1.46

$$\frac{6a^6b \tan(dx+c) - 60(a^4b^3 + 10a^2b^5 + 14b^7) \tan(dx+c)^7 - 3a^7 - 150(a^5b^2 + 10a^3b^4 + 14ab^6) \tan(dx+c)^6 - 110(a^6b + 10a^4b^3 + 14a^2b^5) \tan(dx+c)^5 - 15(a^7 + 10a^5b^2 + 14a^3b^4) \tan(dx+c)^4 + 6*(5a^6b + 7a^4b^3) \tan(dx+c)^3 - 2*(5a^7 + 7a^5b^2) \tan(dx+c)^2}{a^8b^3 \tan(dx+c)^8 + 3a^9b^2 \tan(dx+c)^7 + 3a^{10}b \tan(dx+c)^6 + a^{11} \tan(dx+c)^5} - 60*(a^4*b^3 + 10*a^2*b^5 + 14*b^7)*\tan(dx+c)^7 - 3*a^7 - 150*(a^5*b^2 + 10*a^3*b^4 + 14*a*b^6)*\tan(dx+c)^6 - 110*(a^6*b + 10*a^4*b^3 + 14*a^2*b^5)*\tan(dx+c)^5 - 15*(a^7 + 10*a^5*b^2 + 14*a^3*b^4)*\tan(dx+c)^4 + 6*(5*a^6*b + 7*a^4*b^3)*\tan(dx+c)^3 - 2*(5*a^7 + 7*a^5*b^2)*\tan(dx+c)^2 / (a^8*b^3*\tan(dx+c)^8 + 3*a^9*b^2*\tan(dx+c)^7 + 3*a^{10}*b*\tan(dx+c)^6 + a^{11}*\tan(dx+c)^5) + 60*(a^4*b + 10*a^2*b^3 + 14*b^5)*\log(b*\tan(dx+c) + a)/a^9 - 60*(a^4*b + 10*a^2*b^3 + 14*b^5)*\log(\tan(dx+c))/a^9 / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)^6/(a+b*tan(dx+c))^4,x, algorithm="maxima")

[Out] 1/15*((6*a^6*b*tan(dx + c) - 60*(a^4*b^3 + 10*a^2*b^5 + 14*b^7)*tan(dx + c)^7 - 3*a^7 - 150*(a^5*b^2 + 10*a^3*b^4 + 14*a*b^6)*tan(dx + c)^6 - 110*(a^6*b + 10*a^4*b^3 + 14*a^2*b^5)*tan(dx + c)^5 - 15*(a^7 + 10*a^5*b^2 + 14*a^3*b^4)*tan(dx + c)^4 + 6*(5*a^6*b + 7*a^4*b^3)*tan(dx + c)^3 - 2*(5*a^7 + 7*a^5*b^2)*tan(dx + c)^2)/(a^8*b^3*tan(dx + c)^8 + 3*a^9*b^2*tan(dx + c)^7 + 3*a^10*b*tan(dx + c)^6 + a^11*tan(dx + c)^5) + 60*(a^4*b + 10*a^2*b^3 + 14*b^5)*log(b*tan(dx + c) + a)/a^9 - 60*(a^4*b + 10*a^2*b^3 + 14*b^5)*log(tan(dx + c))/a^9)/d

Fricas [B] time = 4.43096, size = 3507, normalized size = 11.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(dx+c)^6/(a+b*tan(dx+c))^4,x, algorithm="fricas")

[Out] -1/15*(110*a^6*b^4 + 510*a^4*b^6 + 420*a^2*b^8 - 4*(2*a^10 + 81*a^8*b^2 + 2*9*a^6*b^4 - 660*a^4*b^6 - 630*a^2*b^8)*cos(dx + c)^8 + 2*(10*a^10 + 423*a^8*b^2 - 47*a^6*b^4 - 4320*a^4*b^6 - 3990*a^2*b^8)*cos(dx + c)^6 - 15*(a^10 + 47*a^8*b^2 - 44*a^6*b^4 - 658*a^4*b^6 - 588*a^2*b^8)*cos(dx + c)^4 + 20*(9*a^8*b^2 - 28*a^6*b^4 - 219*a^4*b^6 - 189*a^2*b^8)*cos(dx + c)^2 + 30*(a^6*b^4 + 11*a^4*b^6 + 24*a^2*b^8 + 14*b^10 - (3*a^8*b^2 + 32*a^6*b^4 + 61*a^4*b^6 + 18*a^2*b^8 - 14*b^10)*cos(dx + c)^8 + (9*a^8*b^2 + 95*a^6*b^4 + 172*a^4*b^6 + 30*a^2*b^8 - 56*b^10)*cos(dx + c)^6 - 3*(3*a^8*b^2 + 31*a^6*b^4 + 50*a^4*b^6 - 6*a^2*b^8 - 28*b^10)*cos(dx + c)^4 + (3*a^8*b^2 + 29*a^6*b^4 + 28*a^4*b^6 - 54*a^2*b^8 - 56*b^10)*cos(dx + c)^2 + ((a^9*b + 8*a^7*b^3 - 9*a^5*b^5 - 58*a^3*b^7 - 42*a*b^9)*cos(dx + c)^7 - (2*a^9*b + 13*a^7*b^3 - 51*a^5*b^5 - 188*a^3*b^7 - 126*a*b^9)*cos(dx + c)^5 + (a^9*b + 2*a^7*b^3 - 75*a^5*b^5 - 202*a^3*b^7 - 126*a*b^9)*cos(dx + c)^3 + 3*(a^7*b^3 + 11*a^5*b^5 + 24*a^3*b^7 + 14*a*b^9)*cos(dx + c))*sin(dx + c))*log(2*a*b*cos(dx + c)*sin(dx + c) + (a^2 - b^2)*cos(dx + c)^2 + b^2) - 30*(a^6*b^4 + 11*a^4*b^6 + 24*a^2*b^8 + 14*b^10 - (3*a^8*b^2 + 32*a^6*b^4 + 61*a^4*b^6 + 18*a^2*b^8 - 14*b^10)*cos(dx + c)^8 + (9*a^8*b^2 + 95*a^6*b^4 + 172*a^4*b^6 + 30*a^2*b^8 - 56*b^10)*cos(dx + c)^6 - 3*(3*a^8*b^2 + 31*a^6*b^4 + 50*a^4*b^6 - 6*a^2*b^8 - 28*b^10)*cos(dx + c)^4 + (3*a^8*b^2 + 29*a^6*b^4 + 28*a^4*b^6 - 54*a^2*b^8 - 56*b^10)*cos(dx + c)^2 + ((a^9*b + 8*a^7*b^3 - 9*a^5*b^5 - 58*a^3*b^7 - 42*a*b^9)*cos(dx + c)^7 - (2*a^9*b + 13*a^7*b^3 - 51*a^5*b^5 - 188*a^3*b^7 - 126*a*b^9)*cos(dx + c)^5 + (a^9*b + 2*a^7*b^3 - 75*a^5*b^5 - 202*a^3*b^7 - 126*a*b^9)*cos(dx + c)^3 + 3*(a^7*b^3 + 11*a^5*b^5 + 24*a^3*b^7 + 14*a*b^9)*cos(dx + c))*sin(dx + c))*log(-1/4*cos(dx + c))

$$x + c)^2 + 1/4) - 2*(2*(6*a^9*b + 259*a^7*b^3 + 783*a^5*b^5 + 340*a^3*b^7 - 210*a*b^9)*\cos(d*x + c)^7 - (15*a^9*b + 1141*a^7*b^3 + 3546*a^5*b^5 + 1270*a^3*b^7 - 1260*a*b^9)*\cos(d*x + c)^5 + 5*(151*a^7*b^3 + 483*a^5*b^5 + 100*a^3*b^7 - 252*a*b^9)*\cos(d*x + c)^3 - 15*(9*a^7*b^3 + 29*a^5*b^5 - 6*a^3*b^7 - 28*a*b^9)*\cos(d*x + c))*\sin(d*x + c))/((3*a^13*b + 2*a^11*b^3 - a^9*b^5)*d*\cos(d*x + c)^8 - (9*a^13*b + 5*a^11*b^3 - 4*a^9*b^5)*d*\cos(d*x + c)^6 + 3*(3*a^13*b + a^11*b^3 - 2*a^9*b^5)*d*\cos(d*x + c)^4 - (3*a^13*b - a^11*b^3 - 4*a^9*b^5)*d*\cos(d*x + c)^2 - (a^11*b^3 + a^9*b^5)*d - ((a^14 - 2*a^12*b^2 - 3*a^10*b^4)*d*\cos(d*x + c)^7 - (2*a^14 - 7*a^12*b^2 - 9*a^10*b^4)*d*\cos(d*x + c)^5 + (a^14 - 8*a^12*b^2 - 9*a^10*b^4)*d*\cos(d*x + c)^3 + 3*(a^12*b^2 + a^10*b^4)*d*\cos(d*x + c))*\sin(d*x + c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**4,x)

[Out] Timed out

Giac [A] time = 1.36148, size = 578, normalized size = 1.93

$$\frac{60(a^4b+10a^2b^3+14b^5)\log(|\tan(dx+c)|)}{a^9} - \frac{60(a^4b^2+10a^2b^4+14b^6)\log(|b\tan(dx+c)+a|)}{a^9b} + \frac{5(22a^4b^4\tan(dx+c)^3+220a^2b^6\tan(dx+c)^3+308b^8\tan(dx+c)^3+75a^5b^3\tan(dx+c)^2+720a^3b^5\tan(dx+c)^2+987a*b^7\tan(dx+c)^2+87a^6b^2\tan(dx+c)+792a^4b^4\tan(dx+c)+1059a^2b^6\tan(dx+c)+35a^7b+294a^5b^3+381a^3b^5)}{(b\tan(dx+c)+a)^3a^9} - \frac{(137a^4b\tan(dx+c)^5+1370a^2b^3\tan(dx+c)^5+1918b^5\tan(dx+c)^5-15a^5\tan(dx+c)^4-300a^3b^2\tan(dx+c)^4-525a*b^4\tan(dx+c)^4+60a^4b\tan(dx+c)^3+150a^2b^3\tan(dx+c)^3-10a^5\tan(dx+c)^2-50a^3b^2\tan(dx+c)^2+15a^4b\tan(dx+c)-3a^5)}{(a^9\tan(dx+c)^5)} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="giac")

[Out]
$$-1/15*(60*(a^4*b + 10*a^2*b^3 + 14*b^5)*\log(\text{abs}(\tan(d*x + c)))/a^9 - 60*(a^4*b^2 + 10*a^2*b^4 + 14*b^6)*\log(\text{abs}(b*\tan(d*x + c) + a))/(a^9*b) + 5*(22*a^4*b^4*\tan(d*x + c)^3 + 220*a^2*b^6*\tan(d*x + c)^3 + 308*b^8*\tan(d*x + c)^3 + 75*a^5*b^3*\tan(d*x + c)^2 + 720*a^3*b^5*\tan(d*x + c)^2 + 987*a*b^7*\tan(d*x + c)^2 + 87*a^6*b^2*\tan(d*x + c) + 792*a^4*b^4*\tan(d*x + c) + 1059*a^2*b^6*\tan(d*x + c) + 35*a^7*b + 294*a^5*b^3 + 381*a^3*b^5)/((b*\tan(d*x + c) + a)^3*a^9) - (137*a^4*b*\tan(d*x + c)^5 + 1370*a^2*b^3*\tan(d*x + c)^5 + 1918*b^5*\tan(d*x + c)^5 - 15*a^5*\tan(d*x + c)^4 - 300*a^3*b^2*\tan(d*x + c)^4 - 525*a*b^4*\tan(d*x + c)^4 + 60*a^4*b*\tan(d*x + c)^3 + 150*a^2*b^3*\tan(d*x + c)^3 - 10*a^5*\tan(d*x + c)^2 - 50*a^3*b^2*\tan(d*x + c)^2 + 15*a^4*b*\tan(d*x + c) - 3*a^5)/(a^9*\tan(d*x + c)^5))/d$$

$$3.78 \quad \int \frac{\csc(x)}{1+\tan(x)} dx$$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}} - \tanh^{-1}(\cos(x))$$

[Out] -ArcTanh[Cos[x]] + ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2]

Rubi [A] time = 0.0744782, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3518, 3110, 3770, 3074, 206}

$$\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}} - \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Csc[x]/(1 + Tan[x]),x]

[Out] -ArcTanh[Cos[x]] + ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2]

Rule 3518

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*cos[e + f*x] + b*sin[e + f*x])^n)/Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))

Rule 3110

Int[(cos[(c_.) + (d_.)*(x_.)]^(m_.)*sin[(c_.) + (d_.)*(x_.)]^(n_.))/(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)]), x_Symbol] :> Int[ExpandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3074

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)]^(-1), x_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*cos[c + d*x] - a*sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{1 + \tan(x)} dx &= \int \frac{\cot(x)}{\cos(x) + \sin(x)} dx \\
&= \int \left(\csc(x) + \frac{1}{-\cos(x) - \sin(x)} \right) dx \\
&= \int \csc(x) dx + \int \frac{1}{-\cos(x) - \sin(x)} dx \\
&= -\tanh^{-1}(\cos(x)) - \text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, -\cos(x) + \sin(x) \right) \\
&= -\tanh^{-1}(\cos(x)) - \frac{\tanh^{-1} \left(\frac{-\cos(x) + \sin(x)}{\sqrt{2}} \right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.0413579, size = 41, normalized size = 1.58

$$\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + (1+i)(-1)^{3/4} \tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right) - 1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]/(1 + Tan[x]), x]

[Out] (1 + I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - Log[Cos[x/2]] + Log[Sin[x/2]]

Maple [A] time = 0.025, size = 26, normalized size = 1.

$$-\sqrt{2} \operatorname{Arctanh}\left(\frac{\sqrt{2}}{4}(2 \tan(x/2) - 2)\right) + \ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)/(1+tan(x)), x)

[Out] -2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))+ln(tan(1/2*x))

Maxima [B] time = 1.85027, size = 68, normalized size = 2.62

$$\frac{1}{2} \sqrt{2} \log\left(-\frac{\sqrt{2} - \frac{\sin(x)}{\cos(x)+1} + 1}{\sqrt{2} + \frac{\sin(x)}{\cos(x)+1} - 1}\right) + \log\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(1+tan(x)), x, algorithm="maxima")

[Out] 1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(cos(x) + 1) - 1)) + log(sin(x)/(cos(x) + 1))

Fricas [B] time = 2.35363, size = 203, normalized size = 7.81

$$\frac{1}{4} \sqrt{2} \log \left(\frac{2(\sqrt{2} + \cos(x)) \sin(x) - 2\sqrt{2} \cos(x) - 3}{2 \cos(x) \sin(x) + 1} \right) - \frac{1}{2} \log \left(\frac{1}{2} \cos(x) + \frac{1}{2} \right) + \frac{1}{2} \log \left(-\frac{1}{2} \cos(x) + \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(1+tan(x)),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log((2*(sqrt(2) + cos(x))*sin(x) - 2*sqrt(2)*cos(x) - 3)/(2*cos(x)*sin(x) + 1)) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(x)}{\tan(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(1+tan(x)),x)

[Out] Integral(csc(x)/(tan(x) + 1), x)

Giac [A] time = 1.29647, size = 59, normalized size = 2.27

$$\frac{1}{2} \sqrt{2} \log \left(\frac{\left| -2\sqrt{2} + 2 \tan\left(\frac{1}{2}x\right) - 2 \right|}{\left| 2\sqrt{2} + 2 \tan\left(\frac{1}{2}x\right) - 2 \right|} \right) + \log \left(\left| \tan\left(\frac{1}{2}x\right) \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(1+tan(x)),x, algorithm="giac")

[Out] 1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(1/2*x) - 2)) + log(abs(tan(1/2*x)))

3.79 $\int \sin^m(c + dx)(a + b \tan(c + dx))^3 dx$

Optimal. Leaf size=229

$$\frac{3a^2b \sin^{m+2}(c + dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; \sin^2(c + dx)\right)}{d(m+2)} + \frac{a^3 \cos(c + dx) \sin^{m+1}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \sin^2(c + dx)\right)}{d(m+1)\sqrt{\cos^2(c + dx)}}$$

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2*b*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 + m)) + (3*a*b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m)) + (b^3*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(4 + m))/(d*(4 + m))
```

Rubi [A] time = 0.449863, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4401, 2643, 2564, 364, 2577}

$$\frac{3a^2b \sin^{m+2}(c + dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; \sin^2(c + dx)\right)}{d(m+2)} + \frac{a^3 \cos(c + dx) \sin^{m+1}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \sin^2(c + dx)\right)}{d(m+1)\sqrt{\cos^2(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2*b*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 + m)) + (3*a*b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m)) + (b^3*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(4 + m))/(d*(4 + m))
```

Rule 4401

```
Int[u_, x_Symbol] := With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /; !InertTrigFreeQ[u]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)^(n_.)]*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] := Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned} \int \sin^m(c + dx)(a + b \tan(c + dx))^3 dx &= \int (a^3 \sin^m(c + dx) + 3a^2 b \sec(c + dx) \sin^{1+m}(c + dx) + 3ab^2 \sec^2(c + dx) \sin^{2+m}(c + dx) + b^3 \sec^3(c + dx) \sin^{3+m}(c + dx)) dx \\ &= a^3 \int \sin^m(c + dx) dx + (3a^2 b) \int \sec(c + dx) \sin^{1+m}(c + dx) dx + (3ab^2) \int \sec^2(c + dx) \sin^{2+m}(c + dx) dx + b^3 \int \sec^3(c + dx) \sin^{3+m}(c + dx) dx \\ &= \frac{a^3 \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1+m}{2}; \frac{3+m}{2}; \sin^2(c + dx)\right) \sin^{1+m}(c + dx)}{d(1+m)\sqrt{\cos^2(c + dx)}} + \frac{3ab^2 \sqrt{\cos^2(c + dx)} {}_2F_1\left(\frac{1}{2}, \frac{1+m}{2}; \frac{3+m}{2}; \sin^2(c + dx)\right) \sin^{1+m}(c + dx)}{d(1+m)\sqrt{\cos^2(c + dx)}} + \frac{3a^2 b {}_2F_1\left(1, \frac{2+m}{2}; \frac{3+m}{2}; \sin^2(c + dx)\right) \sin^{2+m}(c + dx)}{d(1+m)\sqrt{\cos^2(c + dx)}} + \frac{b^3 {}_2F_1\left(1, \frac{2+m}{2}; \frac{3+m}{2}; \sin^2(c + dx)\right) \sin^{2+m}(c + dx)}{d(1+m)\sqrt{\cos^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 2.51971, size = 205, normalized size = 0.9

$$\frac{\sin^{m+1}(c + dx) \left(b \sin(c + dx) \left(\frac{3a^2 {}_2F_1\left(1, \frac{m+2}{2}, \frac{m+4}{2}; \sin^2(c + dx)\right)}{m+2} + b \left(\frac{3a \sqrt{\cos^2(c + dx)} \tan(c + dx) {}_2F_1\left(\frac{3}{2}, \frac{m+3}{2}, \frac{m+5}{2}; \sin^2(c + dx)\right)}{m+3} + \frac{b \sin^2(c + dx) {}_2F_1\left(2, \frac{m+4}{2}, \frac{m+6}{2}; \sin^2(c + dx)\right)}{m+4} \right) \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sin[c + d*x]^(1 + m)*((a^3*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x])/(1 + m) + b*Sin[c + d*x]*
(3*a^2*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2])/(2 + m)
+ b*((b*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*Sin[c +
d*x]^2)/(4 + m) + (3*a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/
2, (5 + m)/2, Sin[c + d*x]^2]*Tan[c + d*x])/(3 + m))))/d
```

Maple [F] time = 0.392, size = 0, normalized size = 0.

$$\int (\sin(dx + c))^m (a + b \tan(dx + c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x)
```

```
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^3 \sin(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((b*tan(d*x + c) + a)^3*sin(d*x + c)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \tan(dx + c)^3 + 3ab^2 \tan(dx + c)^2 + 3a^2b \tan(dx + c) + a^3\right) \sin(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] integral((b^3*tan(d*x + c)^3 + 3*a*b^2*tan(d*x + c)^2 + 3*a^2*b*tan(d*x + c) + a^3)*sin(d*x + c)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^3 \sin(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] integrate((b*tan(d*x + c) + a)^3*sin(d*x + c)^m, x)

3.80 $\int \sin^m(c + dx)(a + b \tan(c + dx))^2 dx$

Optimal. Leaf size=179

$$\frac{a^2 \cos(c + dx) \sin^{m+1}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \sin^2(c + dx)\right)}{d(m+1)\sqrt{\cos^2(c + dx)}} + \frac{2ab \sin^{m+2}(c + dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; \sin^2(c + dx)\right)}{d(m+2)} + \frac{b^2 \sqrt{\cos^2(c + dx)}}{d}$$

[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a*b*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 + m)) + (b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m))

Rubi [A] time = 0.268018, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4401, 2643, 2564, 364, 2577}

$$\frac{a^2 \cos(c + dx) \sin^{m+1}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \sin^2(c + dx)\right)}{d(m+1)\sqrt{\cos^2(c + dx)}} + \frac{2ab \sin^{m+2}(c + dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; \sin^2(c + dx)\right)}{d(m+2)} + \frac{b^2 \sqrt{\cos^2(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^2,x]

[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a*b*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 + m)) + (b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m))

Rule 4401

Int[u_, x_Symbol] := With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /; !InertTrigFreeQ[u]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2564

Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt

Q[p, 0] || GtQ[a, 0])

Rule 2577

Int[(cos[(e_.) + (f_.)*(x_.)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*FracPart[(n - 1)/2]))*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2)]/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]

Rubi steps

$$\begin{aligned} \int \sin^m(c + dx)(a + b \tan(c + dx))^2 dx &= \int \left(a^2 \sin^m(c + dx) + 2ab \sec(c + dx) \sin^{1+m}(c + dx) + b^2 \sec^2(c + dx) \sin^{2+m}(c + dx) \right) dx \\ &= a^2 \int \sin^m(c + dx) dx + (2ab) \int \sec(c + dx) \sin^{1+m}(c + dx) dx + b^2 \int \sec^2(c + dx) \sin^{2+m}(c + dx) dx \\ &= \frac{a^2 \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1+m}{2}; \frac{3+m}{2}; \sin^2(c + dx)\right) \sin^{1+m}(c + dx)}{d(1+m)\sqrt{\cos^2(c + dx)}} + \frac{b^2 \sqrt{\cos^2(c + dx)} \sin^{2+m}(c + dx)}{d(1+m)\sqrt{\cos^2(c + dx)}} \\ &= \frac{a^2 \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1+m}{2}; \frac{3+m}{2}; \sin^2(c + dx)\right) \sin^{1+m}(c + dx)}{d(1+m)\sqrt{\cos^2(c + dx)}} + \frac{2ab {}_2F_1\left(1, \frac{2+m}{2}; \frac{3+m}{2}; \sin^2(c + dx)\right) \sin^{1+m}(c + dx)}{d(1+m)\sqrt{\cos^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 1.19385, size = 166, normalized size = 0.93

$$\frac{\sin^{m+1}(c + dx) \left(\frac{a^2 \sqrt{\cos^2(c + dx)} \sec(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}; \sin^2(c + dx)\right)}{m+1} + \frac{b \sin(c + dx) \left(2a(m+3) {}_2F_1\left(1, \frac{m+2}{2}, \frac{m+4}{2}; \sin^2(c + dx)\right) + b(m+2) \sqrt{\cos^2(c + dx)} \right)}{(m+2)(m+3)} \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^2,x]

[Out] (Sin[c + d*x]^(1 + m)*((a^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x])/(1 + m) + (b*Sin[c + d*x]*(2*a*(3 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2] + b*(2 + m)*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[c + d*x]^2]*Tan[c + d*x]))/((2 + m)*(3 + m)))/d

Maple [F] time = 0.248, size = 0, normalized size = 0.

$$\int (\sin(dx + c))^m (a + b \tan(dx + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x)

[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^2 \sin(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((b*tan(d*x + c) + a)^2*sin(d*x + c)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \tan(dx + c)^2 + 2ab \tan(dx + c) + a^2\right) \sin(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] integral((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)*sin(d*x + c)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^2 \sin(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] integrate((b*tan(d*x + c) + a)^2*sin(d*x + c)^m, x)

3.81 $\int \sin^m(c + dx)(a + b \tan(c + dx)) dx$

Optimal. Leaf size=109

$$\frac{a \cos(c + dx) \sin^{m+1}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \sin^2(c + dx)\right)}{d(m+1)\sqrt{\cos^2(c + dx)}} + \frac{b \sin^{m+2}(c + dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; \sin^2(c + dx)\right)}{d(m+2)}$$

[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 + m))

Rubi [A] time = 0.14461, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {4401, 2643, 2564, 364}

$$\frac{a \cos(c + dx) \sin^{m+1}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \sin^2(c + dx)\right)}{d(m+1)\sqrt{\cos^2(c + dx)}} + \frac{b \sin^{m+2}(c + dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; \sin^2(c + dx)\right)}{d(m+2)}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x]), x]

[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 + m))

Rule 4401

Int[u_, x_Symbol] :=> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /; !InertTrigFreeQ[u]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :=> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2564

Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :=> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*SIN[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :=> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int \sin^m(c+dx)(a+b\tan(c+dx))dx &= \int (a\sin^m(c+dx)+b\sec(c+dx)\sin^{1+m}(c+dx))dx \\
&= a\int \sin^m(c+dx)dx+b\int \sec(c+dx)\sin^{1+m}(c+dx)dx \\
&= \frac{a\cos(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1+m}{2}; \frac{3+m}{2}; \sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{b\operatorname{Subst}\left(\int \frac{x^{1+m}}{1-x^2}dx\right)}{d} \\
&= \frac{a\cos(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1+m}{2}; \frac{3+m}{2}; \sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{b {}_2F_1\left(1, \frac{2+m}{2}; \frac{4+m}{2}; \sin^2(c+dx)\right)}{d}
\end{aligned}$$

Mathematica [A] time = 0.257188, size = 109, normalized size = 1.

$$\frac{a\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \sin^2(c+dx)\right)}{d(m+1)} + \frac{b\sin^{m+2}(c+dx) {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; \sin^2(c+dx)\right)}{d(m+2)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x]), x]

[Out] (a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + m))/(d*(1 + m)) + (b*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 + m))

Maple [F] time = 0.488, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^m (a+b\tan(dx+c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c)), x)

[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c)), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)\sin(dx+c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)), x, algorithm="maxima")

[Out] integrate((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((b\tan(dx+c)+a)\sin(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] integral((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx)) \sin^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c)),x)

[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**m, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a) \sin(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] integrate((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)

$$3.82 \quad \int \frac{\sin^m(c+dx)}{a+b \tan(c+dx)} dx$$

Optimal. Leaf size=765

result too large to display

```
[Out] (2^(1+m)*Hypergeometric2F1[(1+m)/2, 1+m, (3+m)/2, -Tan[(c+d*x)/2]^2]*Tan[(c+d*x)/2]*(Tan[(c+d*x)/2]/(1+Tan[(c+d*x)/2]^2))^m*(1+Tan[(c+d*x)/2]^2)^m/(a*d*(1+m)) + (2^(1+m)*b*AppellF1[(2+m)/2, 1+m, 1, (4+m)/2, -Tan[(c+d*x)/2]^2, (a^2*Tan[(c+d*x)/2]^2)/(b-Sqrt[a^2+b^2])^2]*Tan[(c+d*x)/2]^2*(Tan[(c+d*x)/2]/(1+Tan[(c+d*x)/2]^2))^m*(1+Tan[(c+d*x)/2]^2)^m/(Sqrt[a^2+b^2]*(b-Sqrt[a^2+b^2])*d*(2+m)) - (2^(1+m)*b*AppellF1[(2+m)/2, 1+m, 1, (4+m)/2, -Tan[(c+d*x)/2]^2, (a^2*Tan[(c+d*x)/2]^2)/(b+Sqrt[a^2+b^2])^2]*Tan[(c+d*x)/2]^2*(Tan[(c+d*x)/2]/(1+Tan[(c+d*x)/2]^2))^m*(1+Tan[(c+d*x)/2]^2)^m/(Sqrt[a^2+b^2]*(b+Sqrt[a^2+b^2])*d*(2+m)) + (2^(1+m)*a*b*AppellF1[(3+m)/2, 1+m, 1, (5+m)/2, -Tan[(c+d*x)/2]^2, (a^2*Tan[(c+d*x)/2]^2)/(b-Sqrt[a^2+b^2])^2]*Tan[(c+d*x)/2]^3*(Tan[(c+d*x)/2]/(1+Tan[(c+d*x)/2]^2))^m*(1+Tan[(c+d*x)/2]^2)^m/(Sqrt[a^2+b^2]*(b-Sqrt[a^2+b^2])^2*d*(3+m)) - (2^(1+m)*a*b*AppellF1[(3+m)/2, 1+m, 1, (5+m)/2, -Tan[(c+d*x)/2]^2, (a^2*Tan[(c+d*x)/2]^2)/(b+Sqrt[a^2+b^2])^2]*Tan[(c+d*x)/2]^3*(Tan[(c+d*x)/2]/(1+Tan[(c+d*x)/2]^2))^m*(1+Tan[(c+d*x)/2]^2)^m/(Sqrt[a^2+b^2]*(b+Sqrt[a^2+b^2])^2*d*(3+m))
```

Rubi [A] time = 4.17316, antiderivative size = 765, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {12, 6719, 6728, 364, 959, 510}

$$\frac{ab2^{m+1} \tan^3\left(\frac{1}{2}(c+dx)\right) \left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m \left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m F_1\left(\frac{m+3}{2}; m+1, 1; \frac{m+5}{2}; -\tan^2\left(\frac{1}{2}(c+dx)\right)\right), \frac{a^2 \tan^2\left(\frac{1}{2}(c+dx)\right)}{(b-\sqrt{a^2+b^2})^2}}{d(m+3)\sqrt{a^2+b^2}(b-\sqrt{a^2+b^2})^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c+d*x]^m/(a+b*Tan[c+d*x]),x]
```

```
[Out] (2^(1+m)*Hypergeometric2F1[(1+m)/2, 1+m, (3+m)/2, -Tan[(c+d*x)/2]^2]*Tan[(c+d*x)/2]*(Tan[(c+d*x)/2]/(1+Tan[(c+d*x)/2]^2))^m*(1+Tan[(c+d*x)/2]^2)^m/(a*d*(1+m)) + (2^(1+m)*b*AppellF1[(2+m)/2, 1+m, 1, (4+m)/2, -Tan[(c+d*x)/2]^2, (a^2*Tan[(c+d*x)/2]^2)/(b-Sqrt[a^2+b^2])^2]*Tan[(c+d*x)/2]^2*(Tan[(c+d*x)/2]/(1+Tan[(c+d*x)/2]^2))^m*(1+Tan[(c+d*x)/2]^2)^m/(Sqrt[a^2+b^2]*(b-Sqrt[a^2+b^2])*d*(2+m)) - (2^(1+m)*b*AppellF1[(2+m)/2, 1+m, 1, (4+m)/2, -Tan[(c+d*x)/2]^2, (a^2*Tan[(c+d*x)/2]^2)/(b+Sqrt[a^2+b^2])^2]*Tan[(c+d*x)/2]^2*(Tan[(c+d*x)/2]/(1+Tan[(c+d*x)/2]^2))^m*(1+Tan[(c+d*x)/2]^2)^m/(Sqrt[a^2+b^2]*(b+Sqrt[a^2+b^2])*d*(2+m)) + (2^(1+m)*a*b*AppellF1[(3+m)/2, 1+m, 1, (5+m)/2, -Tan[(c+d*x)/2]^2, (a^2*Tan[(c+d*x)/2]^2)/(b-Sqrt[a^2+b^2])^2]*Tan[(c+d*x)/2]^3*(Tan[(c+d*x)/2]/(1+Tan[(c+d*x)/2]^2))^m*(1+Tan[(c+d*x)/2]^2)^m/(Sqrt[a^2+b^2]*(b-Sqrt[a^2+b^2])^2*d*(3+m)) - (2^(1+m)*a*b*AppellF1[(3+m)/2, 1+m, 1, (5+m)/2, -Tan[(c+d*x)/2]^2, (a^2*Tan[(c+d*x)/2]^2)/(b+Sqrt[a^2+b^2])^2]*Tan[(c+d*x)/2]^3*(Tan[(c+d*x)/2]/(1+Tan[(c+d*x)/2]^2))^m*(1+Tan[(c+d*x)/2]^2)^m/(Sqrt[a^2+b^2]*(b+Sqrt[a^2+b^2])^2*d*(3+m))
```

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 6719

$\text{Int}[(u_)*((a_)*(v_)^{(m_)}*(w_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[p]}*(a*v^m*w^n)^{\text{FracPart}[p]})/(v^{(m*\text{FracPart}[p])}*w^{(n*\text{FracPart}[p])}), \text{Int}[u*v^{(m*p)}*w^{(n*p)}, x], x] /; \text{FreeQ}[\{a, m, n, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ !\text{FreeQ}[v, x] \ \&\& \ !\text{FreeQ}[w, x]$

Rule 6728

$\text{Int}[(u_)/((a_)+(b_)*(x_)^{(n_)}+(c_)*(x_)^{(n2_)}), x_Symbol] \rightarrow \text{With}[\{v = \text{RationalFunctionExpand}[u/(a + b*x^n + c*x^{2*n})], x\}, \text{Int}[v, x] /; \text{SumQ}[v]] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{IGtQ}[n, 0]$

Rule 364

$\text{Int}[(c_)*(x_)^{(m_)}*((a_)+(b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a^p*(c*x)^{(m+1)}*\text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -((b*x^n)/a)])/(c*(m+1)), x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \ \&\& \ !\text{IGtQ}[p, 0] \ \&\& \ (\text{ILtQ}[p, 0] \ || \ \text{GtQ}[a, 0])$

Rule 959

$\text{Int}[(g_)*(x_)^{(n_)}*((a_)+(c_)*(x_)^2)^{(p_)}]/((d_)+(e_)*(x_)), x_Symbol] \rightarrow \text{Dist}[(d*(g*x)^n)/x^n, \text{Int}[(x^n*(a + c*x^2)^p]/(d^2 - e^2*x^2), x], x] - \text{Dist}[(e*(g*x)^n)/x^n, \text{Int}[(x^{n+1}*(a + c*x^2)^p]/(d^2 - e^2*x^2), x], x] /; \text{FreeQ}[\{a, c, d, e, g, n, p\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ !\text{IntegersQ}[n, 2*p]$

Rule 510

$\text{Int}[(e_)*(x_)^{(m_)}*((a_)+(b_)*(x_)^{(n_)})^{(p_)}*((c_)+(d_)*(x_)^{(n_)})^{(q_)}, x_Symbol] \rightarrow \text{Simp}[(a^p*c^q*(e*x)^{(m+1)}*\text{AppellF1}[(m+1)/n, -p, -q, 1 + (m+1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m+1)), x] /; \text{FreeQ}[\{a, b, c, d, e, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{NeQ}[m, n - 1] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0]) \ \&\& \ (\text{IntegerQ}[q] \ || \ \text{GtQ}[c, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{\sin^m(c+dx)}{a+b \tan(c+dx)} dx &= \frac{2 \operatorname{Subst} \left(\int \frac{2^m (1-x^2) \left(\frac{x}{1+x^2}\right)^{1+m}}{x(a+2bx-ax^2)} dx, x, \tan \left(\frac{1}{2}(c+dx)\right) \right)}{d} \\
&= \frac{2^{1+m} \operatorname{Subst} \left(\int \frac{(1-x^2) \left(\frac{x}{1+x^2}\right)^{1+m}}{x(a+2bx-ax^2)} dx, x, \tan \left(\frac{1}{2}(c+dx)\right) \right)}{d} \\
&= \frac{\left(2^{1+m} \tan^{-m} \left(\frac{1}{2}(c+dx)\right) \left(\frac{\tan \left(\frac{1}{2}(c+dx)\right)}{1+\tan^2 \left(\frac{1}{2}(c+dx)\right)} \right)^m \left(1 + \tan^2 \left(\frac{1}{2}(c+dx)\right) \right)^m \right) \operatorname{Subst} \left(\int \frac{x^m (1-x^2) (1+x^2)^{-1}}{a+2bx-ax^2} dx \right)}{d} \\
&= \frac{\left(2^{1+m} \tan^{-m} \left(\frac{1}{2}(c+dx)\right) \left(\frac{\tan \left(\frac{1}{2}(c+dx)\right)}{1+\tan^2 \left(\frac{1}{2}(c+dx)\right)} \right)^m \left(1 + \tan^2 \left(\frac{1}{2}(c+dx)\right) \right)^m \right) \operatorname{Subst} \left(\int \left(\frac{x^m (1+x^2)^{-1-m}}{a} \right) dx \right)}{d} \\
&= \frac{\left(2^{1+m} \tan^{-m} \left(\frac{1}{2}(c+dx)\right) \left(\frac{\tan \left(\frac{1}{2}(c+dx)\right)}{1+\tan^2 \left(\frac{1}{2}(c+dx)\right)} \right)^m \left(1 + \tan^2 \left(\frac{1}{2}(c+dx)\right) \right)^m \right) \operatorname{Subst} \left(\int x^m (1+x^2)^{-1-m} dx \right)}{ad} \\
&= \frac{2^{1+m} {}_2F_1 \left(\frac{1+m}{2}, 1+m; \frac{3+m}{2}; -\tan^2 \left(\frac{1}{2}(c+dx)\right) \right) \tan \left(\frac{1}{2}(c+dx)\right) \left(\frac{\tan \left(\frac{1}{2}(c+dx)\right)}{1+\tan^2 \left(\frac{1}{2}(c+dx)\right)} \right)^m \left(1 + \tan^2 \left(\frac{1}{2}(c+dx)\right) \right)^m}{ad(1+m)} \\
&= \frac{2^{1+m} {}_2F_1 \left(\frac{1+m}{2}, 1+m; \frac{3+m}{2}; -\tan^2 \left(\frac{1}{2}(c+dx)\right) \right) \tan \left(\frac{1}{2}(c+dx)\right) \left(\frac{\tan \left(\frac{1}{2}(c+dx)\right)}{1+\tan^2 \left(\frac{1}{2}(c+dx)\right)} \right)^m \left(1 + \tan^2 \left(\frac{1}{2}(c+dx)\right) \right)^m}{ad(1+m)} \\
&= \frac{2^{1+m} {}_2F_1 \left(\frac{1+m}{2}, 1+m; \frac{3+m}{2}; -\tan^2 \left(\frac{1}{2}(c+dx)\right) \right) \tan \left(\frac{1}{2}(c+dx)\right) \left(\frac{\tan \left(\frac{1}{2}(c+dx)\right)}{1+\tan^2 \left(\frac{1}{2}(c+dx)\right)} \right)^m \left(1 + \tan^2 \left(\frac{1}{2}(c+dx)\right) \right)^m}{ad(1+m)} \\
&= \frac{2^{1+m} {}_2F_1 \left(\frac{1+m}{2}, 1+m; \frac{3+m}{2}; -\tan^2 \left(\frac{1}{2}(c+dx)\right) \right) \tan \left(\frac{1}{2}(c+dx)\right) \left(\frac{\tan \left(\frac{1}{2}(c+dx)\right)}{1+\tan^2 \left(\frac{1}{2}(c+dx)\right)} \right)^m \left(1 + \tan^2 \left(\frac{1}{2}(c+dx)\right) \right)^m}{ad(1+m)}
\end{aligned}$$

Mathematica [F] time = 12.7916, size = 0, normalized size = 0.

$$\int \frac{\sin^m(c+dx)}{a+b \tan(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[c + d*x]^m/(a + b*Tan[c + d*x]), x]

[Out] Integrate[Sin[c + d*x]^m/(a + b*Tan[c + d*x]), x]

Maple [F] time = 0.164, size = 0, normalized size = 0.

$$\int \frac{(\sin(dx+c))^m}{a+b \tan(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(d*x+c)^m/(a+b*tan(d*x+c)),x)`

[Out] `int(sin(d*x+c)^m/(a+b*tan(d*x+c)),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^m}{b \tan(dx+c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="maxima")`

[Out] `integrate(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(dx+c)^m}{b \tan(dx+c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="fricas")`

[Out] `integral(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^m(c+dx)}{a+b \tan(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)**m/(a+b*tan(d*x+c)),x)`

[Out] `Integral(sin(c + d*x)**m/(a + b*tan(c + d*x)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^m}{b \tan(dx+c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="giac")`

[Out] `integrate(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)`

3.83 $\int \sin^m(c + dx)(a + b \tan(c + dx))^n dx$

Optimal. Leaf size=23

$$\text{CannotIntegrate}(\sin^m(c + dx)(a + b \tan(c + dx))^n, x)$$

[Out] CannotIntegrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]

Rubi [A] time = 2.46307, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \sin^m(c + dx)(a + b \tan(c + dx))^n dx$$

Verification is Not applicable to the result.

[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]

[Out] Defer[Int][Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]

Rubi steps

$$\int \sin^m(c + dx)(a + b \tan(c + dx))^n dx = \int \sin^m(c + dx)(a + b \tan(c + dx))^n dx$$

Mathematica [A] time = 3.18417, size = 0, normalized size = 0.

$$\int \sin^m(c + dx)(a + b \tan(c + dx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]

[Out] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]

Maple [A] time = 0.644, size = 0, normalized size = 0.

$$\int (\sin(dx + c))^m (a + b \tan(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^n, x)

[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^n, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \sin(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \tan(dx + c) + a)^n \sin(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.84 $\int \sin^4(c + dx)(a + b \tan(c + dx))^n dx$

Optimal. Leaf size=435

$$\frac{\left(\sqrt{-b^2} \left(a^2 b^2 (-n^2 + 6n + 6) + 3a^4 + b^4 (n^2 + 4n + 3)\right) + ab^2 n (5a^2 + b^2 (2n + 3))\right) (a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n + 1; \dots\right)}{16bd(n+1)(a^2 + b^2)^2 \left(a - \sqrt{-b^2}\right)}$$

```
[Out] -((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) + Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 + 6*n -
n^2) + b^4*(3 + 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan
[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(16*b*(a^2 + b^2
)^2*(a - Sqrt[-b^2])*d*(1 + n)) - ((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) - Sqrt[
-b^2]*(3*a^4 + a^2*b^2*(6 + 6*n - n^2) + b^4*(3 + 4*n + n^2)))*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c
+ d*x])^(1 + n))/(16*b*(a^2 + b^2)^2*(a + Sqrt[-b^2])*d*(1 + n)) + (Cos[c +
d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/(4*(a^2 + b^2)*d
) - (Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b*(a^2*(7 - n) + b^2*(5 +
n)) + a*(5*a^2 + b^2*(3 + 2*n))*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rubi [A] time = 0.803032, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3516, 1649, 831, 68}

$$\frac{\left(\sqrt{-b^2} \left(a^2 b^2 (-n^2 + 6n + 6) + 3a^4 + b^4 (n^2 + 4n + 3)\right) + ab^2 n (5a^2 + b^2 (2n + 3))\right) (a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n + 1; \dots\right)}{16bd(n+1)(a^2 + b^2)^2 \left(a - \sqrt{-b^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) + Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 + 6*n -
n^2) + b^4*(3 + 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan
[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(16*b*(a^2 + b^2
)^2*(a - Sqrt[-b^2])*d*(1 + n)) - ((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) - Sqrt[
-b^2]*(3*a^4 + a^2*b^2*(6 + 6*n - n^2) + b^4*(3 + 4*n + n^2)))*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c
+ d*x])^(1 + n))/(16*b*(a^2 + b^2)^2*(a + Sqrt[-b^2])*d*(1 + n)) + (Cos[c +
d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/(4*(a^2 + b^2)*d
) - (Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b*(a^2*(7 - n) + b^2*(5 +
n)) + a*(5*a^2 + b^2*(3 + 2*n))*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 1649

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
x], x, 1]}, -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^(p + 1)*(a*(e*f - d*g) +
(c*d*f + a*e*g)*x))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)
*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p +
```



```

1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3))
+ e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, c, d, e, m},
x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && !(IGtQ[m, 0]
&& RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

Rule 831

```

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && !RationalQ[m
]

```

Rule 68

```

Int[(((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)), x_Symbol] := Simp[(((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]

```

Rubi steps

$$\begin{aligned}
\int \sin^4(c + dx)(a + b \tan(c + dx))^n dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^4(a+x)^n}{(b^2+x^2)^3} dx, x, b \tan(c + dx)\right)}{d} \\
&= \frac{\cos^4(c + dx)(b + a \tan(c + dx))(a + b \tan(c + dx))^{1+n}}{4(a^2 + b^2)d} - \operatorname{Subst}\left(\int \frac{(a+x)^n(b^4(a^2+x^2))}{(b^2+x^2)^3} dx, x, b \tan(c + dx)\right) \\
&= \frac{\cos^4(c + dx)(b + a \tan(c + dx))(a + b \tan(c + dx))^{1+n}}{4(a^2 + b^2)d} - \frac{\cos^2(c + dx)(a + b \tan(c + dx))^{1+n}}{4(a^2 + b^2)d} \\
&= \frac{\cos^4(c + dx)(b + a \tan(c + dx))(a + b \tan(c + dx))^{1+n}}{4(a^2 + b^2)d} - \frac{\cos^2(c + dx)(a + b \tan(c + dx))^{1+n}}{4(a^2 + b^2)d} \\
&= \frac{\cos^4(c + dx)(b + a \tan(c + dx))(a + b \tan(c + dx))^{1+n}}{4(a^2 + b^2)d} - \frac{\cos^2(c + dx)(a + b \tan(c + dx))^{1+n}}{4(a^2 + b^2)d} \\
&= \frac{\cos^4(c + dx)(b + a \tan(c + dx))(a + b \tan(c + dx))^{1+n}}{4(a^2 + b^2)d} - \frac{\cos^2(c + dx)(a + b \tan(c + dx))^{1+n}}{4(a^2 + b^2)d} \\
&= \frac{(ab^2n(5a^2 + b^2(3 + 2n)) + \sqrt{-b^2}(3a^4 + a^2b^2(6 + 6n - n^2) + b^4(3 + 4n + 4n^2))}{16b(a^2 + b^2)^2(a - \sqrt{-b^2})}
\end{aligned}$$

Mathematica [B] time = 6.5794, size = 910, normalized size = 2.09

$$b \left(\frac{{}_2F_1\left(1, n+1; n+2; \frac{a+b \tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b \tan(c+dx))^{n+1}}{2\sqrt{-b^2}(a-\sqrt{-b^2})(n+1)} - \frac{{}_2F_1\left(1, n+1; n+2; \frac{a+b \tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b \tan(c+dx))^{n+1}}{2\sqrt{-b^2}(a+\sqrt{-b^2})(n+1)} + \frac{\cos^4(c+dx)(b^2+a \tan(c+dx)b)(a+b \tan(c+dx))^{1+n}}{4b^2(a^2+b^2)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]

[Out] (b*((Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])])*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])])*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*(1 + n)) - (Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c + d*x]))/(b^2*(a^2 + b^2)) + (Cos[c + d*x]^4*(a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c + d*x]))/(4*b^2*(a^2 + b^2)) + (((Sqrt[-b^2]*(a^2 + b^2*(1 - n)) - a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])])*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(a - Sqrt[-b^2])*(1 + n)) - ((a^2*Sqrt[-b^2] - (-b^2)^(3/2)*(1 - n) + a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])])*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(a + Sqrt[-b^2])*(1 + n)))/(2*(a^2 + b^2)) - (b^2*((Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b^2*(-3*a^2 - b^2*(3 - n)) + a^2*b^2*(2 - n) + b*(a*(-3*a^2 - b^2*(3 - n)) - a*b^2*(2 - n))*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)) - (((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n - Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])])*(a + b*Tan[c + d*x])^(1 + n))/(2*b^2*(a - Sqrt[-b^2])*(1 + n)) + ((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])])*(a + b*Tan[c + d*x])^(1 + n))/(2*b^2*(a + Sqrt[-b^2])*(1 + n)))/(2*b^2*(a^2 + b^2)))/(4*(a^2 + b^2)))/d

Maple [F] time = 0.556, size = 0, normalized size = 0.

$$\int (\sin(dx + c))^4 (a + b \tan(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x)

[Out] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \sin(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((\cos(dx + c)^4 - 2 \cos(dx + c)^2 + 1)(b \tan(dx + c) + a)^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")

[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*tan(d*x + c) + a)^n, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c))**n,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \sin(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")

[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^4, x)

3.85 $\int \sin^2(c + dx)(a + b \tan(c + dx))^n dx$

Optimal. Leaf size=276

$$\frac{\left(\sqrt{-b^2}(a^2 + b^2(n+1)) + ab^2n\right)(a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{a+b \tan(c+dx)}{a-\sqrt{-b^2}}\right)}{4bd(n+1)(a^2 + b^2)\left(a - \sqrt{-b^2}\right)} - \frac{\left(ab^2n - \sqrt{-b^2}(a^2 + b^2(n+1))\right)}{4}$$

[Out] $-\left((a*b^2*n + \text{Sqrt}[-b^2]*(a^2 + b^2*(1 + n)))\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (a + b*\text{Tan}[c + d*x])/(a - \text{Sqrt}[-b^2])]\right)*(a + b*\text{Tan}[c + d*x])^{(1 + n)}/(4*b*(a^2 + b^2)*(a - \text{Sqrt}[-b^2])*d*(1 + n)) - \left((a*b^2*n - \text{Sqrt}[-b^2]*(a^2 + b^2*(1 + n)))\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (a + b*\text{Tan}[c + d*x])/(a + \text{Sqrt}[-b^2])]\right)*(a + b*\text{Tan}[c + d*x])^{(1 + n)}/(4*b*(a^2 + b^2)*(a + \text{Sqrt}[-b^2])*d*(1 + n)) - (\text{Cos}[c + d*x]^2*(b + a*\text{Tan}[c + d*x]))*(a + b*\text{Tan}[c + d*x])^{(1 + n)}/(2*(a^2 + b^2)*d)$

Rubi [A] time = 0.367371, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3516, 1649, 831, 68}

$$\frac{\left(\sqrt{-b^2}(a^2 + b^2(n+1)) + ab^2n\right)(a + b \tan(c + dx))^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{a+b \tan(c+dx)}{a-\sqrt{-b^2}}\right)}{4bd(n+1)(a^2 + b^2)\left(a - \sqrt{-b^2}\right)} - \frac{\left(ab^2n - \sqrt{-b^2}(a^2 + b^2(n+1))\right)}{4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[c + d*x]^2*(a + b*\text{Tan}[c + d*x])^n, x]$

[Out] $-\left((a*b^2*n + \text{Sqrt}[-b^2]*(a^2 + b^2*(1 + n)))\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (a + b*\text{Tan}[c + d*x])/(a - \text{Sqrt}[-b^2])]\right)*(a + b*\text{Tan}[c + d*x])^{(1 + n)}/(4*b*(a^2 + b^2)*(a - \text{Sqrt}[-b^2])*d*(1 + n)) - \left((a*b^2*n - \text{Sqrt}[-b^2]*(a^2 + b^2*(1 + n)))\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (a + b*\text{Tan}[c + d*x])/(a + \text{Sqrt}[-b^2])]\right)*(a + b*\text{Tan}[c + d*x])^{(1 + n)}/(4*b*(a^2 + b^2)*(a + \text{Sqrt}[-b^2])*d*(1 + n)) - (\text{Cos}[c + d*x]^2*(b + a*\text{Tan}[c + d*x]))*(a + b*\text{Tan}[c + d*x])^{(1 + n)}/(2*(a^2 + b^2)*d)$

Rule 3516

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] :> \text{Dist}[b/f, \text{Subst}[\text{Int}[(x^m*(a + x)^n)/(b^2 + x^2)^{(m/2 + 1)}], x], x, b*\text{Tan}[e + f*x], x] /;$ FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 1649

$\text{Int}[(\text{Pq}_.)*((d_.) + (e_.)*(x_.))^{(m_.)}*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] :> \text{With}[\{Q = \text{PolynomialQuotient}[\text{Pq}, a + c*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[\text{Pq}, a + c*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[\text{Pq}, a + c*x^2, x], x, 1]\}, -\text{Simp}[\left((d + e*x)^{(m+1)}*(a + c*x^2)^{(p+1)}*(a*(e*f - d*g) + (c*d*f + a*e*g)*x)\right)/(2*a*(p+1)*(c*d^2 + a*e^2)), x] + \text{Dist}[1/(2*a*(p+1)*(c*d^2 + a*e^2)), \text{Int}[(d + e*x)^m*(a + c*x^2)^{(p+1)}*\text{ExpandToSum}[2*a*(p+1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p+3) - a*e*(d*g*m - e*f*(m+2*p+3)) + e*(c*d*f + a*e*g)*(m+2*p+4)*x, x], x] /;$ FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))]/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned} \int \sin^2(c + dx)(a + b \tan(c + dx))^n dx &= \frac{b \operatorname{Subst}\left(\int \frac{x^{2(a+x)^n}}{(b^2+x^2)^2} dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{\cos^2(c + dx)(b + a \tan(c + dx))(a + b \tan(c + dx))^{1+n}}{2(a^2 + b^2)d} - \frac{\operatorname{Subst}\left(\int \frac{(a+x)^n(-b)}{b^2+x^2} dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{\cos^2(c + dx)(b + a \tan(c + dx))(a + b \tan(c + dx))^{1+n}}{2(a^2 + b^2)d} - \frac{\operatorname{Subst}\left(\int \left(\frac{ab^4n - b^2}{b^2+x^2}\right) dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{\cos^2(c + dx)(b + a \tan(c + dx))(a + b \tan(c + dx))^{1+n}}{2(a^2 + b^2)d} - \frac{(ab^2n - \sqrt{-b^2}(a^2 + b^2))}{d} \\ &= -\frac{(ab^2n + \sqrt{-b^2}(a^2 + b^2(1 + n)))}{4b(a^2 + b^2)(a - \sqrt{-b^2})d(1 + n)} {}_2F_1\left(1, 1 + n; 2 + n; \frac{a + b \tan(c + dx)}{a - \sqrt{-b^2}}\right)(a + b \tan(c + dx))^{1+n} \end{aligned}$$

Mathematica [A] time = 1.08754, size = 270, normalized size = 0.98

$$\frac{(a + b \tan(c + dx))^{n+1} \left((a^2 b^2 (n-1) + a^3 \sqrt{-b^2} - a(-b^2)^{3/2} (2n+1) + b^4(-n+1) \right) {}_2F_1\left(1, n+1; n+2; \frac{a+b \tan(c+dx)}{a-\sqrt{-b^2}}\right)}{4b(a^2 + b^2)(a - \sqrt{-b^2})d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^n, x]
```

```
[Out] (((a^3*Sqrt[-b^2] + a^2*b^2*(-1 + n) - b^4*(1 + n) - a*(-b^2)^(3/2)*(1 + 2*
n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]
)] - (a^3*Sqrt[-b^2] - a^2*b^2*(-1 + n) + b^4*(1 + n) - a*(-b^2)^(3/2)*(1 +
2*n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b
^2])]) + 2*b*(a^2 + b^2)*(1 + n)*Cos[c + d*x]*(b*Cos[c + d*x] + a*Sin[c + d*
x]))*(a + b*Tan[c + d*x])^(1 + n))/(4*b*(a^2 + b^2)*(-a + Sqrt[-b^2])*(a +
Sqrt[-b^2])*d*(1 + n))
```

Maple [F] time = 0.473, size = 0, normalized size = 0.

$$\int (\sin(dx + c))^2 (a + b \tan(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x)`

[Out] `int(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \sin(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")`

[Out] `integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-(\cos(dx + c)^2 - 1)(b \tan(dx + c) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")`

[Out] `integral(-(cos(d*x + c)^2 - 1)*(b*tan(d*x + c) + a)^n, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**n,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \sin(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")`

[Out] `integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^2, x)`

3.86 $\int \csc^2(c + dx)(a + b \tan(c + dx))^n dx$

Optimal. Leaf size=48

$$\frac{b(a + b \tan(c + dx))^{n+1} {}_2F_1\left(2, n + 1; n + 2; \frac{b \tan(c + dx)}{a} + 1\right)}{a^2 d (n + 1)}$$

[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c + d*x])^(1 + n))/(a^2*d*(1 + n))

Rubi [A] time = 0.0548995, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3516, 65}

$$\frac{b(a + b \tan(c + dx))^{n+1} {}_2F_1\left(2, n + 1; n + 2; \frac{b \tan(c + dx)}{a} + 1\right)}{a^2 d (n + 1)}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]

[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c + d*x])^(1 + n))/(a^2*d*(1 + n))

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 65

Int[((b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c]/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])

Rubi steps

$$\begin{aligned} \int \csc^2(c + dx)(a + b \tan(c + dx))^n dx &= \frac{b \operatorname{Subst}\left(\int \frac{(a+x)^n}{x^2} dx, x, b \tan(c + dx)\right)}{d} \\ &= \frac{b {}_2F_1\left(2, 1 + n; 2 + n; 1 + \frac{b \tan(c + dx)}{a}\right)(a + b \tan(c + dx))^{1+n}}{a^2 d (1 + n)} \end{aligned}$$

Mathematica [A] time = 0.875393, size = 48, normalized size = 1.

$$\frac{b(a + b \tan(c + dx))^{n+1} {}_2F_1\left(2, n + 1; n + 2; \frac{b \tan(c + dx)}{a} + 1\right)}{a^2 d (n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]

[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c + d*x])^(1 + n))/(a^2*d*(1 + n))

Maple [F] time = 0.239, size = 0, normalized size = 0.

$$\int (\csc(dx + c))^2 (a + b \tan(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x)

[Out] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \csc(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \tan(dx + c) + a)^n \csc(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")

[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**n,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \csc(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.87 $\int \csc^4(c + dx)(a + b \tan(c + dx))^n dx$

Optimal. Leaf size=140

$$\frac{b(6a^2 + b^2(n^2 - 3n + 2))(a + b \tan(c + dx))^{n+1} {}_2F_1\left(2, n + 1; n + 2; \frac{b \tan(c + dx)}{a} + 1\right)}{6a^4 d(n + 1)} + \frac{b(2 - n) \cot^2(c + dx)(a + b \tan(c + dx))^n}{6a^2 d}$$

[Out] (b*(2 - n)*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n))/(6*a^2*d) - (Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(1 + n))/(3*a*d) + (b*(6*a^2 + b^2*(2 - 3*n + n^2))*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c + d*x])^(1 + n))/(6*a^4*d*(1 + n))

Rubi [A] time = 0.12994, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3516, 950, 78, 65}

$$\frac{b(6a^2 + b^2(n^2 - 3n + 2))(a + b \tan(c + dx))^{n+1} {}_2F_1\left(2, n + 1; n + 2; \frac{b \tan(c + dx)}{a} + 1\right)}{6a^4 d(n + 1)} + \frac{b(2 - n) \cot^2(c + dx)(a + b \tan(c + dx))^n}{6a^2 d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]

[Out] (b*(2 - n)*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n))/(6*a^2*d) - (Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(1 + n))/(3*a*d) + (b*(6*a^2 + b^2*(2 - 3*n + n^2))*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c + d*x])^(1 + n))/(6*a^4*d*(1 + n))

Rule 3516

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]

Rule 950

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c]/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned} \int \csc^4(c + dx)(a + b \tan(c + dx))^n dx &= \frac{b \operatorname{Subst}\left(\int \frac{(a+x)^n(b^2+x^2)}{x^4} dx, x, b \tan(c + dx)\right)}{d} \\ &= -\frac{\cot^3(c + dx)(a + b \tan(c + dx))^{1+n}}{3ad} - \frac{b \operatorname{Subst}\left(\int \frac{(a+x)^n(b^2(2-n)-3ax)}{x^3} dx, x, b \tan(c + dx)\right)}{3ad} \\ &= \frac{b(2-n) \cot^2(c + dx)(a + b \tan(c + dx))^{1+n}}{6a^2d} - \frac{\cot^3(c + dx)(a + b \tan(c + dx))^{1+n}}{3ad} \\ &= \frac{b(2-n) \cot^2(c + dx)(a + b \tan(c + dx))^{1+n}}{6a^2d} - \frac{\cot^3(c + dx)(a + b \tan(c + dx))^{1+n}}{3ad} \end{aligned}$$

Mathematica [A] time = 1.24389, size = 78, normalized size = 0.56

$$\frac{b(a + b \tan(c + dx))^{n+1} \left(a^2 {}_2F_1\left(2, n + 1; n + 2; \frac{b \tan(c + dx)}{a} + 1\right) + b^2 {}_2F_1\left(4, n + 1; n + 2; \frac{b \tan(c + dx)}{a} + 1\right) \right)}{a^4 d(n + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*(a^2*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a] + b^2*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a])*(a + b*Tan[c + d*x])^(1 + n))/(a^4*d*(1 + n))
```

Maple [F] time = 0.229, size = 0, normalized size = 0.

$$\int (\csc(dx + c))^4 (a + b \tan(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \csc(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \tan(dx + c) + a\right)^n \csc(dx + c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")

[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**n,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \csc(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")

[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)

$$3.88 \quad \int \sin^3(c + dx)(a + b \tan(c + dx))^n dx$$

Optimal. Leaf size=23

$$\text{CannotIntegrate}(\sin^3(c + dx)(a + b \tan(c + dx))^n, x)$$

[Out] CannotIntegrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]

Rubi [A] time = 1.97873, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \sin^3(c + dx)(a + b \tan(c + dx))^n dx$$

Verification is Not applicable to the result.

[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]

[Out] Defer[Int][Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]

Rubi steps

$$\int \sin^3(c + dx)(a + b \tan(c + dx))^n dx = \int \sin^3(c + dx)(a + b \tan(c + dx))^n dx$$

Mathematica [A] time = 3.06851, size = 0, normalized size = 0.

$$\int \sin^3(c + dx)(a + b \tan(c + dx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]

[Out] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]

Maple [A] time = 0.542, size = 0, normalized size = 0.

$$\int (\sin(dx + c))^3 (a + b \tan(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^n, x)

[Out] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^n, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \sin(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos(dx+c)^2-1\right)(b\tan(dx+c)+a)^n\sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")

[Out] integral(-(cos(d*x + c)^2 - 1)*(b*tan(d*x + c) + a)^n*sin(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**n,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx+c) + a)^n \sin(dx+c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")

[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^3, x)

$$3.89 \quad \int \sin(c + dx)(a + b \tan(c + dx))^n dx$$

Optimal. Leaf size=21

CannotIntegrate(sin(c + dx)(a + b tan(c + dx))^n, x)

[Out] CannotIntegrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]

Rubi [A] time = 0.855052, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \sin(c + dx)(a + b \tan(c + dx))^n dx$$

Verification is Not applicable to the result.

[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]

[Out] Defer[Int][Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]

Rubi steps

$$\int \sin(c + dx)(a + b \tan(c + dx))^n dx = \int \sin(c + dx)(a + b \tan(c + dx))^n dx$$

Mathematica [A] time = 2.18546, size = 0, normalized size = 0.

$$\int \sin(c + dx)(a + b \tan(c + dx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]

[Out] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]

Maple [A] time = 0.041, size = 0, normalized size = 0.

$$\int \sin(dx + c)(a + b \tan(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^n, x)

[Out] int(sin(d*x+c)*(a+b*tan(d*x+c))^n, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \sin(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((b \tan(dx + c) + a)^n \sin(dx + c), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")

[Out] integral((b*tan(d*x + c) + a)^n*sin(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b \tan(c + dx))^n \sin(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**n,x)

[Out] Integral((a + b*tan(c + d*x))**n*sin(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \sin(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")

[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c), x)

3.90 $\int \csc(c + dx)(a + b \tan(c + dx))^n dx$

Optimal. Leaf size=21

CannotIntegrate(csc(c + dx)(a + b tan(c + dx))^n, x)

[Out] CannotIntegrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]

Rubi [A] time = 0.486547, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \csc(c + dx)(a + b \tan(c + dx))^n dx$$

Verification is Not applicable to the result.

[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]

[Out] Defer[Int][Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]

Rubi steps

$$\int \csc(c + dx)(a + b \tan(c + dx))^n dx = \int \csc(c + dx)(a + b \tan(c + dx))^n dx$$

Mathematica [A] time = 1.52046, size = 0, normalized size = 0.

$$\int \csc(c + dx)(a + b \tan(c + dx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]

[Out] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]

Maple [A] time = 0.23, size = 0, normalized size = 0.

$$\int \csc(dx + c)(a + b \tan(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^n, x)

[Out] int(csc(d*x+c)*(a+b*tan(d*x+c))^n, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \csc(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((b \tan(dx + c) + a)^n \text{csc}(dx + c), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")

[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**n,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \text{csc}(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")

[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c), x)

3.91 $\int \csc^3(c + dx)(a + b \tan(c + dx))^n dx$

Optimal. Leaf size=23

$$\text{CannotIntegrate}(\csc^3(c + dx)(a + b \tan(c + dx))^n, x)$$

[Out] CannotIntegrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]

Rubi [A] time = 1.69981, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \csc^3(c + dx)(a + b \tan(c + dx))^n dx$$

Verification is Not applicable to the result.

[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]

[Out] Defer[Int][Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]

Rubi steps

$$\int \csc^3(c + dx)(a + b \tan(c + dx))^n dx = \int \csc^3(c + dx)(a + b \tan(c + dx))^n dx$$

Mathematica [A] time = 2.85234, size = 0, normalized size = 0.

$$\int \csc^3(c + dx)(a + b \tan(c + dx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]

[Out] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]

Maple [A] time = 0.189, size = 0, normalized size = 0.

$$\int (\csc(dx + c))^3 (a + b \tan(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^n, x)

[Out] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^n, x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \csc(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \tan(dx + c) + a\right)^n \csc(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")

[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**n,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \tan(dx + c) + a)^n \csc(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")

[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```

```

38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46   If[AtomQ[expn],
47     1,
48     If[ListQ[expn],
49       Max[Map[ExpnType,expn]],
50       If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52           ExpnType[expn[[1]]],
53           If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55               1,
56               Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58             If[Head[expn]===Plus || Head[expn]===Times,
59               Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60             If[ElementaryFunctionQ[Head[expn]],
61               Max[3,ExpnType[expn[[1]]],
62             If[SpecialFunctionQ[Head[expn]],
63               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64             If[HypergeometricFunctionQ[Head[expn]],
65               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66             If[AppellFunctionQ[Head[expn]],
67               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68             If[Head[expn]===RootSum,
69               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
70             If[Head[expn]===Integrate || Head[expn]===Int,
71               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
72             9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp,Log,
78     Sin,Cos,Tan,Cot,Sec,Csc,
79     ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
80     Sinh,Cosh,Tanh,Coth,Sech,Csch,
81     ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
82   },func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   },func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
99
100

```

```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```



```

119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'`^`') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'`+`') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150     max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152     max(8,apply(max,map(ExpnType,[op(expn)]))) else
153     9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159   member(func,[
160     exp,log,ln,
161     sin,cos,tan,cot,sec,csc,
162     arcsin,arccos,arctan,arccot,arcsec,arccsc,
163     sinh,cosh,tanh,coth,sech,csch,
164     arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168   member(func,[
169     erf,erfc,erfi,
170     FresnelS,FresnelC,
171     Ei,Ei,Li,Si,Ci,Shi,Chi,
172     GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173     EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181   member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #         Port of original Maple grading function by
3 #         Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #         added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```

```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124     else:
125         return "C"

```

4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))]
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34             #since this estimate of leaf count is bit lower than

```

```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```